ELSEVIER

Journal of Monetary Economics 49 (2002) 761–795

Journal of MONETARY ECONOMICS

www.elsevier.com/locate/econbase

On inflation as a regressive consumption tax ☆

Andrés Erosa[a,]*, Gustavo Ventura[b]

[a] Departament d'Economia i d'Història Econòmica, Universitat Autònoma de Barcelona, Bellaferra, Barcelona 08193, Spain

[b] Department of Economics, Penn State University, University Park, PA 16802, USA

Received 14 September 2000; received in revised form 24 May 2001; accepted 25 July 2001

Abstract

Evidence on the portfolio holdings and transaction patterns of households suggests that the burden of inflation is not evenly distributed. We build a monetary growth model consistent with key features of cross-sectional household data and use this framework to study the distributional impact of inflation. At the aggregate level, our model economy behaves similar to standard monetary growth models within the representative agent abstraction. Inflation has, however, important distributional effects since it is effectively a *regressive* consumption tax. Thus, neglecting the distributional consequences of inflation may prove misleading in assessing the effects of inflation in our economy. © 2002 Elsevier Science B.V. All rights reserved.

JEL classification: E31; D30

Keywords: Inflation; Heterogeneity; Distribution

☆ Earlier versions of the paper circulated under the title "Inflation, Heterogeneity and Costly Credit: How Regressive is the Inflation Tax?". We have benefited substantially from the comments of Fernando Alvarez, David Andolfatto, Paul Beaudry, Roland Benabou, Dan Bernhardt, Jeff Campbell, Harold Cole, John Knowles, Narayana Kocherlakota, Per Krusell, and Victor Ríos-Rull. We also thank seminar participants at the 1999 Northwestern Summer Macro Conference, Cleveland Fed, Rochester, Western Ontario, Minneapolis Fed, Southern California, SUNY-Buffalo, Illinois, Toronto, Pompeu Fabra, ITAM, British Columbia, Richmond Fed, Atlanta Fed, Universidad San Andrés, 1999 SED Meetings, 2000-ITAM Conference on the Consequences of Financial Crises on Income Distribution, and 1999 Econometric Society Meetings – Cancun. Both authors thank financial support from the Social Science and Humanities Research Council. Part of this paper was completed while Erosa was affiliated with Western Ontario. He thanks financial support from the Spanish Ministry of Science and Technology through grant SEC 2000-0684. Shannon Seitz provided excellent research assistance. All errors are ours.

*Corresponding author. Tel.: +34-93-581-1215; fax: +34-93-581-2012.

E-mail address: andres.erosa@uab.es (A. Erosa).

PII: S0304-3932(02)00115-0

1. Introduction

The literature on the welfare cost of inflation has largely ignored the distributional effects of inflation. However, the heterogeneity in household wealth composition and transaction patterns observed in the data suggest that the burden of inflation borne by poor individuals may be significantly higher than for rich individuals. Consider the following facts regarding wealth composition and transaction patterns for households in the US:

- Observation 1: High income individuals use cash and cash plus checks for a smaller fraction of their total transactions than low income individuals (Avery et al., 1987).
- Observation 2: The fraction of household wealth held in liquid assets decreases with income and wealth (Wolff, 1983; Kessler and Wolff, 1991; Kennickell and Starr-McCluer, 1996).
- Observation 3: A non-trivial fraction of households do not own a checking account and/or do not own or use credit cards to perform transactions (Avery et al., 1987; Kennickell et al., 1997; Mulligan and Sala-i-Martin, 2000).

We develop a monetary growth model that is consistent with the evidence on heterogeneity in transaction patterns and portfolio holdings across individuals to assess the distributional impact of inflation. Our model economy behaves, at an aggregate level, in a manner similar to standard monetary growth models within the representative agent abstraction. Inflation has, however, important distributional effects. We find that the burden of inflation is substantially higher for individuals at the bottom of the income distribution than for those at the top. Moreover, inflation leads to an important redistribution of assets across individuals. These findings are robust to various alternative specifications of costly credit transactions. Thus, in evaluating the impact of inflation in our economy, neglecting the distributional consequences of inflation can be quite misleading.

In our economy, individuals allocate assets between capital and money and perform transactions using either cash or costly credit. Money is a poor store of value since it is dominated in rate of return by capital; nevertheless, individuals hold money because they value a large number of consumption goods and purchasing goods with credit entails buying credit services. If the technology for transacting with credit exhibits economies of scale, inflation may have important distributional effects. In particular, if the amount of credit services required is a non-increasing function of the total amount of goods purchased, inflation implies that the per-unit cost of transacting is inversely related to the level of consumption. High income households face a lower per unit cost of credit purchases than their low income counterparts since they consume more than low income households. As a result, they pay a higher fraction of their purchases with credit and they hold less money as a fraction of total assets than low income households. In this respect, inflation operates as a non-linear *regressive* consumption tax, for high income households are better able to avoid the inflation tax than those with low incomes. Alternatively, if the technology for transacting does not exhibit scale economies, the model is *inconsistent* with the cross-sectional evidence on transaction patterns and portfolio holdings and inflation does not have a distributional impact.

Through its role as a non-linear consumption tax, inflation may have important consequences for saving behavior when consumption varies over time due to income risk. In this case, individuals use savings to smooth consumption fluctuations. Since a non-linear consumption tax affects consumption smoothing decisions, inflation has an impact on savings behavior. Our numerical experiments show that inflation may lead to a substantial concentration in the distribution of wealth. Interestingly, inflation does not affect the distribution of wealth in the absence of income risk. When income is certain, individuals use their savings to finance a constant level of consumption in steady-state. A non-linear consumption tax schedule does not affect any intertemporal trade-off because consumption is constant across periods; thus, savings rates are unaffected by inflation. Nevertheless, inflation does have a distributional impact on welfare.

There is a large literature on the effects of inflation in economies where costly credit services provide an alternative means of payment to money. Important contributions include Prescott (1987), Schreft (1992), Gillman (1993), Cole and Stockman (1991), Dotsey and Ireland (1996), and Aiyagari et al. (1998) among others. All of these studies consider economies with a representative agent since they are concerned with the aggregate implications of inflation. Our primary contribution is to show, in an environment with costly credit transactions, that inflation has non-trivial distributional consequences that have not been explored previously. Chatterjee and Corbae (1992) consider an economy where money is essentially a store of value that competes with bonds as a vehicle for saving. They show that inflation redistributes wealth between borrowers and lenders by affecting the real interest rate. In our paper inflation is essentially a tax on *monetary transactions* rather than on savings; therefore, inflation can have large distributional implications even when its impact on real interest rate is small.[1] Imrohoroglu (1992) and Imrohoroglu and Prescott (1991) also study the impact of inflation in an economy where individuals face uninsurable idiosyncratic income risk. However, they abstract from the transactions role of money since money is treated only as a store of value in their papers. Within their framework, inflation acts as a flat tax on savings rather than a non-linear consumption tax.

In a recent paper, Mulligan and Sala-i-Martin (2000) use cross-sectional data to estimate the interest elasticity of money demand at low interest rates. They argue that "the relevant monetary decision for the majority of U.S. households is not the fraction of assets to be held in interest bearing form, but whether to hold any such assets at all". In our framework, most households own capital (which, given our broad interpretation of capital, includes consumer durables). The relevant decision we model is whether, and to what extent, to purchase with credit. In contrast to

[1] The tax code in the US and many other countries, features less than perfect indexation of capital income. This may have important distributional consequences. We abstract from this issue since we concentrate on inflation as a tax on monetary transactions. See Altig and Carlstrom (1991) and Cavalcanti and Erosa (1999) for studies of inflation and capital income taxation.

Mulligan and Sala-i-Martin (2000), our focus is on the distributive effects of inflation.

The remainder of this paper is organized as follows. Section 2 presents the model economy and provides some analytical intuition for the results that we report in our numerical experiments. Section 3 discusses the calibration of the model, and shows that the model is broadly consistent with evidence regarding patterns of transaction and portfolio holdings. The distributional impact of inflation is discussed in Section 4. Section 5, explores the sensitivity of our findings to the specification of the transaction technology. Section 6, considers the distributive impact of inflation in an economy where individuals do not face uninsurable income risk. Finally, Section 7 concludes.

2. An economy with heterogeneous transaction patterns and portfolio holdings

2.1. The economic environment

The economy is populated by a continuum of infinitely lived individuals who face uninsurable idiosyncratic risk in labor productivity. This assumption allows us to study the effects of inflation in a world where agents smooth consumption and it has important consequences for the effects of inflation on saving decisions. An agent with labor productivity shock $z \in \mathscr{Z}$ receives labor income $\hat{w}z$, where $\hat{w}$ is a wage common to all agents, and the random variable z is assumed to follow a finite-state Markov process with support in the set $\mathscr{Z}$. In order to highlight the distributional impact of inflation, we assume that the population is partitioned in two subsets. Type-H individuals have the support of their productivity shocks on the set $\mathscr{Z}_{\mathrm{H}}$ and individuals of Type-L have the support of their productivity shocks on the set $\mathscr{Z}_{\mathrm{L}}$, where $\mathscr{Z}_{\mathrm{L}}$ and $\mathscr{Z}_{\mathrm{H}}$ form a partition of $\mathscr{Z}$. Type-H individuals are assumed to have a higher labor productivity on average than Type-L individuals.

