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A theory of capital gains taxation and business turnover

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Abstract We present a theory concerning the realization of capital gains where ownership and control are linked as in Holmes and Schmitz (1995). The model developed is a version of a Lucas-tree economy in which the productivity of a technology depends on the ownership of the technology. The existence and uniqueness of equilibrium follow from the Contraction Mapping Theorem. The theory implies that impediments to asset trading, such as capital gains taxation, negatively affect production efficiency. Moreover, we calibrate the model economy to U.S. data on small-business turnover and find that indexing deductions for inflation is capable of increasing capital-gains tax revenues.

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1 Introduction

In this paper we present a theory concerning the realization of capital gains where ownership and control are linked as in Holmes and Schmitz (1995). The model is a version of a Lucas-tree economy in which the productivity of a technology depends on the ownership of the technology. By changing the ownership of productive assets, trade becomes a potentially important mechanism for improving productivity. The theory thus implies that impediments to trading assets, such as capital gains taxation, negatively affect production efficiency. We use the theory to investigate the impact of taxing capital gains on the allocation of productive assets and the efficiency of production.

There has been much concern in the public debate that the capital gains tax generates a "lock-in effect" that interferes with the efficient turnover of financial and productive assets. Because the tax is levied when an asset is sold at a gain, individuals are discouraged from trading assets in order to avoid paying taxes on them. Taxation then effectively locks individuals to their assets. As a result, the distortions produced by the capital gains tax go beyond the reduction in the savings rates typically studied in the literature on the taxation of capital income.

There is a lack of coherent models that can be used to evaluate how trade distortions affect the productivity of assets in a general equilibrium context. This might be due to technical difficulties in formulating general equilibrium models that address ownership and control. Previous studies of capital gains taxation have focused on how taxation affects risk sharing and savings decisions (see our literature review in Section 6). In this paper, we abstract from risk sharing and savings decisions since we model individuals with a linear utility function on consumption. Instead, we focus on how the distortions to trade caused by capital gains taxation affect the ownership and productivity of assets.

The main mechanism driving trade in the model is related to non-market goods in the spirit of Holmes and Schmitz (1995). The idea is that every business can be viewed as an asset possessing some qualities that can be transferred to new owners, but also featuring other qualities that are idiosyncratic to the current ownership and cannot be transferred in the market for businesses. When businesses are sold, only their market qualities are transferred, while their non-market qualities are drawn by the new owners from a fixed distribution. From society's perspective, trade is an important mechanism for renewing non-market qualities, but it is not the only one. Another such mechanism in the model is the creation of new businesses which can be pursued at a cost. We let the extent to which these two mechanisms are used to be determined by general-equilibrium conditions. We believe that these margins are important for the measurement of the effects of capital gains taxation on entrepreneurship and therefore should be endogenous in the model.

We build on Lucas (1978) by introducing to his framework shocks that are idiosyncratic to the owner. We thus study a dynamic economy with heterogeneous agents in which the distribution of shocks across owners is part of the state of the economy. We also introduce inflation into Lucas' model. Inflation is assumed to erode tax deductions in the computation of capital gains by the tax authority. Thus, as in the U.S. tax code, taxation applies to nominal gains. As a result, tax deductions imply an additional source of heterogeneity since they depend on ownership tenure. Although inflation causes a reduction of tax liabilities, its effect on tax revenues is non-trivial, as the pattern of transactions is endogenous in our general equilibrium framework.

We show that one of the virtues of the model is its tractability. A competitive equilibrium is shown to be a fixed point of a mapping that displays the contraction property. As a result, the existence and uniqueness of equilibrium are direct applications of the contraction mapping theorem, which enables the computation of competitive equilibrium by simple value function iterations. Moreover, the computation of transitional dynamics between steady states characterized by alternative tax policies is also straightforward. From the theoretical and computational points of view we conclude that the model is a powerful tool for studying the effects of capital gains taxation on the productivity of the unincorporated sector.

Despite its simplicity, the model can be parameterized to mimic U.S. data on the turnover of small businesses. Simulations of the calibrated model economy indicate that indexing gains for inflation will quite likely increase, not decrease, tax revenues.

The paper is divided as follows. Section 2 lays out the basic environment. Section 3 presents a version of the model without taxation. Section 4 introduces the taxation of capital gains. Section 5 presents the numerical experiments. Section 6 positions our paper in the public finance literature and concludes. The appendix contains an overview of the computation method.

2 The Environment

Time is discrete and the horizon is infinite. There is one perishable consumption good per date and a $[0, 1]$ continuum of infinitely-lived individuals with linear preferences for consumption. The discount factor is $\beta \in (0, 1)$. There is a $[0, 1]$ continuum of technologies for the production of the consumption good, and each person is endowed with one technology with access to spot markets at all dates for trading technologies. We also refer to a technology as a businesses that is productive only if it is owned and operated by a single individual. This feature corresponds to the assumption that ownership is an input into the production function. In what follows we anticipate that in equilibrium each technology is owned and controlled by a single individual.

Each technology is indexed by a pair of productivity shocks (z, s) with support in the finite set $Z \times S \subset \mathbb{R}^2$. The shock z represents a transferable quality that survives changes of ownership of the technology. The shock

s represents a non-transferable quality that is idiosyncratic to the current owner of the technology and, hence, it does not survive changes in ownership. Since s is related to the match between a business and its owner, while z is independent of ownership, we also refer to s as an *ownership quality* and to z as a *market quality*. The output $y_t(z,s)$ of the business (z,s) at date t, in units of the contemporaneous consumption good, is related to its qualities according to

$$y_t(z,s) = A_t\ f(z,s),$$

where f is a positive, strictly increasing and bounded function, and A_t is a productivity index that grows exogenously at a rate g per period.

The physical environment so far described is essentially a Lucas-tree economy with linear preferences, but appended with productivity shocks to ownership. We assume that at each period there are three stages governing the law of motion for the productivity shocks. First, each owner learns his or her initial business qualities (z,s). At a second stage, business qualities can be modified as technologies can be traded (change ownership). At the end of the period there is a production stage when the output of each technology is realized.

An owner who does not sell his technology has two options in the second stage: He can keep the technology as it is and enjoy output $y_t(z,s)$, or he can discontinue the current features of the technology and make a new draw of business qualities. The option to discontinue a business entails the exit of an old business and the creation of a new one. We assume that the new business draw is made from a fixed distribution F on $Z \times S$, and that there is a cost of creating a new business expressed in terms of foregone output of the consumption good. If the new business at the second stage is (z_0,s_0), output at the third stage is $(1-\lambda)y_t(z_0,s_0)$, where $\lambda \in (0,1)$ is a parameter specifying the cost of creating the new business as a fraction of the level of output.

An owner who sells her technology and buys a different technology with market quality z_1 has to make a draw for the ownership quality from the distribution G on S. If the draw is s_1 then output at the production stage is $y_t(z_1,s_1)$. After production takes place, the next period's market and ownership qualities evolve stochastically according to a first-order Markov process described by a joint transition function Q.

We also assume the existence of a government taxing business transfers at the flat rate τ according to the realization of nominal capital gains. Tax revenues finance government expenditures that enter the utility function in an additively-separable fashion. For tax purposes, a capital gain is defined as the sale price of a technology minus the value of deductions. We assume that the only deduction accepted by the government is, in nominal terms, the previous price paid by the current seller when the business was purchased, or the book value of the business when the business was created by the current seller. We assume that the real value of deductions falls from one period to the next at the constant rate π which we interpret as the inflation rate. The specifics of the tax scheme are laid out in section 4.

3 Asset pricing with no taxation

Here we assume that $\tau = 0$ so that there is no taxation. Let the expected discounted utility of the owner of a business (z, s) at the beginning of period t from behaving optimally be denoted as $v_t(z, s)$. Then v_t satisfies

$$v_t(z,s) = \max\{k_t(z,s),\ p_t(z),\ e_t\}, \tag{1}$$

where k_t, p_t and e_t denote, respectively, the period t expected discounted value of the technology when the action on the second stage is to keep, sell/buy and exit/create. Note that we are implicitly anticipating the outcome that the value of selling does not depend on the non-transferable quality s and that the value of exiting does not depend on any initial quality. The value of keeping the business at date t satisfies

$$k_t(z,s) = y_t(z,s) + \beta \int v_{t+1}(z', s')Q(z, s, dz', ds'), \tag{2}$$

where the integral represents the expectation taken over the next period business qualities (z', s') according to the function Q.

The value of exiting and creating a new business from the existing technology satisfies

$$e_t = \int [k_t(z,s) - \lambda y_t(z,s)]F(dz, ds), \tag{3}$$

where the integral represents the expectation taken over the new draws for both the market quality and the ownership quality.

Since preferences are linear, the value of selling the technology, $p_t(z)$, also denotes the price, in units of the date t consumption good, of a technology with market quality z. Notice that in this environment the value of a technology for its owner, $v_t(z, s)$, is always greater or equal to its market price, $p_t(z)$.[1] In order to prevent the existence of arbitrage opportunities, the price function p_t and the value of keeping businesses at t, k_t, shall satisfy the no-arbitrage condition

$$p_t(z) = \int k_t(z,s)G(ds), \tag{4}$$

for all z, where the integral represents the expectation taken over new draws for the ownership quality.

Another consequence of linear preferences for this economy is that the marginal rates of intertemporal substitution in consumption are constant along with the interest rate. As a result, asset prices do not depend on the law of motion of the distribution of business qualities across owners and thus

[1] The market price does not take into account the quality of the current ownership, s, since this quality is not transferable in the market for businesses. As a result, $v_t \geq p_t$, so that the price of a business can be less than the value of the business for its owner.

there is no need to index the functions in (1)-(4) by any economy-wide state variable. Another consequence of asset prices being invariant to this law of motion is that value functions grow over time at the rate of productivity growth in the economy (g). Defining $\widehat{\beta} = \beta(1+g)$ and substituting $\widehat{\beta}$ for β in equation (2) we can drop the time indexes in equations (1)-(4).

The state of the economy with no taxation is described by a measure x_t of businesses across qualities. We represent the optimal decision at t by the policy functions g_t^k for keep, g_t^s for sell/buy and g_t^e for exit/create. For each $i \in \{k, s, e\}$, g_t^i is an indicator function that takes the value 1 when the action i is optimal. The law of motion for x_t is given by

$$\begin{aligned} x_{t+1}(z', s') = &\int g_t^k(z, s) Q(z, s, z', s')\ x_t(dz, ds) + \\ &\int g_t^s(z_1, s) Q(z_1, s_{1,} z', s') G(ds_1)\, x_t(dz_1, ds)\ + \\ &\int g_t^e(z, s) Q(z_0, s_0, z', s') F(dz_0, ds_0) x_t(dz, ds)\,. \end{aligned} \tag{5}$$

In the absence of arbitrage opportunities, each buyer will be indifferent regarding distinct market qualities. Hence we may assume, without loss of generality, that each trader selling a business with market quality z moves on to buy a business with the same market quality z. This property reflects the fact that interactions of individuals in a competitive equilibrium for this economy are inessential. The feasibility condition for trade in this economy can thus be omitted with the understanding that each trader buys the same quality that he or she sells. We proceed with the following definition of a recursive competitive equilibrium.

Definition. *A competitive equilibrium for the economy without taxation and with an initial distribution of business x_0 is a value function v and a price function p such that*

(i) There exists functions k, g^k, g^s, g^e and a constant e such that equations (1)-(5) are satisfied for all business qualities;

(ii) g^k, g^s and g^e above are the optimal policy functions for v and there exists a sequence of distributions $\{x_t\}_{t=1}^{\infty}$ implied by x_0 and (g^k, g^s, g^e) in the obvious way.

We now prove the existence and uniqueness of a competitive equilibrium.

Proposition 1 *There exists a unique competitive equilibrium for the economy with no taxation.*

Proof The proof follows from showing that a competitive equilibrium is analogous to a fixed point of an operator which satisfies the assumptions of the contraction mapping theorem. We use (1)-(4) to define a mapping on the complete metric space of non-negative functions $\mathcal{B} = \{v : Z \times S \to \mathbb{R}_+\}$ that are bounded in the sup norm. Define the operator $T : \mathcal{B} \to \mathcal{B}$ by the recursion

$$[Tv](z, s) = \max\{[T^k v](z, s), [T^s v](z), T^e v\}, \tag{6}$$

$$[T^k v](z,s) = y(z,s) + \beta \int v(z',s')Q(z,s,dz',ds'), \tag{7}$$

$$[T^s v](z) = \int \{y(z,s) + \beta \int v(z',s')Q(z,s,dz',ds')\}G(ds) \tag{8}$$

and

$$T^e v = \int \{y(z,s) - \lambda y_0 + \beta \int v(z',s')Q(z,s,dz',ds')\}F(dz,ds). \tag{9}$$