Individuals consume a continuum of non-perishable commodities, indexed by $i \in [0, 1]$, and are endowed with one unit of time per period devoted entirely to market work. Preferences are given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

with $\beta \in (0, 1)$, where the period utility function is of the form

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

Following Schreft (1992), Aiyagari et al. (1998) and others, we assume that the consumption aggregator, denoted c, adopts the form $c = \inf_i \{c(i)\}$. Note that this assumption of perfect complementary in consumption implies that individuals will consume the same amount of all goods.

Economic activity within a period is divided in three stages or subperiods. Individuals supply labor and capital services to firms during the first subperiod and receive wages and interest income at the end of the period. Consumers purchase consumption goods from firms in the second subperiod which can be purchased either with cash or (costly) credit. In the third stage, households participate in a centralized asset market where they receive the income for the factor services supplied in the first subperiod, they pay for the goods bought on credit during the second stage, and they use remaining funds to acquire cash or unsold output to accumulate as capital into the next period.

Households optimally choose whether to purchase consumption goods with credit or cash as in Schreft (1992), Gillman (1993), Dotsey and Ireland (1996) and others. In order to buy an amount c of good i with credit, the consumer must purchase $\gamma(c,i)$ units of financial services. The function $\gamma(\cdot,\cdot)$ is weakly increasing in c, strictly increasing in i, and satisfies $\lim_{i\to 1}\gamma(c,i)=\infty$ for all $c\geqslant 0$. The latter assumption guarantees that some goods will be purchased with cash so that there is a well-defined demand for money. We assume that there is a large number of intermediaries in the economy and that the technology to produce financial services requires one (efficiency) unit of labor per unit of service produced. Intermediaries charge a fee q per unit of financial service sold. Competition ensures that in equilibrium intermediaries will make zero profits, that is, q is equal to the wage rate ($\hat{w}$).

We restrict our analysis to steady-state behavior. For simplicity of notation we do not index the aggregate variables in our economy with a time subscript. Goods are produced with a technology that transforms capital and labor inputs according to a constant return to scale production function $F(K,L^g)$, where K and L^g denote aggregate capital and labor inputs in the goods production sector and capital depreciates at the rate δ. Investment and the different types of consumption goods are assumed to be perfect substitutes in production so that their relative price is equal to 1. It should be emphasized that, if labor productivity growth were introduced in the goods production sector, the associated balanced growth path would have an aggregate money demand with a unitary income elasticity (provided there is no technological change in the finance sector), while the cross-sectional income elasticity of money could well be less than one.

The government is assumed to consume a constant amount G of goods per period and to balance its budget by printing currency at a constant rate π, and by taxing capital and labor income at constant rates τ_k and τ_l, respectively. Taxation of factor income is introduced into the model in order to emphasize the public finance aspect of inflation. Specifically, we will compare the inflation tax to a labor income tax by performing revenue neutral experiments in which the tax on capital income is fixed and the tax on labor income is adjusted so as to satisfy the government's budget constraint. The proceeds of the inflation tax are used to finance government consumption rather than be distributed as a lump sum transfer to consumers.

In equilibrium, after tax prices satisfy $w=(1-\tau_l)\,f_2(K,L^g)$ and $r=(1-\tau_k)(f_1(K,L^g)-\delta)$ and the government budget constraint is given by

$$G=\pi M+(\hat{w}-w)(L^f+L^g)+(\hat{r}-r)K,$$

where M represents aggregate real money holdings, $\hat{r}$ is the before tax interest rate, and L^f denotes the aggregate labor input in the finance industry.

We now describe the agent's problem in the language of dynamic programming. At the beginning of the period, the state of an individual (x) is summarized by the labor endowment shock z, asset holdings a, and real money holdings m (i.e. $x = (a, m, z)$). We denote by $\mathscr{X}$ the set of all possible values of x. The decision problem is represented by the following Bellman equation

$$v(z, a, m) = \max_{c,s,m',a'} \{u(c) + \beta E[v(z', a', m')|z]\}$$

subject to

$$c(1 - s) \leqslant m,$$

$$c + q \int_0^s \gamma(c, i)\, \mathrm{d}i + a' + m'(1 + \pi) \leqslant (1 + r)a + wz + m,$$

$$a' \geqslant 0,$$

where $s \in [0, 1]$ stands for the fraction of consumption goods purchased with credit. Observe that the first restriction is a cash-in-advance constraint, while the second one is the budget constraint. The final restriction indicates that households cannot borrow. Let $g_c(x), g_s(x), g_a(x), g_m(x)$ denote the optimal decision rules for consumption, the fraction of transactions using credit, assets, and money, respectively, that solve the above dynamic program.

The decision rules of individuals and the stochastic process for the shocks define an invariant distribution of agents (λ) over shocks, money, and asset holdings. Thus, the market clearing conditions for goods, capital, money, financial services, and labor markets are given by

$$\int_{\mathscr{X}} g_c(x)\, \mathrm{d}\lambda + \delta K + G = f(K, L^g),$$

$$\int_{\mathscr{X}} g_a(x)\, \mathrm{d}\lambda = K,$$

$$\int_{\mathscr{X}} g_m(x)\, \mathrm{d}\lambda = M,$$

$$\int_{\mathscr{X}} \int_0^{g_s(x)} \gamma(c, i)\, \mathrm{d}i\, \mathrm{d}\lambda = L^f,$$

$$\int_{\mathscr{X}} z\, \mathrm{d}\lambda = L^f + L^g.$$

2.2. Patterns of transactions and the credit technology

The model developed in the preceding section can generate cross-sectional transaction patterns consistent with those observed in the data. In particular, when there exist scale economies in the transactions technology, individuals with higher consumption levels will make relatively more transactions with credit than those with low consumption levels. As a result, inflation has differential effects on individuals with different levels of consumption. Alternatively, when the transactions technology does not feature economies of scale, the fraction of consumption good purchased with credit is independent of the level of consumption and inflation does not have a distributional impact.

In order to provide some analytical intuition we consider a simplified version of our economy where individuals do not face income risk. We assume that the economy is in steady-state and that individuals are heterogeneous in income, and thus, in consumption levels. Notice that there are a continuum of distributions of income that are consistent with our steady-state assumption in the absence of idiosyncratic shocks to labor productivity. Denote by $\Gamma(c)$ an arbitrary c.d.f. of consumption in steady-state. From the consumer's FOCs we can obtain

$$c(1-s) = m,$$
$$cR \leqslant \hat{w}\gamma(c,s), \quad \text{with} = \text{if } s > 0,$$

where R denotes the nominal interest rate and s represents the fraction of goods that are purchased with credit. The first equation is the CIA constraint which binds when the nominal interest rate is positive. The second equation states that when households purchase goods with credit, they equate the opportunity cost of money with the cost of credit services for the marginal good purchased with credit. We allow for the possibility of corner solutions to capture the notion that it may be optimal for some individuals to perform all their transactions with money ($s = 0$). Combining the above two equations, we can obtain an expression for the inverse-money demand for a household with consumption level c:

$$R \leqslant \frac{\hat{w}\gamma(c, 1-m/c)}{c}, \quad \text{with " = " if } s = 1 - \frac{m}{c} > 0. \tag{1}$$

Individuals face transactions cost when purchasing consumption goods. These costs are given by the sum of expenditures on financial services associated with credit purchases and the cost of monetary transactions ($R \times m$). Transaction costs per unit of consumption, or average transaction costs (*atc*) are given by

$$atc(R,c) \equiv \frac{R(1-s)c + \int_0^s \hat{w}\gamma(c,i)\,\mathrm{d}i}{c},$$

where the fraction of credit purchases (s) is a function of the nominal interest rate (R) and of the amount of goods transacted (c). When an individual faces a nominal interest rate R and purchases with cash a fraction m/c of his total expenditures in consumption goods, average transactions costs are given by

$$atc(R,c) = R\frac{m}{c} + \int_{m/c}^{1} \frac{\hat{w}\gamma(c,1-z)}{c}\,\mathrm{d}z = \int_0^1 \min\left\{R, \frac{\hat{w}\gamma(c,1-z)}{c}\right\}\mathrm{d}z. \tag{2}$$

There is a very simple geometric interpretation for (2) as illustrated in Figs. 1(a) and (b): average transaction costs are given by the area under the inverse money demand function ($A + B$ in Fig. 1(a), and C in Fig. 1(b)). Notice that average