Every (v,p) that is a competitive equilibrium must satisfy (1)-(4) with $v_t = v_{t+1} = v$ and $p_t = p$, for some constant sequence $\{k_t, e_t\}$. Given (v,p), the equilibrium sequence of distributions $\{x_t\}$ is trivially determined by equation (5) and the initial distribution x_0. Since the right hand sides of (2)-(4) agree with $T^k v$, $T^s v$ and $T^e v$ then (v,p) is a competitive equilibrium if and only if $v = Tv$ and $p = T^s v$. It is clear that T maps $\mathcal{B}$ into itself. We now show that T is a contraction. Suppose that $v_1,\ v_2 \in \mathcal{B}$ with $v_1 \leq v_2$. Since Q is a transition function and (F,G) are probability distribution functions then $T^i v_1 \leq T^i v_2$ for $i \in \{k,s,e\}$. Hence $Tv_1 \leq Tv_2$ and thus T is monotone. If $v \in \mathcal{B}$ and a is a positive and constant function, then properties of (Q,F,G) imply $T^i(v+a) \leq T^i v + \beta a$ for $i \in \{k,s,e\}$. Hence, $T(v+a) \leq Tv + \beta a$ so that T satisfies discounting. We conclude that T satisfies the Blackwell's sufficient conditions for a contraction with modulus β. Then, by the contraction mapping theorem, T has a unique fixed point v^* in $\mathcal{B}$. Setting $p^* = T^s v^*$, (v^*, p^*) is the unique competitive equilibrium for the economy with no taxes. The proof is now complete.

In order to prove the existence of a unique steady state we impose the following structure on transition function Q.

Assumption 1. Q is strictly positive.

This assumption suffices to guarantee that the only ergodic set for the process governing business qualities on $Z \times S$ is $Z \times S$ itself.[2] We now prove the convergence of the proposition 1 equilibrium to a unique steady state.

Proposition 2 *Let $\{x_t\}_{t=0}^{\infty}$ denote a sequence of business distributions attaining the proposition 1 equilibrium for the economy with no taxation. Then $\{x_t\}_{t=0}^{\infty}$ converges to a unique invariant distribution x^*.*

Proof See theorem 11.4 (page 332) of Stokey *et al.* (1989).

As noted before, a competitive equilibrium for this economy resembles individuals closing their businesses in order to start new ones, or selling their businesses in order buy other ones with the same market qualities. Although the state of the economy includes a distribution of businesses across heterogenous attributes, interaction among individuals is inessential, and

[2] The same result would follow if we assume instead that for some integer $n \geq 1$, every element of the n-product matrix Q^n is positive.

the aggregate thus resembles a collection of fishermen living in a continuum of Robinson Crusoe islands. It should therefore not be surprising that the contraction mapping theorem can be used to establish the existence of equilibrium as it is standard in models with a representative agent and no distortions. We shall see that this property is preserved when we introduce taxation of capital gains.

4 Asset pricing with taxation

The model discussed above provides a natural setup for the study of capital gains taxation when ownership matters for the productivity of a technology. Scholars in public finance have conjectured that such taxation has welfare costs associated with discouraging trade and the consequent reduction of average business productivity (see Atkison and Stiglitz (1980) and Stiglitz (1983)). Moreover, such welfare costs are believed to be aggravated by the presence of inflation under a lack of indexation of tax deductions. In this section, we lay out the foundations for the study of the welfare effects of taxing nominal capital gains in the environment presented above.

We assume that the capital gains realized by sellers of businesses are taxed at a flat rate $\tau \in (0,1)$. Since the quality of a particular business evolves over time, its price also evolves over time, which gives scope for the existence of real capital gains. By assuming that deductions are not indexed for the constant rate of inflation π, nominal gains can also build up regardless the existence of real gains. As nominal gains accumulate, so do potential tax liabilities, which in turn tends to discourage trade.

Capital gains are taxed upon realization as in the U.S. economy. We thus assume that capital gains are paid when a business is sold. When a business is transferred, the seller pays a tax based on the difference between the price at which the business is sold and the value of deductions in terms of the contemporaneous consumption good. We shall index each business (z,s) by a pair (w,n), where $w \in W \equiv Z$ pins down its initial book value, and $n \in \mathbb{N}$ specifies the tenure of the business with the current owner. If the business (z,s,w,n) is sold when the price function is p then the tax liability is

$$\tau[p_t(z) - \frac{p_{t-n}(w)}{(1+\pi)^n}], \text{ if } w \in Z \tag{10}$$

In the case where the business was not created by the current owner, w represents its market quality when acquired n periods ago. Hence, $p_t(w)$ is the price paid and $p_t(z) - p_{t-n}(w)$ is the real capital gain for tax purposes. However, in the presence of a constant rate of increase in nominal prices, π, the current nominal sale price is $(1+\pi)^n p_t(z)$, and the nominal gain is $(1+\pi)^n p_t(z) - p_{t-n}(w)$, if, as assumed, deductions are not adjusted for inflation. As a result, expressed in terms of the contemporaneous consumption good, the tax base becomes $p_t(z) - p_{t-n}(w)/(1+\pi)^n$, which implies the tax liability (10). In the case where the current owner did create the business n periods ago, we assume that the tax authority considers as a valid (nominal) deduction an amount equal to $p_{t-n}(w_0)$, where w_0 represents an arbitrary business quality.

Notice that even when the market quality remains constant ($z = w$) and when there is no productivity growth ($g = 0$), the higher the tenure n the higher the tax liability for $\pi > 0$. The taxation of nominal gains thus have the potential for sizable distortions even when real gains accrue moderately.

Let the expected discounted utility of the owner of a business (z, s, w, n), at the beginning of period t be denoted by $v_t(z, s, w, n)$. Now v_t satisfies

$$v_t(z, s, w, n) = \max\{k_t(z, s, w, n),\ q_t(z, w, n),\ e_t\}, \tag{11}$$

where k_t and e_t denote, respectively, the expected discounted value of the technology when the action in period t are to keep and exit/create. Now q_t denotes the after-tax value of the business in units of the date t consumption good. The value of keeping the business at date t satisfies

$$k_t(z, s, w, n) = y(z, s) + \beta \int v_{t+1}(z', s', w, n + 1)Q(z, s, dz', ds'). \tag{12}$$

When a business is kept, the third-state variable remains fixed at the value w and the tenure of the owner increases by one unity.

The value of exiting and creating a new business from the existing technology satisfies

$$e_t = \int [k_t(z, s, w_0, 0) - \lambda y_t(z, s)]F(dz, ds), \tag{13}$$

where the integral represents the expectation taken over the new draws for both the market quality and the ownership quality. Notice that exiting and creating a new business becomes a device for avoiding the realization of capital gains at the cost of forfeiting potential good qualities (z, s) and incurring the creation cost λy_t. When a business is sold by its creator, tax liabilities, if any, are computed according to $w = w_0$. The tenure of the owner with the new business is initialized with $n = 0$.

The after-tax value of a business (z, s, w, n) is

$$q_t(z, w, n) = \min\{p_t(z), p_t(z) - \ \tau(p_t(z) - \frac{p_{t-n}(w)}{(1+\pi)^n})\}, \text{ if } w \in Z; \tag{14}$$

According to equation (14), the after-tax value of a business is its sale price minus its tax liability when the tax liability is positive. We assume that the tax authority does not allow a negative tax payment since $q_t(z, w, n) \leq p_t(z)$ for all (z, w, n). This assumption is motivated by the limited *loss offset* specified by the U.S. tax code. For simplicity, the maximum capital-loss allowance is set to zero in the model economy. In the U.S., however, it has been historically a positive value, albeit small.[3]

As before, in order to prevent the existence of arbitrage opportunities, the price function p_t and the value of keeping businesses at t, k_t, shall satisfy

$$p_t(z) = \int k_t(z, s, z, 0) G(ds), \tag{15}$$

for all z, where the integral represents the expectation taken over the new draw for the ownership quality. Notice that $w = z$ and $n = 0$ are the tax parameters for the buyer of a business with market quality z.

The asset price function (15) is independent of the distribution of business across productivities and tax attributes. Although the distribution of businesses at a point in time is part of the state of the economy, asset prices and policy functions are independent from this distribution. As a result, time subscripts for the value functions can be dropped, provided we normalize the discount factor with the economy's productivity growth rate. The counterpart here to the law of motion (5) is a straightforward but uninformative extension of the no-taxation case. We avail ourselves of the spelling out of this law of motion and proceed to define a competitive equilibrium with taxation of capital gains.

Definition. *A competitive equilibrium for the economy with capital gains taxation and with an initial distribution of business x_0 is a value function v and a price function p such that*

(i) There exists functions k, q, g^k, g^s, g^e and a constant e such that equations (11)-(15) are satisfied for all business qualities;

(ii) g^k, g^s and g^e above are the optimal policy functions for v and there exists a sequence of distributions $\{x_t\}_{t=1}^{\infty}$ implied by x_0 and (g^k, g^s, g^e) in the obvious way.

We now show that the contraction mapping theorem can still be applied to demonstrate the existence and uniqueness of a competitive equilibrium.

Proposition 3 *There exists a unique competitive equilibrium for the economy with capital gains taxation.*

Proof As in the proof idea for proposition 1, define the operator $T : \mathcal{B} \to \mathcal{B}$ by the recursion

$$[Tv](z, s, w, n) = \max\{[T^k v](z, s, w, n), [T^s v](z, w, n), T^e v\}, \tag{16}$$

$$[T^k v](z, s, w, n) = y(z, s) + \beta \int v(z', s', w, n+1) Q(z, s, dz', ds'), \tag{17}$$

$$p(z) = \int \{y(z, s) + \beta \int v(z', s', z, 1) Q(z, s, dz', ds')\} G(ds), \tag{18}$$

[3] In the U.S. tax code, a capital loss that exceeds the cap can be *carried over* to the next period's tax computation. We do not consider this kind of deduction.

$$[T^s v](z,w,n) = \min\{p(z), (1-\tau)p(z) + \frac{\tau}{[(1+\lambda)(1+\pi)]^n} p(w)\} \quad (19)$$

and

$$T^e v = \int \{y(z,s) - \lambda y_0 + \beta \int v(z',s',w_0,1) Q(z,s,dz',ds')\} F(dz,ds). \quad (20)$$

Since the right hand sides of (12), (13), and (14) agree, respectively, with $T^k v$, $T^e v$ and $T^s v$, then (v,p) is a competitive equilibrium if and only if $v = Tv$ and (v,p) satisfies (18). It suffices to show that T^s preserves the monotonicity and discounting properties specified by Blackwell's sufficient conditions, as in the proof of Proposition 1. Suppose that $v_1, v_2 \in \mathcal{B}$ with $v_1 \le v_2$. Clearly (18) associates to (v_1, v_2) a pair (p_1, p_2) with $p_1 \le p_2$. From (19), $\tau \in (0,1)$ implies $T^s v_1 \le T^s v_2$ so that monotonicity is preserved. If $v \in \mathcal{B}$ and $a \ge 0$ then the prices associated with $(v, v+a)$ by (18) are (p_v, p_{v+a}) satisfying $p_{v+a} = p_v + \beta a$. As a result, equation (19) implies

$$T^s(v+a) \le T^s v + \max\{1, 1-\tau + \frac{\tau}{(1+\pi)^n}\}\beta a. \quad (21)$$

Since $\tau \in (0,1)$ and $\pi \ge 0$, (21) implies $T^s(v+a) \le T^s v + \beta a$ so that discounting is preserved. Thus, the contraction mapping theorem applies and T has a unique fixed point v^* in $\mathcal{B}$. Therefore, there exists a unique competitive equilibrium for the economy with taxation of capital gains where the value function is v^* and the asset price function is p^*, as given by (18) with $v = v^*$. The proof is now complete.

We now prove the convergence of equilibrium to a steady state. We assume that the transition function Q governing productivity shocks satisfies a form of stochastic dominance. Let $\{z_i, s_i\}$ be an enumeration of $Z \times S$, ordered by output levels according to $i \ge j \Leftrightarrow f(z_i, s_i) \ge f(z_j, s_j)$. Let $h_i \equiv \sum_j Q(z_i, s_i, z_j, s_j) f(z_j, s_j)$ denote the expected next-period output, conditional on the business being kept next period, given that the state at the end of the current period is (z_i, s_i). We make the following assumption on Q.

Assumption 2. The expected output h_i is strictly increasing in i.

Under assumption 2, there is a realization of the productivity shocks for which the optimal decision in equilibrium is always to keep the business. This fact, stated as a lemma below, is used to show the existence of a unique ergodic set for the state space $Z \times S \times W \times \mathbb{N}$.