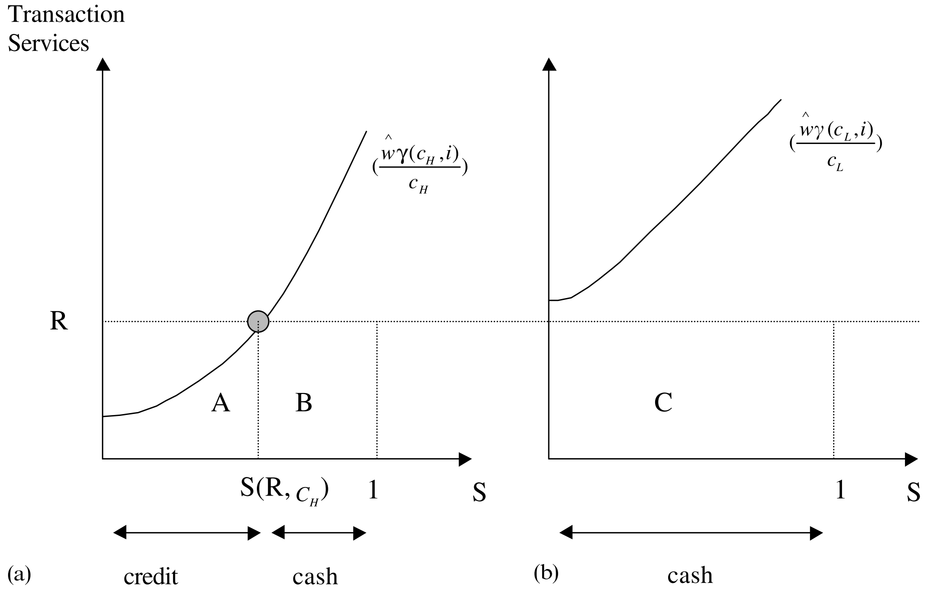

Fig. 1. Average transaction costs and the inverse money demand.

transaction costs are bounded above by the nominal interest rate (R) and they are strictly below R when the fraction of goods purchased with credit is strictly positive.

In the following sections, we consider how average transaction costs vary with different patterns of transactions, depending on whether the transaction technology exhibits economies of scale.

2.2.1. No economies of scale

The transactions technology is said to not exhibit economies of scale when the per unit cost of transacting goods is independent of the volume transacted; more formally, when the transactions technology can be expressed as $\gamma(c, i) = cv(i)$, for all $c \geqslant 0$, $i \in [0, 1]$ and for some function $v(\cdot)$. In this case, the inverse money demand function satisfies

$$R \leqslant \hat{w}v\left(1 - \frac{m}{c}\right), \quad \text{with `` = '' if } s > 0.$$

Denoting by $\varphi(\cdot)$ the inverse of the function $v(\cdot)$, we obtain an expression relating patterns of transactions to the nominal interest rate and the price of financial services

$$\frac{m}{c} = \min\{1 - \varphi(R/\hat{w}), 1\} \equiv \psi(R/\hat{w}). \tag{3}$$

The above expression states that the fractions of consumption goods that are purchased with money and credit are independent of the level of consumption, as in the cases studied by Aiyagari et al. (1998), Schreft (1992), Gillman (1993) and others. As a result, average transaction costs faced by individuals are independent of the amount of goods transacted.

The aggregate money demand, M, is obtained from (3) by solving for m and aggregating across individuals

$$M(R/\hat{w}) = \int c\psi(R/\hat{w})\, \mathrm{d}\Gamma(c) = \psi(R/\hat{w}) \int c\, \mathrm{d}\Gamma(c) = \psi(R/\hat{w})C,$$

where C denotes aggregate or average consumption. It is clear from the above expression that the aggregate money demand is also independent of the distribution of income:

It is interesting to note that we can specify the transactions technology $v(\cdot)$ to obtain individual, and hence aggregate, money demand functions that are commonly used in the empirical literature. For instance, consider the "semi-log specification", $m = cA\mathrm{e}^{-\alpha R}$, where the parameter α represents the interest "semi-elasticity" of money demand. Solving for R we obtain $R = -\log[(m/c)/A]/\alpha$, and substituting into (1), we obtain

$$-\frac{\log[(m/c)/A]}{\alpha} \leqslant \hat{w}v\left(1 - \frac{m}{c}\right), \quad \text{with} = \text{if } s > 0. \tag{4}$$

Assuming (1) holds with equality, Eq. (4) is satisfied when

$$v(i) = -\frac{\log[(1-i)/A]}{\alpha\hat{w}}.$$

Since $v(0) = 0$, as long as $R > 0$, the individual will purchase some goods with credit ($s > 0$).[2]

The "log–log" specification, $m/c = AR^{-\alpha}$, where α represents the interest elasticity of money demand, can be obtained in a similar fashion. Solving for R and using Eq. (1) at equality we obtain

$$R = \left\{\frac{m}{Ac}\right\}^{-1/\alpha} = \hat{w}v\left(1 - \frac{m}{c}\right).$$

The second equality holds when v is defined as

$$v(i) = \kappa \frac{1}{(1-i)^{1/\alpha}},$$

where $\kappa = A^{1/\alpha}/\hat{w}$. Notice that the assumption of no corner solutions ($s > 0$) is valid as long as $cR \geqslant \hat{w}v(0)$; that is, $c \geqslant \hat{w}\kappa/R$. In other words, individuals will transact with credit if the nominal interest rate is not too low or if the volume of transactions is sufficiently large.

2.2.2. Economies of scale

The transactions technology is said to exhibit economies of scale when the per unit cost of transacting consumption goods decreases with the volume transacted. Formally, $\gamma(\lambda c, i) \leqslant \lambda\gamma(c, i)$ for all $\lambda, c \geqslant 0$ and $i \in [0, 1]$. From (1),

$$\frac{m}{c} = \min\{1 - \varphi(R/\hat{w}, c), 1\} \equiv \psi(R/\hat{w}, c).$$

[2] This particular specification of the credit technology is the one used by Aiyagari et al. (1998).

In contrast to an environment with no economies of scale in the transaction technology, the patterns of transactions across individuals now depend on the volume of transactions. Money demand, normalized by consumption, is a decreasing function of the volume of consumption. In other words, individuals with high levels of consumption rely relatively more on credit purchases than individuals with low levels of consumption.

The interest elasticity of money demand is also different across individuals. A given money demand with scale economies in the credit technology, for two levels of consumption ($c_H > c_L$), is illustrated in Fig. 2.

Individuals transacting with money exclusively do not, at the margin, change their patterns of transactions when the nominal interest rate changes ($\psi(R/\hat{w}, c) = 1$ for small changes in R). Society in this case does not face a resource loss when R increases; however, individuals face private resource losses due to the increase in the "inflation tax". On the other hand, society does incur a real resource loss when individuals with higher levels of consumption increase credit transactions in order to avoid the inflation tax.

It is straightforward to verify that average transaction costs are decreasing in the amount of goods transacted. For $\lambda > 1$,

$$atc(R, \lambda c) = \int_0^1 \min\left\{R, \frac{\hat{w}\gamma(\lambda c, 1-z)}{\lambda c}\right\} dz \leqslant \int_0^1 \min\left\{R, \frac{\hat{w}\gamma(c, 1-z)}{c}\right\} dz$$
$$= atc(R, c),$$

with strict inequality when $atc(R, \lambda c) < R$. Hence, when individuals are heterogeneous in their levels of consumption, they face different average transaction costs.

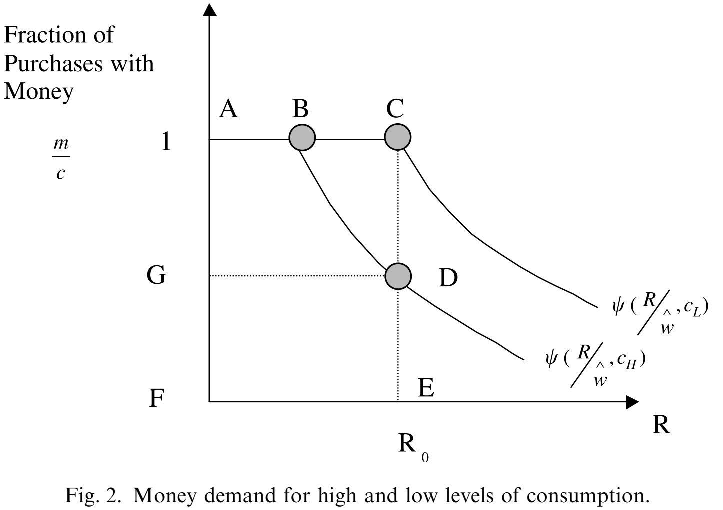

Fig. 2. Money demand for high and low levels of consumption.

Aggregate money demand also depends on the distribution of income when individuals differ in patterns of transactions:

$$M(R,\Gamma) = \int c \min\{1-\varphi(R/\hat{w},c),1\}\,\mathrm{d}\Gamma(c)$$

$$= \int_0^{\underline{c}} c\,\mathrm{d}\Gamma(c) + \int_{\underline{c}}^{\infty} \{1-\varphi(R/\hat{w},c)\}\,\mathrm{d}\Gamma(c),$$

where $\underline{c}$ is such that $\varphi(R/\hat{w},\underline{c}) = 1$. When the interest rate changes, there is a subset of individuals who do not change their money demand, but this subset decreases with the interest rate ($\underline{c}$ is a decreasing function of R).