Lemma 1 *There exists a maximal pair $(\bar{z}, \bar{s})$ in $\{z_i, s_i\}$ so that for any $(w,n) \in W \times \mathbb{N}$ it is optimal to keep the $(\bar{z}, \bar{s}, w, n)$ business in the proposition-3 equilibrium.*

Proof If the productivity pair at the beginning of the current state is $(\bar{z}, \bar{s})$, then any decision other than keep will produce a lower expected output in the current period and, according to assumption 2, in the next period as well. Since an owner who keeps a business pays no capital-gains taxes, then the decision to keep $(\bar{z}, \bar{s}, w, n)$ maximizes future output at the same time that it avoids taxation.

We now prove that equilibrium converges to a (unique) steady state.

Proposition 4 *Let $\{x_t\}_{t=0}^{\infty}$ denote a sequence of business distributions attaining the proposition-3 equilibrium for the economy with taxes. Then $\{x_t\}_{t=0}^{\infty}$ converges to a unique invariant distribution x^*.*

Proof We first choose a suitable truncation of the state space. Notice first that Z, S and W are finite and that the potential tax liability of a business (z, s, w, n) converges to a finite limit as $n \to \infty$ (the value of deductions goes to zero). Since tax deductions approach zero for n large and decisions are discrete, there exists $\bar{n}$ such that the optimum policy for (z, s, w, n) is the same as the optimum policy for $(z, s, w, \bar{n})$ for $n \geq \bar{n}$ and all $w \in W$. Hence we can restrict attention to the law of motion of the business distribution on the finite set $Z \times S \times W \times \{0, 1, ..., \bar{n}\}$ with the understanding that the state $\bar{n}$ represents tenure ages greater or equal to $\bar{n}$ periods. The policy functions for the proposition-3 equilibrium, together with (Q, F, G), now define a Markov chain which we shall represent by the stochastic matrix $\mathcal{T}$. We are thus assuming that $\mathcal{T}$ maps x_t into x_{t+1}. In what follows, let $\{w_i\}$ denote the subset of W which contains the market qualities that are traded, if trade is optimal for some state, and contains the creation index w_0, if exit is optimal for some state. If trade is never optimal, then no taxation takes place and the convergence of $\{x_t\}$ to a unique limit follows from proposition 2.

There are now two other cases characterizing the law of motion for x_t. In the first case, the only optimal policy for all businesses with tenure $\bar{n}$ is keep. Notice that according to lemma 4, for all $n < \bar{n}$ there is always a state (z, s, w, n) for which the optimal policy is also to keep. Since, according to assumption 1, Q is strictly positive, every state evolves with a positive probability to $\{(z, s, w, \bar{n})\}$ in a finite number of steps. Hence, for this case, there is only one ergodic set described $\{(z, s, w, \bar{n})\}$, where all businesses have tenure $\bar{n}$, $w \in \{w_i\}$ and no taxation takes place in the limit. For this case, the convergence of $\{x_t\}$ is also implied by proposition 2.

In the second case, there exists (z, s) in $Z \times S$ for which the optimal policy for state $(z, s, w, \bar{n})$ is to sell or to exit. Thus such a state is a renewal state in the sense that assumptions 1-2 imply that every productivity shock (z, s) can be visited with a positive probability in a finite number of steps from any state in $Z \times S \times W \times \{0, 1, ..., \bar{n}\}$. In particular, "condition M" in Stokey *et al.* (1989), page 348, applies, so that according to their theorem 11.12 (page 350), $\{x_t\}$ converges to a unique x^* in the total variation norm.

We now discuss the transition across different tax regimes. Notice that the proof of proposition 3 does not require any additional assumptions on the tax parameters other than that τ and π are non-negative with τ less than one. One crucial feature of the tax policy used in the proof is that the pair (w, n) suffices to pin down the capital-gains tax liability whenever the asset price function p is time invariant. There are situations in which the price function might not be time invariant. As we show below, this is the case with a transition from a high capital gains tax regime to a low capital gains tax regime. For such transitions, equations (1.17-1.21) may not fully describe the equilibrium. For the quantitative experiments of the next section we use an algorithm which replaces the state (w, n) by a single variable b, the nominal value of deductions (or "book value"). This change of variables may allow for the computation of a larger class of transitions, by value function iterations, in the same spirit as the operator T above. The algorithm is outlined in the appendix. We have not been able to show that the operator defined by the algorithm is a contraction for high values of β.

There is nevertheless at least one class of transitions in tax regimes for which equations (16)-(20) are relevant. A change in tax policies from a capital gains tax to a lump-sum tax makes the state (w, n) irrelevant after the new regime starts. We are thus interested in the transition from a proposition-3 steady state with $\tau > 0$, call it (v^τ, p^τ), to a proposition-1 steady state here denoted (v^0, p^0). The next proposition describes this transition.

Proposition 5 *The transitional dynamics from a steady state with $\tau > 0$, represented by (v^τ, p^τ) at $t = 0$, to a steady state with no capital gains taxation, denoted (v^0, p^0), is characterized by*

$$(v_t, p_t) = \begin{cases} (v^\tau, p^\tau), & \text{for } t = 0; \\ (v^0, p^0), & \text{for all } t \geq 1; \end{cases}$$

with asymptotic convergence of the distribution of businesses.

Proof The result is implied by the invariance of prices to business distributions. Suppose that the steady-state distribution associated with (v^τ, p^τ) is defined by $x_0(z, s, w, n)$ for all (z, s, w, n). When the tax regime changes (date $t = 1$), the distribution at the beginning of the period, x_1, is defined by $x_1(z, s) = \int x(z, s, dw, dn)$ for all (z, s), where we used that the state variables (w, n) become irrelevant when the taxation of capital gains is removed. Then, the competitive equilibrium under the new regime ought to be described by equations (1)-(4). The new value and asset pricing functions are thus independent from x_1 and ought to be given by the proposition-1 equilibrium, which is described by $(v_t, p_t) = (v^0, p^0)$ for all $t \geq 1$. The law of motion (5) determines the convergence of x_t to the new steady state from the initial distribution x_1.