We can now obtain the money demands implied by alternative credit technologies. For instance, if $\gamma(c,i) = \gamma(i/(1-i))^{\theta}$,

$$\psi(R/\hat{w},c) = \frac{1}{1+[Rc/\hat{w}\gamma]^{1/\theta}}.$$

This transaction technology is considered by Dotsey and Ireland (1996), and is used below in our benchmark specification. We can also specify the transaction technology to obtain an individual money demand function that, as c increases, behaves asymptotically as the constant elasticity or ‘‘log–log’’ specification. Setting $\gamma(c,i) = \underline{\gamma} + c\kappa(1/(1-i)^{1/\alpha})$ we obtain

$$\psi(R/\hat{w},c) = \min\left\{1, \left\{\frac{\kappa}{R/\hat{w}-\underline{\gamma}/c}\right\}^{1/\alpha}\right\}.$$

3. The benchmark economy

In this section we parameterize our benchmark economy and discuss its key properties. The consequences of inflation hinge crucially on two factors: the fraction of transactions made with cash (i.e. the size of the inflation tax base), and the relative responsiveness of the demand for money to changes in the inflation rate. In our economy both factors are determined by the parameterization of the transactions technology.

3.1. Transaction technology

Our strategy is to calibrate our model to match selected statistics on monetary aggregates. Since this is the standard approach in the literature, we can easily relate our results to other papers. We show below that our specification implies sensible cross-sectional statistics for portfolio composition and patterns of transactions.

Table 1

(a) *Calibration of model period*

US data		Model economy	
M1/GNP	M1/GNP (adj.)	M1/GNP quarterly	M1/GNP bimonthly
0.152	0.118	0.109	0.072

(b) *Calibration of transaction technology*

	Semi-elasticity (0–10%)	Cash transaction (%) (5%)	θ	γ
Quarterly specification	5.95	82	0.3232	0.0421
Bimonthly specification	5.95	82	0.3112	0.0279

Following Dotsey and Ireland (1996), the function $\gamma(c,i)$ is independent of the amount transacted (c) and is parameterized as

$$\gamma(c,i) = \gamma\left[\frac{i}{(1-i)}\right]^{\theta}.$$

A virtue of this parameterization is that it matches two important statistics in our model with the data: the interest sensitivity of money demand, measured by the semi-elasticity of money demand, and the fraction of purchases made with credit.

The annual inflation rate for the benchmark economy is set at 5% and our specification of money is M1. Avery et al. (1987, Fig. 1) report that US households performed about 82% of their purchases with M1. We use this figure as the target value for the fraction of transactions made with cash for the benchmark economy to match. Using the velocity equations estimated by Dotsey and Ireland (1996, p. 38), the target semi-elasticity is set to 5.95.[3] We thus search across parameter values for θ and γ so that: (1) the steady-state equilibria of the benchmark economy reproduce the target fraction of transactions made with cash; and (2) the model economy displays the target interest semi-elasticity of money demand for inflation rates between 0% and 10%. The targets and the parameter values selected for the transaction technology are summarized in Tables 1(a) and (b).

3.2. Model period

Given an average fraction of transactions performed with credit, the choice of the model period determines the relation of money to output. We choose the model period in order to generate in our benchmark economy a reasonable inflation tax base. For the period 1980–1996, the ratio of M1/GNP averaged about 0.152 at the

[3] We approximate the inflation semi-elasticity between inflation rates π' and π by $\log(V(\pi')/V(\pi))/(\pi'-\pi)$, where $V(.)$ stands for income velocity.

annual level. This constitutes an upper bound on the size of the inflation tax base, since it is well known that a substantial fraction of US currency is not in the hands of US residents. Sprenkle (1993) estimates that only 21–22% of currency is actually in the US.Taking this into account, the relevant M1/GNP ratio is approximately 0.118. Choosing a model period equal to one quarter generates a M1/GNP of about 0.109. Thus, we select one quarter as the model period for our first specification. Nevertheless, a model period equal to one quarter may still overestimate the base of the inflation tax, as an important fraction of checking deposits pay interest and thus escape part of the inflation tax.[4] For instance, assuming 40% of deposits are not subject to the inflation tax, the M1/GNP ratio is further reduced to 0.074. Based upon these considerations, we also report statistics for a model period equal to two months. For the latter, our model economy generates a M1/GNP ratio of approximately 0.072 at the annual level.

3.3. Preferences, production technology, and government consumption

The production technology for goods is given by a Cobb–Douglas specification with a capital income share of 0.36, as we consider a broad measure of capital. The depreciation rate of capital is selected so that the annual investment to capital ratio is about 0.08. The coefficient of relative risk aversion σ and the discount rate β (at the annual level) are fixed at 2 and 0.96, respectively. Government expenditures are set so that they are 0.195 of GDP in the benchmark economy.[5] The tax rate on capital income is fixed at 0.25.[6] The tax rate on labor income is endogenously determined in equilibrium as that which balances the government budget constraint.

3.4. Labor endowments

Data on the ratio of the average labor income of college graduates relative to non-college graduates for the US economy are used to determine the relative value of the average labor endowment for both types of individuals. We assume that individuals within each group can experience two possible labor income levels: $z_i^1 = \bar{z}_i + \Delta_i$ and $z_i^2 = \bar{z}_i - \Delta_i$, for $i = \mathrm{L}, \mathrm{H}$. Using data from the US Bureau of the Census, we divide the population in two groups according to education levels, and compute the mean labor endowment (labor earnings) for each group ($\bar{z}_i$) and the average fraction of the population that belongs to each group.[7] For the period 1991–1997, the ratio of labor

[4] Avery et al. (1987, p. 183) report that 61% of demand deposits paid no interest in 1986.

[5] Government consumption is defined as federal, state, and local government consumption. See Survey of Current Business 1994, Table 1 and 1995, Table 1.1.

[6] Recall that we are using a broad measure of capital which includes, for instance, consumer durables whose returns are virtually untaxed.

[7] The first group includes those individuals with elementary, high school, and some college education, and accounts on average for 68.966% of the sample in the period considered. The second group contains those individuals in the data with a college level education or higher (31.034% of the sample). Source: US Bureau of Census, Historical Income Tables. Individuals considered are full time, year round, male workers, who are 25 years or older (Table P-23).

Table 2

(a) *Selected statistics* (*Benchmark case*)

	Quarterly specification		Bimonthly specification	
	Type-L	Type-H	Type-L	Type-H
Transactions using cash (%)	87.9	58.8	88.4	58.4
Liquidity ($m/(a+m)$) (%)	7.42	6.16	4.90	3.75
Mean income	0.80	1.45	0.79	1.45
Mean asset holdings	0.80	1.44	0.80	1.44
Mean money holdings	0.94	1.13	0.94	1.13

(b) *Representative agent and heterogeneous agents economies* (*quarterly specification*)

	r	Consumption	Cash use (%)	Finance share (%)	Velocity
Inflation 0%					
Repr. agent	1.01	2.862	98.1	0.02	6.9
Heter. agents	1.00	2.860	98.0	0.02	7.0
Inflation 5%					
Repr. agent	1.01	2.852	81.4	0.18	8.3
Heter. agents	1.00	2.848	78.9	0.24	9.2
Inflation 10%					
Repr. agent	1.01	2.825	54.6	0.63	12.5
Heter. agents	1.00	2.823	58.3	0.66	13.4

earnings of both groups averaged 1.837. Thus we set $\bar{z}_{\rm H}/\bar{z}_{\rm L} = 1.837$. We restrict the transition probabilities to satisfy $p(z_i^1, z_i^1) = p(z_i^2, z_i^2) = \phi$ for $i = {\rm L, H}$. In our benchmark economy we set $\phi = 0.90$ and $\Delta_i = 0.15 \times \bar{z}_i$ for $i = {\rm L, H}$.

3.5. Properties of the benchmark economy

Table 2 reports statistics for our benchmark economy. The capital to output ratios are 2.7 and 2.6, at the annual level, for the quarterly and bimonthly specifications, respectively. These figures are in the range of available measures for the capital to labor ratio in the US economy when a broad notion of capital is used. The fraction of resources devoted to transactions services in the benchmark economy is 0.24% of GDP when the model period is one quarter, and about 0.16% when it equals two months. These fractions are below the 0.5% costs of transacting services estimated by Aiyagari et al. (1998) using data on commercial banks for the US economy. Using the fact that the mean household income in the US economy in 1994 was about \$43,000 as well as a 5% inflation rate, a typical household in our economy is predicted to spend *in a year* about \$105 in transactions services when the model period is a quarter and \$70 when it equals two months. These figures should be interpreted as the cost to a typical household of using credit cards (notice households may well have more than one member, with each of them holding more than one credit card), savings accounts, money market accounts, and the cash management services of a broker. Part of these costs may be non-monetary: individuals spend resources traveling or learning about financial markets as well as complying with government regulations (filing tax forms).[8]

Our benchmark economy also displays heterogeneity in patterns of transactions and portfolio composition that is consistent with the available evidence. We measure credit purchases in the U.S. economy as the fraction of expenditures financed with credit cards and interest bearing deposits, such as savings accounts and money market accounts (MMA). Using data from Avery et al. (1986), we find that the amount of credit purchases increases with the education of individuals: while individuals with less than 12 years of education pay, on average, 8.7% of their transactions with credit, average credit purchases are 14.4% for high school graduates, 21% for individuals with some college education, and 32% for college graduates.[9] We view these figures as lower bounds to the actual amount of credit purchases, especially for individuals with high levels of education. This is because, as reported by Avery et al., high income families are more likely to be paid (some) interest on their checking account balances than low income families. In addition, we believe that high income families are more likely to perform cash management activities that reduce their exposure to the inflation tax per dollar transacted with money. Having said this, our view is that our benchmark economy produces very reasonable transaction patterns: for both choices of the model period, we find that Type-H (college) agents use credit on 41% of their purchases while Type-L (non-college) agents rely on credit on 12% of their transactions.

Table 2 also shows that in our benchmark economy Type-L individuals have a more liquid portfolio of financial assets than Type-H individuals. At a more disaggregated level, the fraction of liquid assets is about 12.0% for the bottom 10% of the overall income distribution and about 2.0%, 1.6%, and 1.1% for the top 10%, 5%, and 1% percentiles of the income distribution, respectively.[10] It is worth emphasizing that these figures are consistent with calculations provided by some authors. Wolff (1983) for instance, using net worth data, finds that the fraction of

[8] For example, using estimates from the IRS of the time cost of preparing and filing Schedule D, Mulligan and Sala-i-Martin (2000, p. 988) calculate an annual cost of \$88 of preparing Schedule D!

[9] The data is taken from Table 1 in Avery et al. (1986). These authors recognize that there is some double counting in their measures of expenditures because credit card balances are paid with checks at the end of the month (p. 92). We then compute purchases with money by adding the share of expenditures with cash and checks drawn from a main checking account minus the share of payments with credit cards. Hence, the fraction of credit purchases is calculated as one minus the fraction of money purchases.

[10] The figures for liquidity reported in Table 2(a) represent the average liquidity across individuals of Type-L and Type-H. That is, $\int [m/(a+m)]\,d\lambda$ for each category of education. This statistic is quite different from $\int m\,d\lambda / \int (m + a\,d\lambda)$ since capital holdings are substantially more concentrated than money holdings in our economy.

currency and demand deposits out of a broad notion of wealth ranged from about 8%, for the poorest households, to less than 1%, for the richest families considered.

4. Findings

In this section, we present findings on the aggregate and individual-level effects of inflation. We are able to replicate, qualitatively and quantitively, the aggregate effects of inflation documented in the literature. However, we also illustrate important distributive implications of inflation: through its role as a regressive consumption tax, inflation affects the welfare and asset holdings across the income distribution differentially. The findings therefore suggest that neglecting the distributional consequences of inflation can lead to an incomplete assessment of the impact of inflation.

4.1. Aggregate impact of inflation

A representative agent economy where individuals are endowed with the average labor endowment of the benchmark economy is considered in Table 2(b). Aggregate consumption, the fraction of transactions performed with cash, the velocity of money, and the share of the finance sector in output vary in a similar way with changes in the inflation rate across the representative agent economy and our benchmark economy with heterogeneous agents. Thus, the introduction of heterogeneity in an economy with costly transaction services does not lead to different predictions regarding the impact of inflation on the aggregate statistics of the economy.

4.2. Inflation as a regressive consumption tax

In our economy, inflation operates as a regressive consumption tax because the extent to which individuals are affected by the inflation tax depends on their level of consumption. While an increase in the inflation rate raises the cost of cash purchases, it does not affect the cost of credit transactions. Since individuals are more likely to transact with credit when their level of consumption is relatively high, an increase in inflation raises the cost of transacting for individuals at the bottom of the income distribution to a greater extent than for those at the top. To illustrate this point we compute the transaction costs, per unit of consumption, faced by Type-L and Type-H individuals in our model economy under different inflation rates. As in Section 2.2, transactions costs are defined as the sum of expenditures in financial services plus the opportunity costs of the money balances held. For each type i, $i = \mathrm{L, H}$, we compute the following statistic

$$\int_{X_i=\{x\in X\,:\,z\in Z_i\}} \frac{\hat{w}S(x) + Rm(x)}{c(x)}\,\mathrm{d}\lambda(x),$$

where $S(x) \equiv \int_0^{g_s(x)} \gamma(j)\,\mathrm{d}(j)$ denotes the amount of credit purchases by an individual in state x, $m(x)$ and $c(x)$ stand, respectively, for money holdings and consumption at x, $\hat{w}$ is the price of credit services, and R is the nominal interest rate. Notice that the numerator in the integrand represents the transaction costs incurred by an individual in state x, both in credit and money purchases. The above expression yields the transaction costs per unit of consumption purchased, averaged across individuals of a given type.

As illustrated in Table 3, average transaction costs are higher for individuals of Type-L than for those of Type-H for any inflation rate. More importantly, the difference between average transaction costs across types grows with the rate of inflation. In our quarterly specification, average transaction costs increase by about 0.01 for Type-L individuals when the inflation rate rises from 5% to 10%, while they only increase 0.005 for those of Type-H. These findings are a clear indication that inflation affects individuals across the income distribution very differently.

Table 3
Distributional effects

	Quarterly specification		Bimonthly specification	
	Type-L	Type-H	Type-L	Type-H
Welfare cost				
Inflation 5%	1.48	− 1.04	1.25	− 0.55
Inflation 10%	2.77	− 1.11	1.86	− 0.45
Welfare cost (fixed distribution)				
Inflation 5%	0.199	0.035	0.203	0.093
Inflation 10%	0.704	0.214	0.539	0.202
Capital holdings				
Inflation 0%	0.88	1.26	0.85	1.33
Inflation 5%	0.81	1.41	0.80	1.44
Inflation 10%	0.78	1.48	0.79	1.46
Consumption				
Inflation 0%	2.317	4.066	2.799	4.999
Inflation 5%	2.272	4.128	2.755	5.037
Inflation 10%	2.234	4.134	2.732	5.032
Credit purchases (%)				
Inflation 0%	0.11	6.1	0.1	6.4
Inflation 5%	12.1	41.1	11.6	41.6
Inflation 10%	28.6	71.1	28.3	71.9
Average transaction costs				
Inflation 0%	0.0100	0.0099	0.0068	0.0068
Inflation 5%	0.0218	0.0196	0.0147	0.0132
Inflation 10%	0.0316	0.0246	0.0212	0.0164
Money demand elasticity				
0–5% inflation	− 0.075	− 0.228	− 0.070	− 0.232
5–10% inflation	− 0.350	− 1.04	− 0.337	− 1.06

4.3. Average transaction costs and patterns of transactions

At the root of the distributional impact of inflation is that the ability to substitute credit for money transactions, as the inflation rate rises, differs substantially across individuals. Since purchasing goods with credit entails a fixed cost, those individuals whose consumption is relatively high, make a higher fraction of their purchases with credit than those whose consumption is low. Table 3 shows that Type-H individuals perform a larger fraction of their transactions using credit than people of Type-L. For the quarterly specification, we have that for the benchmark economy (annual inflation of 5%) Type-H individuals use credit in 41% of their transactions while Type-L individuals rely on credit purchases for about 12% of their transactions. Although both types of individuals increase their credit purchases significantly when the inflation rate rises, high income individuals are better able to avoid the "inflation tax" than low income individuals. In this regard, the results in Table 3 indicate that money demand tends to be more interest-elastic for high versus low income individuals. Consequently, seignorage collected as a fraction of income is significantly smaller for Type-H than for Type-L individuals. For instance, when the annual inflation rate is 10% in the quarterly specification, seignorage collected as a fraction of income for Type-H people is 0.5% while it is 1.26% for individuals of Type-L.

4.4. Inflation and the distribution of capital

Our numerical findings show that inflation can have large effects on the long-run distribution of capital holdings. When the inflation rate increases, Type-H individuals increase their capital holdings relative to individuals of Type-L. Table 3 shows that when the inflation rate increases from 0% to 10% in the quarterly specification, average capital holdings for Type-H individuals increase from 1.26 to 1.48 of the aggregate capital stock in the economy (measured in per capita terms). On the other hand, Type-L individuals reduce their capital holdings from 0.88 to 0.78 of the aggregate capital stock. A similar observation applies for the bimonthly specification (see Table 3).[11] Thus, one interesting implication of our framework is that inflation affects the savings decisions of individuals facing idiosyncratic risk in a non-trivial way through its role as a non-linear consumption tax.