Proposition 5 shows that transitions in this environment are typically described by a one-shot adjustment in asset prices, followed by a long-lasting adjustment in the distribution of businesses. In the next section we study a parameterized version of the model designed to capture the welfare effects of removing the taxation of inflationary gains. We proceed by comparing policies that raise the same discounted value of tax revenues. If $r(z,s,w,n)$ is the tax revenue from business (z,s,w,n), then the discounted value of revenues, R, is computed according to

$$R = \sum_{t=0}^{\infty} \sum_{z,s,w,n} \beta^t x_t(z,s,w,n) r(z,s,w,n),$$

while our welfare criteria, U, is

$$U = \sum_{z,s,w,n} x_0(z,s,w,n) v(z,s,w,n).$$

The welfare measure U is equivalent to the expected discounted value of a business, an expectation with regards to the underlying equilibrium distribution of businesses.

5 The welfare costs of taxing capital gains

The link between ownership and control of technologies in this paper follows closely the work of Holmes and Schmitz (1995). They propose an overlapping generations structure in which some productivity shocks are idiosyncratic to the match between a technology and its manager. They assume that the owner and the manager of a technology are the same person. They go on to show that their structure has no difficulty in matching the turnover properties of a survey of small businesses undertaken by the U.S. Department of Commerce (1987). In this section, we calibrate a simple version of the model economy to some of the salient features of the data. We then perform tax policy experiments and compute welfare effects of taxing inflationary gains.

We choose functional forms as follows. The support for the market quality is $Z = \{z_1, z_2\}$, and for the ownership quality is $S = \{s_1, s_2\}$. The transition matrix governing z at the end of the period is

$$Q^z = \begin{bmatrix} Q^z_{11} & Q^z_{12} \\ Q^z_{21} & Q^z_{22} \end{bmatrix},$$

while that for s is

$$Q^s = \begin{bmatrix} Q^s_{11} & Q^s_{12} \\ Q^s_{21} & Q^s_{22} \end{bmatrix},$$

so that Q is represented by the Kronecker product $Q^z \otimes Q^s$. The distributions governing new draws for z and s are simply the lists $F_z = (1 - P_z, P_z)$, $F_s = (1 - P_{s0}, P_{s0})$ and $G = (1 - P_{s1}, P_{s1})$, where P_z, P_{s0} and P_{s1} are, respectively, the probability of drawing the high market quality z_2, the high ownership quality s_2 when creating a business, and the high ownership quality s_2 when buying a business. We assume that output is the sum of a constant y_0 and a stochastic component $f(z_i, s_j) = z_i + s_j$ for $i, j \in \{1, 2\}$.

{Insert Table 1}

We now describe the salient features of the data that we want the model to mimic. Holmes and Schmitz (1995) present cross-tabulations for the 1982 Characteristics of Business Owners Survey conducted by the U.S. Census Bureau (U.S. Department of Commerce 1987). They report data for 15,737 non-minority males who owned businesses in 1982. For tax purposes, these businesses were classified as proprietorship, partnership, and subchapter S corporations. Their tabulations display the business distribution across age of the business, across tenure of the owner with the business, and across a founder status. The founder status separates a business created by the 1982 owner from a business purchased by the 1982 owner from somebody else. Age and tenure are aggregated into ranges of years in order to satisfy U.S. Census Bureau disclosure requirements (see Tables 1-2).[4] Holmes and Schmitz also report the business age distribution for the total population and the fraction of non-founder business firms in each age category, which we reproduce in Table 3.[5] They also document that the fraction of businesses that exited and the fraction that was sold in 1982 was 13.64% and 1.96%, respectively.

{Insert Table 2}

We compute statistics in our model economy that are the counterpart of the U.S. data displayed (see Tables 1-3). This computation involves selecting parameter values, simulating the model economy, and recording the age, the tenure, and the founder status of each business. We can then obtain the invariant distribution of businesses across these categories (see the appendix).[6]

{Insert Table 3}

Parameter values were chosen as follows. The discount factor β was set at .95, the same value chosen by Holmes and Schmitz (1995), which is consistent with the model period being a year. The tax rate τ was set

[4] Tenure categories are years of ownership. The numbers in parenthesis are statistics from the model.

[5] Holmes and Schmitz also report, for some years, data on the distribution of age, tenure and founder status across businesses that exited and businesses that were traded. We consider these marginal distributions of less importance and, for ease of exposition, abstract from them here.

[6] Notice that while the number of non-founder businesses with zero age and zero tenure in the model is zero, this is not true in the data (see first row and second column in Table 2). The explanation is that while every nonfounder business in the model is at least one year old, some businesses in the data are resold in less than a year.

at .28. Notice that the choice of two possible levels for the market quality, $Z = \{z_1, z_2\}$, imply that there are only two possible prices in units of the contemporaneous consumption good ($p(z_1)$ and $p(z_2)$). With this choice of Z, most capital gains in the computed model are inflationary. Although 28% corresponds to the top tax rate on capital gains for most of the 80's, we think that $\tau = .28$ implies a lower bound for the tax liabilities since we are not allowing for real gains to accumulate considerably. We pick the inflation rate for the survey period as 6%, which implies $\pi = .06$. Hence, nominal gains accumulate at the rate of 6%. Notice that it would not be uncommon to observe corporate equity appreciating at an average rate of 7% in real terms, a kind of real gains also possible for small businesses from which the model is abstracting from.

We next choose the parameters in $(Q^z, Q^s, P_z, P_{s0}, P_{s1})$ determining the stochastic process for shocks probabilities, the creation cost parameter λ, the deterministic value of output y_0, and the support for the shocks (z_1, z_2) and (s_1, s_2). We start by noting that the decision of whether to keep, sell, or shut down a business only depends on $z_2 - z_1$ and $s_2 - s_1$ so that we can (exogenously) fix $z_1 = .2$ and $s_1 = 0$. We then have 11 parameters to pin down.[7] The U.S. data on unincorporated businesses in Table 3 (age-distribution of businesses and share of non-founders by age of business) provide 9 (independent) targets. The exit rate and the sale rate for the year 1982 give us two additional targets.

We proceed by minimizing a loss function representing the distance between the selected targets and comparable statistics in the model economy. This procedure implies that the parameters governing the stochastic process for shocks are

$$Q^z = \begin{bmatrix} Q^z_{11} & Q^z_{12} \\ Q^z_{21} & Q^z_{22} \end{bmatrix} = \begin{bmatrix} .99 & .01 \\ .06 & .94 \end{bmatrix},$$

$$Q^s = \begin{bmatrix} Q^s_{11} & Q^s_{12} \\ Q^s_{21} & Q^s_{22} \end{bmatrix} = \begin{bmatrix} .72 & .28 \\ .11 & .89 \end{bmatrix}$$

and

$$(P_z, P_{s0}, P_{s1}) = (.10, .52, .36).$$

The calibration implies that the shocks to the market qualities are more persistent than the shocks to the ownership quality ($Q^z_{22} > Q^s_{22}$ and $Q^z_{11} > Q^s_{11}$). Moreover, a good ownership quality can become a bad one the next period with probability .11 ($Q^s_{21} = .11$). When a business is created, the probability that the market quality is the high one is .10 and the probability that the ownership quality is the high one is .52, i.e., $(P_z, P_{s0}) = (.10, .52)$. The probability that the high ownership quality is drawn when the business is purchased is given by $P_{s1} = .36$. The calibrated value for λ is .045.

[7] Note that for each of Q^z and Q^s there are only 2 independent parameters to be determined out of total of parameters 4. The other 2 parameters are implied by the restriction that a probability measure adds up to 1.

As a result, the cost of creating a business in the model is 4.5% of the deterministic level of output y_0.

We obtain $z_2 = 1.02$ and $s_2 = 12$. These values correspond to productivity differences of $z_2 - z_1 = .82$ and $s_2 - s_1 = 12$, which are close to the estimates obtained by Holmes and Schmitz for the differences between the high and low realizations of their permanent shocks to transferable and non-transferable qualities. They estimate the difference in productivity to be .29 for the transferable quality, and 11.29 for the non-transferable quality. Our calibration sets the parameter representing the deterministic value of output, y_0 to 61. Notice that the parameter y_0 implies a lower bound for the sale price of a business. The choice of $y_0 = 61$ implies a ratio of average sale price to mean aggregate output of about 1. Higher values of y_0 would increase prices and hence increase nominal gains in the absence of indexation of deductions for inflation. The U.S. Department of Commerce (1987) reports the starting capital requirements for the 1982 Characteristics of Business Owners Survey according to the annual receipts of firms in selected categories. The computed average ratio of starting capital requirements to receipts is about 1, with a range of .75 for the Hispanic owners to 1.21 for the non-minority male owners. We conjecture that this ratio should approximate closely the ratio of mean business prices to mean output in the data. Our choice of y_0 is thus consistent with a proxy of business prices to business output in the data. The calibrated model economy displays an exit rate of 12.56% and a sale rate of 1.57%, which are close to the corresponding targets of 13.64% and 1, 96% from the U.S. data.

The computed steady-state turnover statistics for the model under the selected parameters are displayed in parentheses in Tables 1-3. Notice that the model fares well regarding most of the age distribution of businesses reported in Table 3. The main discrepancy is that 19% of all businesses in the survey are 23 years old or older, while such businesses are just 6% of the total in the model. This discrepancy is also reflected in Tables 1-2, which shows that there are more young businesses in the model than in the data for most tenure categories. Regarding the share of non-founder business in the population, as reported in Table 3, we also find that the model fares relatively well, except for the population of businesses that are 23 years old or older. About 20% of those businesses in the model are non-founder businesses, while this share in the data is 51%. We conclude that further progress in matching the data would require generating more non-founder businesses that are relatively old. We conjecture that this would require a larger support for the business qualities, instead of the support with 2 values used here.

5.1 Findings

We find that the aggregate tax revenue is only .03% of GDP in the model economy, which should not be surprising since less than 2% of all businesses are sold in a given period. The aggregate tax revenue represents 1.74% of the output of the businesses actually. We ask if trade could be made more vigorous by cutting the taxation of capital gains. To answer this question we first compute steady-states for the model economy under different tax rates. In a second experiment, we compute transitions across steady states and consider tax policies that generate a value for the present discounted value of tax revenues equal to the one in the calibrated model economy.

When the tax rate τ is changed from the benchmark value of .28 to .14, the fraction of businesses being sold in a steady-state equilibrium increases from 1.6% to 11%. This finding indicates that the tax-elasticity of sales is quite high in the model. The tax reduction increases aggregate tax revenues from .03% of the output in the benchmark economy to .11% of the output in the economy with lower taxes. Moreover, output increases by .32%. Our findings indicate that a tax cut can increase revenues from the taxation of small businesses and, at the same time, produce efficiency gains of about half of a percent of steady-state output.

We now simulate the introduction of full indexation of tax deductions when the tax rate is maintained at the benchmark value of 28%. To this end, we set the inflation rate to 0 ($\pi = 0$) which is equivalent, in our model economy, to assuming that the tax system is fully indexed for inflation. We find that the measure of businesses that are kept per period remains relatively unchanged, while the measure of businesses being sold changes from 1.6% to about 7%. The increase in the number of sales associated to indexing deductions implies that tax revenues increase from .03% to .05% of output. Moreover, output increases by .26%. Finally, we compute the equilibrium transition after replacing the taxation of capital gains by lump-sum taxation. The lump-sum tax is set so that the present value of tax revenues is the same as in the benchmark economy. The main finding is that the present value of output increases by .48%.

To sum up, the results indicate that the tax-elasticitiy of business-sales is high and that, as a result, the taxation of capital gains has sizable efficiency costs. We find that indexing deductions to inflation increases welfare and capital gains tax revenues. Moreover, eliminating the taxation of capital gains increases welfare by about half a percent.

6 Final Remarks

The taxation of capital gains is regarded as highly distortionary and has occupied a central place in tax policy discussions. Since capital gains are taxed upon realization rather than accrual, individuals can defer, free of interest, taxes on accrued capital gains by postponing the sale of assets. This "lock-in effect" is effectively a tax on trade which adds an additional excess burden to the taxation of capital income.

In this paper we have taken a very different perspective from the prevailing view in the capital gains taxation literature. Previous theoretical studies focus on the effects of capital gains taxes on portfolio and savings decisions. This line of work has been followed, among others, by Auerbach (1992), Balcer and Judd (1987) and Cavalcanti (1996). In this paper, instead, we focus on how capital gains taxes affect business turnover and production efficiency. The key idea here is that when ownership of assets affects the uses to which they are put, the impediments to the transfer of ownership of assets to those who can best control them may result in sizable negative effects on production efficiency[8].

The main contribution of this paper is to provide a simple model of the distortions that the taxation of capital gains imposes on business turnover decisions and production efficiency in the unincorporated sector.[9] The model lends itself well to the measurement of distortions caused by the lack of indexation of inflationary gains. Inflation affects asset prices and business turnover through the channel of tax deductions. This follows from the fact that we do not adjust the nominal value of assets by inflation when computing capital gains tax liabilities, which is in accordance with the present tax code in the U.S. economy.