4.5. Consumption and welfare

Inflation has also non-trivial consequences for the steady-state distribution of consumption and welfare. We measure the welfare cost of inflation as the average permanent consumption supplement that makes individuals in the economy with inflation as well off as those in the economy with no inflation, expressed as a fraction

[11] Recent evidence from cross-country data supports the hypothesis that higher inflation rates are associated with higher levels of income concentration. See for instance Easterly and Fischer (2001), Romer and Romer (1998), and Al-Marhubi (1997).

of income. In the case where the model period is one quarter, an increase in inflation from 0% to 10% leads to a decrease in aggregate consumption of goods of about 1.3%. Table 3 shows that this decrease in consumption is not evenly distributed: while individuals of Type-L experience a decrease in consumption of 3.6%, the consumption of Type-H individuals increases by 1.7%. This result explains why the welfare costs of inflation are unevenly distributed across the population. The welfare cost of a 10% inflation rate is equivalent to 2.77% percent of income for Type-L individuals. Alternatively, welfare costs are negative (−1.11%) for individuals of Type-H. Note that these numbers differ substantially from aggregate welfare cost in the economy of 1.57%. Similar conclusions can be drawn for the bimonthly specification of the model economy. These findings clearly illustrate that an analysis focused exclusively on aggregate variables gives an incomplete picture of the effects of inflation in our economy.

The fact that inflation affects differently the well being of individuals across the income distribution does not hinge on the impact of inflation on the long-run distribution of wealth. To emphasize this point, we compute the welfare cost of inflation under the assumption that the distribution of wealth does not change with the inflation rate. That is, we maintain the distribution of wealth of the benchmark economy (inflation rate 5%) as the inflation rate changes.[12] Although Type-H individuals no longer prefer inflation, they do face a welfare cost of inflation that is less than one half of that faced by Type-L individuals (see Table 3). A simple back of the envelope calculation reveals that the difference in welfare costs is well approximated by the difference in average transaction costs across the two types of agents. We therefore conclude that it is the regressive nature of the inflation tax, not the effect of inflation on the wealth distribution, that plays a crucial role in generating the heterogeneous impact of inflation on the welfare of individuals.

When individuals differ in patterns of transactions, as Attanasio et al. (1998) and Mulligan and Sala-i-Martin (2000) have emphasized, the measurement of the welfare cost of inflation can be biased if time series estimates of the interest rate elasticity of money demand are used. Using household-level Italian data, Attanasio et al. (1998) group households according to the type of financial services they use and estimate different money demand functions for each group. The welfare cost of inflation is measured as the weighted sum of the areas under the estimated demand functions. This procedure gives an unbiased estimate of the resources wasted economizing on cash use; however, it can be misleading for analyzing the distributional impact of inflation. In some cases, the authors find that households with higher interest elasticities of money demand, typically high income households, face higher welfare costs of inflation than those with lower interest elasticity of money demand. In contrast, our numerical simulations indicate that those households who have a higher interest elasticity of money demand, and thus, a bigger "welfare" triangle under their money demand function, are the ones who suffer *less* from inflation. To

[12] More precisely, we fix the steady-state distribution of savings in the benchmark economy and we allow agents to choose the allocation of these savings optimally between money and capital as the inflation rate varies.

Table 4
Role of labor income taxation (quarterly case)

	Revenue neutrality		No revenue neutrality	
	Type-L	Type-H	Type-L	Type-H
Welfare cost				
Inflation 5%	1.48	− 1.04	2.09	− 0.25
Inflation 10%	2.77	− 1.11	2.98	− 0.70
Welfare cost (fixed distribution)				
Inflation 5%	0.199	0.035	0.869	0.719
Inflation 10%	0.704	0.214	1.584	1.100
Capital holdings				
Inflation 0%	0.88	1.26	0.88	1.27
Inflation 5%	0.81	1.41	0.81	1.41
Inflation 10%	0.78	1.48	0.79	1.48

put it differently, the deadweight loss of inflation is not borne by high income households.

4.6. Role of labor income taxation

The experiments examined above are revenue neutral in the sense that labor income taxes are adjusted so that the government budget constraint is still balanced when the inflation rate changes. Since Type-H individuals are endowed with significantly more labor income than Type-L individuals, the reader may suspect that Type-H individuals are the primary beneficiaries from a reduction of labor income taxation. In order to evaluate this statement we perform an experiment where the labor income tax rate is held constant as the inflation tax rate varies and report the findings for the quarterly specification in Table 4.[13] The results indicate that the distributional impact of inflation does not depend on whether labor income taxes are adjusted to balance the government budget.[14] Thus, the distributive effects are driven by the non-linear nature of the inflation tax.

4.7. Inflation and self-insurance

In an important paper, Imrohoroglu (1992) finds inflation has non-trivial costs in an environment where individuals use a single asset (money) to smooth out uninsurable fluctuations in labor productivity. As money is a store of value in this environment, inflation acts as a tax on consumption smoothing. Imrohoroglu (1992)

[13] The tax rate on labor income is fixed at the level of our benchmark economy (inflation equal to 5%). Since labor and capital income taxes are fixed, government revenues vary with changes in the inflation rate.

[14] We have also performed experiments where the tax rate on *income* (i.e. capital and labor income) is changed with the inflation rate in order to balance the government budget constraint. The findings are that the distributive effects of inflation are quite similar to the ones described in our benchmark experiments.

illustrates this point by showing that the cross-sectional standard deviation of consumption increases with inflation. In contrast, changes in the aforementioned statistic are negligible in the economy we study. Money is a poor store of value relative to capital since it is dominated in rate of return. Individuals hold money to perform transactions: they prefer to use capital for self-insurance. We therefore conclude that the effect of increases in the inflation rate on the ability of individuals to self-insure is not quantitatively important in the presence of a second asset which dominates money in rate of return.

5. Alternative transaction technologies

In the previous section, it was assumed the cost of purchasing goods with credit was independent of the amount transacted. Our numerical experiments indicate that inflation can have an important distributional impact when there are scale economies in credit transactions. In this section, we study the robustness of this finding to alternative specifications of the credit technology. Two questions motivate this exercise. First, to what extent do our distributional findings depend on a technology that features fixed costs but no variable costs? Second, what is the distributional impact of inflation in a world where there is a large fraction of households who only transact with money? The second question is motivated by cross-sectional evidence on the use and ownership of financial instruments that suggests a substantive fraction of US households do not use a checking account or credit cards to perform transactions. Using the Survey of Consumer Finances, Kennickell et al. (1997) report that the fraction of families without a checking account was 18.9% in 1989, 16.5% in 1992, and 15.1% in 1995, and that the group without checking accounts was comprised disproportionally of low income families. Similarly, Mulligan and Sala-i-Martin (2000) document that about 25% of households in the 1989 Survey of Consumer Finances do not have a checking account with a positive balance. Using data from the Survey of Currency and Transaction Accounts Usage, Avery et al. (1987) report that approximately 46% of households in their sample did not use a credit card. To answer the second question, we consider a transaction technology capable of generating a substantial fraction of agents who use only money as a means of payment.

Two main conclusions emerge from this exercise. First, the distributional impact of inflation is significant as long as the credit technology features sufficiently large-scale economies, regardless of whether credit costs have a proportional component or not. Moreover, in line with the analytical results of Section 2.2, in the absence of scale economies in credit transactions, inflation does not have a distributional impact. However, the model economy is inconsistent with the cross-sectional evidence on portfolio holdings and patterns of transactions in this instance. Second, when scale economies are sufficiently large, and yet transaction costs are empirically plausible, a significant fraction of individuals use money exclusively as a means of payment. In this case, the distributional impact of inflation is even larger than in our benchmark economy.

For ease of exposition we consider two sets of experiments. First, we examine variations of the transactions technology used in our benchmark economy that allow for fixed costs, variable costs, and a combination of the two. Second, we determine whether our findings are robust to alternative functional forms for the transactions technology.

5.1. Experiment 1: fixed costs, variable costs, and fixed and variable costs

We compare three specifications for the transactions technology. The first specification corresponds to that of our benchmark economy. In the second specification, there are no economies of scale in credit purchases: purchasing c units of good i with credit requires $\gamma(c,i) = cv(i)$ units of financial services. The third specification features both fixed and variable costs: purchasing c units of good i with credit requires an amount $cv(i) + \underline{\gamma}$ of financial services. A virtue of the third formulation is that the parameter $\underline{\gamma}$ determines the importance of the scale economies associated when purchasing with credit and allows us to control the fraction of agents who only transact with money in a simple way. In the language of Mulligan and Sala-i-Martin (2000), $\underline{\gamma}$ is the parameter controlling the importance of the "extensive margin" in money demand.

The function $v(\cdot)$, in the last two specifications, is parameterized as $v(i) = \gamma(i/(1-i))^{\theta}$ so that the interest sensitivity of the aggregate money demand and the fraction of purchases made with money in our model economy match the data on these dimensions (see Section 3). We select $\underline{\gamma}$ in the third specification so that about 46% of the population does not transact with credit.[15] Table 5(a) presents the parameter values selected for each specification.