This paper is also related to the industrial organization literature on business turnover, which has mostly focused on the entry and exit of businesses but not on sales.[10] The work of Holmes and Schmitz (1990, 1995 and 1996) have emphasized the importance of business transfers in allocating resources in the economy. Our model economy borrows from their 1995 paper the idea that technologies have a transferable quality that is independent of its owner and a non-transferable quality that is idiosyncratic to an individual-technology pair. Holmes and Schmitz show that a model developed along these lines is consistent with evidence on the small business sector.

A key virtue of our model is its simplicity. Models of capital gains taxation are typically very complex since they not only require heterogeneity among agents, necessary for trade, but they also imply that individuals treat differently assets with identical yields but carrying different tax deductions (book values). The model presented in this paper is sufficiently tractable to enable us to prove existence and uniqueness of equilibrium with a simple contraction-mapping argument. Furthermore, it is possible to compute transitional dynamics between policy regimes and evaluate tax policies that

[8] This idea is not new in the public finance literature. Stiglitz (1983) shows in a standard Austrian capital market model that investment projects may not be terminated at the socially optimal date because of tax-induced distortions. The connection between ownership and control has been emphasized at least since Frank Knight (1921).

[9] Of course, there other taxes that may affect production efficiency in the economy. See Gravelle and Kotlikoff (1995) for a study of how the corporate income tax affects production in the corporate sector. More generally, see Kehoe and Prescott (1995) for a critical history of applied general equilibrium analysis.

[10] See, for instance, Jovanovic (1982), Hoppenhaym (1992), and Ericson and Pakes (1995).

have the same present value of tax revenues. This basic structure can be extended to study transitional dynamics associated to changes in other policies.

The tractability of the model stems from the fact that prices of businesses are independent of the aggregate state of the economy. We emphasize that this property does not hold in Holmes and Schmitz (1995). In their paper, the demand of businesses is fixed by an assumption since the only individuals allowed to purchase are an exogenous fraction of newborns. Individuals that shut down or sell a business are assumed to pursue an outside opportunity that gives them an exogenous fixed income. Equilibrium prices are those at which the number of individuals willing to sell businesses is equal to the exogenous demand. In our model, all individuals are free to choose at each point in time whether they want to keep or discontinue the operation of an existing business, and whether they want to purchase or create a new business. Prices are determined by a no-arbitrage condition that makes individuals indifferent to the quality of business they buy. It turns out that this condition does not depend on the aggregate distribution of businesses in the economy.

The responsiveness of the tax revenue to changes in the capital gains tax rates has been the subject of continuing policy debate. For instance, using microeconomic data Feldstein *et al.* (1980) estimate a very high elasticity of capital gains realizations with respect to tax rates. Their estimated elasticity would imply that a capital gains tax cut would raise tax revenue substantially. Studies by the Treasury's Office of Economic Policy have also arrived at a similar conclusion but these results have been challenged by many economists.[11] The ensuing debates highlight the importance of having a model that can be calibrated to micro observations and used to compute the revenue paths of changes in tax rates.

In our calibrated model, we find that indexing tax deductions to inflation increases the realizations of capital gains sufficiently to increase tax revenues. We also perform the experiment of replacing the capital gains tax by a revenue preserving lump-sum tax. We find that welfare or, equivalently, the output of the unincorporated business sector increases by half a percent with this change in policy. We think that a more complex parameterization of the model, one which increases the number of possible technology qualities, can improve the performance of the model by allowing for the accumulation of real gains. In this regard, our quantitative findings are indicative of how the model could be put to good use. An exhaustive quantitative study is left for future research.

[11] See for example Michael Darby (1988) and Joseph Minarik (1984). For a survey of this literature see Auerbach (1992).

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Appendix: The Computational Method

We compute equilibrium value and price functions by value function iterations. The iterations are generated by an operator inspired by the existence proof with a change of variables. In the presence of capital gains taxation, we replace the pair (w, n) by the state variable b, which represents the value of deductions expressed in units of the contemporaneous consumption good. With this change, the operator maps (v_i, p_i), at iteration i, into (v_{i+1}, p_{i+1}) as follows.

$$v_{i+1}(z,s,b) = \max\{[T^k v_i](z,s,b), [T^s v_i](z,b), T^e v_i\}, \tag{22}$$

$$[T^k v_i](z,s,b) = y(z,s) + \beta \int v_i(z', s', \frac{b}{1+\pi})Q(z,s,dz',ds'), \tag{23}$$

$$p_{i+1}(z) = \int \{y(z,s) + \beta \int v_i(z', s', \frac{p_i(z)}{1+\pi})Q(z,s,dz',ds')\}G(ds), \tag{24}$$

$$[T^s v_i](z,b) = \min\{p_i(z), (1-\tau)p_i(z) + \tau b\} \tag{25}$$

and

$$T^e v_i = \int \{y(z,s) - \lambda y_0 + \beta \int v(z', s', \frac{\lambda y_0}{1+\pi})Q(z,s,dz',ds')\}F(dz,ds). \tag{26}$$

We used a grid of 50 points for b. After convergence of the sequence $\{(v_i, p_i)\}$,we used the policy functions to compute the invariant distribution for steady states, or to compute the transition path.

Business Age	Data (Model)
0	2147 (1977)
1-2	2909 (3152)
3-6	2967 (3497)
7-12	2043 (2882)
13-22	1515 (1880)
23+	1463 (786)

Table 1 Number of Founders by Business Age (in Years)

Age of Business	Tenure of Owner 0	1-2	3-6	7-12	13-22	23+
0	60 (0)					
1-2	56 (103)	108 (64)				
3-6	40 (78)	117 (178)	116 (119)			
7-12	29 (45)	77 (87)	98 (174)	73 (138)		
13-22	31 (18)	70 (35)	106 (62)	106 (140)	75 (127)	
23+	93 (3)	208 (6)	291 (10)	292 (23)	344 (69)	303 (82)

Table 2 Number of Nonfounders by Business Age and Ownership Tenure (In Years)

Age of Business (Years)	Business Distribution	Share of Non-founders
	Data (Model)	Data (Model)
0-2	.34 (.34)	.04 (.03)
3-6	.21 (.25)	.08 (.10)
7-12	.15 (.21)	.12 (.13)
13-22	.12 (.14)	.20 (.17)
23+	.19 (.06)	.51 (.20)

Table 3 Business Age Distribution and Share of Nonfounders