5.1.1. Findings

The impacts of inflation for the three specifications of the transactions technology are compared in Table 5(b). An important observation is that when there are no scale effects in the credit technology (second specification), inflation does not have a distributional impact. Under this formulation there is virtually no heterogeneity in portfolio holdings and patterns of transactions across individuals. As there are no scale economies in the use of credit, the welfare costs of inflation are the same across types and inflation does not have a significant impact on the distribution of assets. Inflation has the highest distributional impact under the third specification of the credit technology, where credit transactions exhibit the highest economies of scale because the fixed cost is bounded away from zero for all goods.

[15] According to Avery et al. (1987), the fraction of agents who report ownership and usage of credit cards is given by 0.71×0.76. If we interpret credit cards as the only alternative means of payment to money, about 46% of households transact only with money.

Table 5

(a) *Parameterization of credit technologies*

	Benchmark	No scale effects	Fixed and variable costs
Credit services (per unit of good i)	$\gamma(i/(1-i))^{\theta}/c(i)$	$\gamma\left(\frac{i}{1-i}\right)^{\theta}$	$\gamma\left(\frac{i}{1-i}\right)^{\theta}+\frac{\underline{\gamma}}{c(i)}$
γ	0.0421	0.015	0.0098
θ	0.3232	0.3272	3.2319
$\underline{\gamma}$	n.a.	n.a.	0.0226
Transactions w/credit by 46th percentile (inflation 5%)	10.6	17.5	0.18

(b) *Distributional effects* (*alternative transactions technology*)

	Variable costs		Benchmark case		Fixed and variable costs	
	Type-L	Type-H	Type-L	Type-H	Type-L	Type-H
Welfare cost						
Inflation 5%	0.4	0.4	1.48	− 1.04	2.64	− 1.96
Inflation 10%	1.1	1.1	2.77	− 1.11	4.00	− 2.13
Welfare cost (fixed distribution)						
Inflation 5%	—	—	0.199	0.035	0.104	0.056
Inflation 10%	—	—	0.704	0.214	0.683	0.305
Assets						
Inflation 0%	0.79	1.45	0.88	1.26	0.94	1.13
Inflation 5%	0.79	1.45	0.81	1.41	0.81	1.41
Inflation 10%	0.79	1.45	0.78	1.48	0.78	1.47
Liquidity (%)						
Inflation 0%	8.40	8.40	7.22	9.51	6.48	10.51
Inflation 5%	7.25	7.25	7.42	6.16	7.82	5.65
Inflation 10%	5.25	5.25	6.85	3.49	5.69	4.79
Credit purchases						
Inflation 0%	0.18	0.18	0.11	6.1	0.09	0.22
Inflation 5%	18.9	18.9	12.1	41.1	5.32	42.18
Inflation 10%	45.6	45.6	28.6	71.1	42.28	48.70
Avg. trans. costs						
Inflation 0%	0.0100	0.0100	0.0100	0.0099	0.0099	0.0099
Inflation 5%	0.0213	0.0213	0.0218	0.0196	0.0223	0.0193
Inflation 10%	0.0296	0.0296	0.0316	0.0246	0.0311	0.0258

5.2. Experiment 2: log–log transaction technology with fixed costs

In this experiment we consider a transactions technology that generates a log–log money demand. This case is particularly interesting because, as shown by Lucas

(2000), the log–log money demand with elasticity equal to 1/2 fits US aggregate time series well. In Section 2, we showed that the log–log money demand is obtained in our model economy when the transactions technology is specified as $\gamma(c, i) = \gamma[1/(1-i)^2]c$. All costs are proportional in this formulation; therefore, we also consider a fixed cost $\underline{\gamma}$ in order to generate heterogeneity in patterns of transactions. We consider three possible values for the fixed costs parameter: $\underline{\gamma} = 0.0,\ 0.01$, and 0.0226. For each specification, we choose the parameter γ so that the fraction of transactions performed with credit is 0.18. The parameter values used in the experiment are reported in Table 6(a).

5.2.2. Findings

As illustrated in Table 6(b), inflation does not have any distributional implications in an environment without economies of scale in transactions (see the results for $\underline{\gamma} = 0.0$). When $\underline{\gamma} = 0.01$, the distributional effects of inflation are quite large, and when $\underline{\gamma} = 0.0226$ the largest distributional impact of all the experiments reported in the paper is obtained. Using the fact that the mean household income in the US economy in 1994 was about \$43,000, a typical household in our economy with $\underline{\gamma} = 0.01$ under a 5% inflation rate is predicted to spend \$102 in transaction services during a year (\$42 in fixed transaction costs). When $\underline{\gamma} = 0.0226$ and the inflation rate is 5%, households spend on average \$118 on transaction services per year (\$96 in fixed transaction costs). Thus, as in the benchmark economy, the distributional impact of inflation does not hinge on implausibly large costs of conducting credit transactions.

6. An economy with no income risk

In this section, we study the distributive impact of inflation in an economy where agents do not face uninsurable income risk: individuals are heterogeneous because they are endowed with different amounts of labor and capital. Contrary to the economy with uninsurable income risk, we find that inflation has only negligible effects on the distribution of wealth. We also find, however, that the burden of inflation is unevenly distributed across individuals. We conclude that it is the regressive nature of the inflation tax, not the effect of inflation on the wealth distribution that plays a crucial role in determining the distributional impact of inflation on welfare.

6.1. Parameterization

We consider an economy with two types of individuals which, as in our benchmark economy, we interpret as individuals with college education (Type-H) and non-college education (Type-L). A crucial difference from our baseline economy is that individuals do not face uninsurable income risk. We use data to parameterize

Table 6

(a) *Log–log specification of credit technology*

	No scale effects	"Small" fixed costs	"Large" fixed cost
Credit services (per unit of good i)	$\gamma \frac{1}{(1-i)^{\theta}}$	$\gamma \frac{1}{(1-i)^{\theta}} + \frac{\chi}{c(i)}$	$\gamma \frac{1}{(1-i)^{\theta}} + \frac{\chi}{c(i)}$
γ	0.00614	0.003434	0.0005899
θ	2	2	2
χ	0	0.0100	0.0226
Transactions w/credit by 46th percentile (inflation 5%)	18.6	12.1	0.51

(b) *Distributional effects* (*log–log transactions technology*)

	Variable costs		Small fixed costs ($\chi = 0.01$)		Large fixed costs ($\chi = 0.0226$)	
	Type-L	Type-H	Type-L	Type-H	Type-L	Type-H
Welfare cost						
Inflation 5%	0.401	0.401	1.60	− 0.78	2.28	− 1.28
Inflation 10%	0.710	0.710	2.13	− 0.31	4.44	− 1.72
Welfare cost (fixed distribution)						
Inflation 5%	—	—	0.250	0.148	0.387	0.143
Inflation 10%	—	—	0.467	0.276	0.999	0.456
Assets						
Inflation 0%	0.79	1.45	0.86	1.31	0.94	1.14
Inflation 5%	0.79	1.45	0.80	1.45	0.84	1.37
Inflation 10%	0.79	1.45	0.80	1.45	0.78	1.48
Credit purchases						
Inflation 0%	0.12	0.12	0.11	0.14	0.09	0.17
Inflation 5%	18.8	18.8	14.6	28.7	1.97	58.5
Inflation 10%	33.2	33.2	40.3	45.6	57.3	73.6
Avg. trans. costs						
Inflation 0%	0.0099	0.0099	0.00996	0.00996	0.00993	0.00993
Inflation 5%	0.0217	0.0217	0.0222	0.0211	0.0224	0.0194
Inflation 10%	0.0305	0.0305	0.0307	0.0285	0.0311	0.0232

their labor endowment and capital holdings.[16] To this end, we use estimates of the distribution of net worth by education level provided by Kennickell et al. (1997) for 1992. The transaction technology is the one used earlier for the benchmark case, and

[16] Notice that the steady-state equilibrium aggregates of an economy with no idiosyncratic risk are consistent with a continuum of distributions of asset holdings. The distribution of money holdings is determined uniquely by the steady-state equilibrium conditions once the distributions of labor endowment and asset holdings are given.

the parameters γ and θ are calibrated as in the economy with uninsurable risk.[17] The rest of the parameters are the same as in previous sections.

6.2. Inflation as a non-linear consumption tax

The following numerical experiment illustrates interesting properties of the inflation tax. We compute, for a fixed distribution of capital holdings, steady-state equilibria for a large number of inflation rates. Average transaction costs, for each type of consumer, as a function of the nominal interest rate in the economy, are presented in Fig. 3(a). As expected, average transaction costs increase with the nominal interest rate. More importantly, average transaction costs increase more rapidly for Type-L than for Type-H individuals, which explains why the burden of inflation is unevenly distributed. Changes in the per unit cost of transacting with the volume of consumption are illustrated in Fig. 3(b) (for annual inflation rates of 0% and 10%). Notice that when the inflation rate is 10%, average transaction costs decrease quite dramatically with the volume of consumption, while for an inflation rate of 0% average transaction costs are almost constant as consumption increases. It follows that scale economies in credit transactions combined with a sufficiently high inflation rate can lead to large differences in transacting costs as consumption changes.

6.3. Welfare

We compute the welfare effects of permanent and unexpected increases in inflation from an annual rate of 0% to 10%. The initial steady-state equilibrium is parameterized as described above. A similar exercise is conducted, where the inflation rate changes from 0% to 5%. The welfare costs of changes in the rate of inflation are presented in Table 7 taking into account the transitional dynamics between steady states. The welfare cost of a 10% annual inflation rate for Type-H individuals is about a half of that for Type-L individuals (0.55% versus 1.03% for the quarterly specification and 0.62% versus 0.37 when the period is two months). We conclude that the burden of inflation is far from being evenly distributed across individuals. [18]

6.4. Inflation and the distribution of capital

A striking difference between an economy with no income risk and an economy with uninsurable income risk is that in the former economy inflation has only a negligible effect on the distribution of capital holdings (compare the findings in this section with the ones in Sections 4 and 5). This observation can easily be understood

[17] The resulting values are $\theta = 0.3277$ and $\gamma = 0.0450$ for the quarterly specification, and $\theta = 0.3255$ and $\gamma = 0.0298$ for the bimonthly specification.

[18] Notice that the numbers reported for welfare costs are substantially higher than the ones obtained with a fixed distribution of capital holdings in the economy with uninsurable idiosyncratic risk.

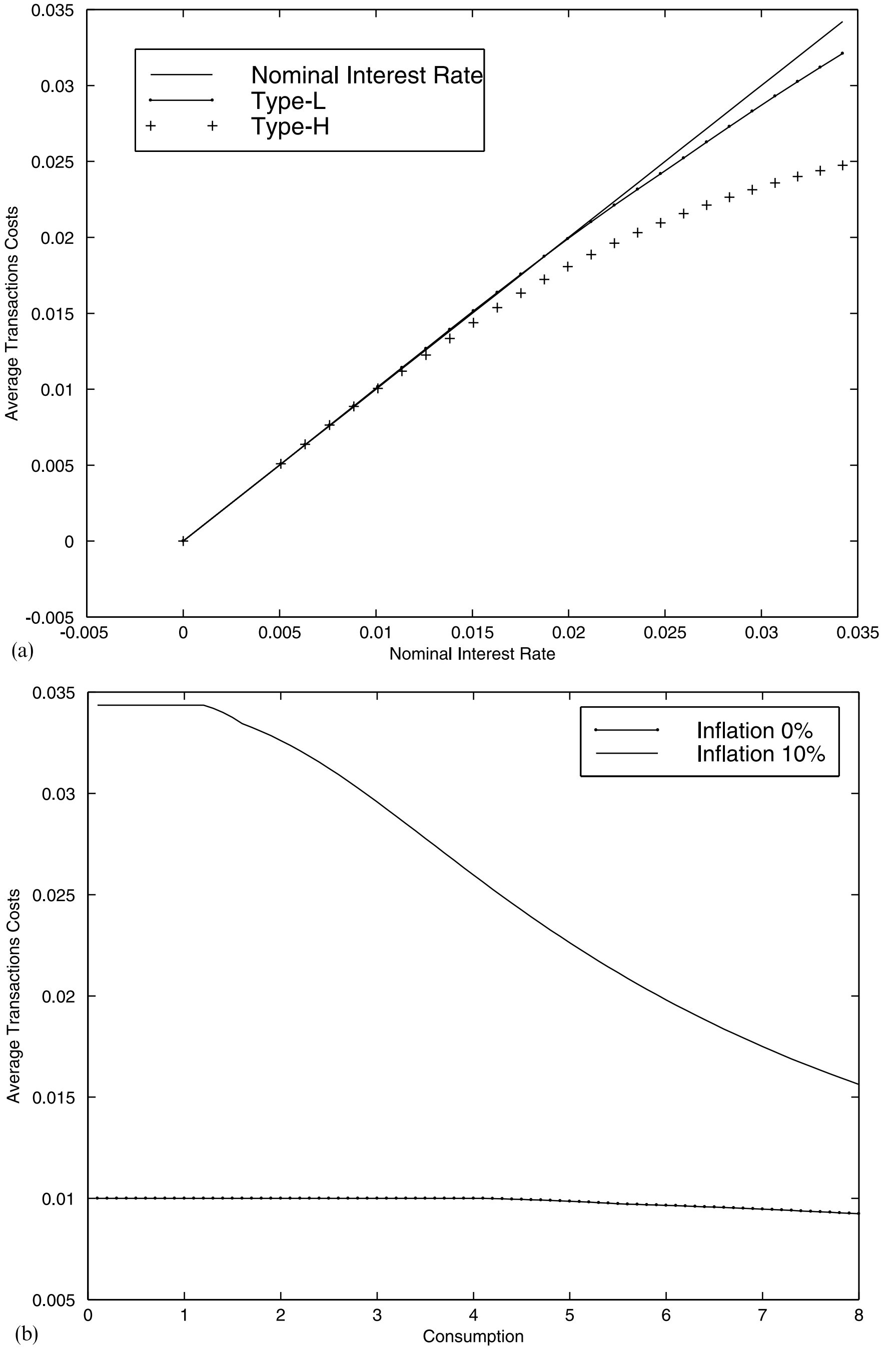

Fig. 3. (a) Average transaction costs and inflation. (b) Inflation as a regressive consumption tax.

Table 7
No income risk

	Quarterly specification		Bimonthly specification	
	Type-L	Type-H	Type-L	Type-H
Welfare cost (w/ transition)				
Inflation 5%	0.411	0.282	0.214	0.139
Inflation 10%	1.026	0.544	0.621	0.366
Welfare cost (steady-state)				
Inflation 5%	0.417	0.287	0.247	0.180
Inflation 10%	1.176	0.693	0.715	0.459
Capital holdings				
Inflation 0%	0.618	1.848	0.618	1.848
Inflation 5%	0.612	1.862	0.615	1.856
Inflation 10%	0.608	1.869	0.612	1.862
Consumption				
Inflation 0%	2.178	4.372	2.732	5.492
Inflation 5%	2.166	4.361	2.723	5.480
Inflation 10%	2.144	4.338	2.705	5.460
Credit purchases (%)				
Inflation 0%	0.7	5.8	0.70	5.70
Inflation 5%	7.6	41.0	7.6	41.1
Inflation 10%	22.8	71.4	22.9	71.8
Average transaction costs				
Inflation 0%	0.0102	0.0101	0.0068	0.0067
Inflation 5%	0.0221	0.0196	0.0147	0.0130
Inflation 10%	0.0321	0.0247	0.0213	0.0164
Money demand elasticity				
0–5% inflation	− 0.038	− 0.232	− 0.037	− 0.233
5–10% inflation	− 0.289	− 1.076	− 0.287	− 1.090

by noticing that a non-linear consumption tax (e.g. inflation) affects savings behavior when individuals face income risk but not otherwise. To this end, consider the Euler equation determining capital accumulation for an individual facing a risky income process, constant interest rate r, and a consumption tax schedule $\tau(c)$:

$$u'(c_t)[1+\tau'(c_t)]^{-1} \geqslant \beta E\{u'(c_{t+1})[1+\tau'(c_{t+1})]^{-1}[1+r]\},\quad \text{with } = \text{ if } a_{t+1} > 0,$$

where $\tau'(c)$ is the derivative of the tax function and E denotes the expectation operator (conditional on date-t information). Notice that $\tau'(c_{t+1})$ represents the *marginal* date $t+1$ tax rate on consumption and is a random variable. As the Euler equations indicates, savings behavior is affected by the probability distribution of the ratio $(1+\tau'(c_t))/(1+\tau'(c_{t+1}))$. This ratio, which affects the relative price of consumption at date $t+1$ in terms of consumption at date t, varies across individuals with different realizations of the income shock (if shocks are Markov) and wealth levels. As a result, savings behavior also varies among individuals. This observation explains the important effects of inflation on the distribution of wealth reported in Sections 4 and 5.[19] In an economy with no income risk, however, the aforementioned ratio is equal to 1 in steady-state because consumption is constant along time. In this case, a non-linear consumption tax schedule affects consumption levels but not savings behavior. Individuals behave like permanent income consumers: they save a constant fraction of their wealth and, since individuals with different wealth face different after tax prices of consumption, they consume a different amount per dollar of expenditure. It follows that the (negligible) effects of inflation on the concentration of assets reported on Table 7 are associated with the transitional dynamics between steady states.

7. Concluding remarks

Our model, while consistent with aggregate effects reported in the literature, provides new insights regarding the distributional effects of inflation. As is well known, inflation represents a flat tax on monetary transactions. In order to evade this tax, individuals purchase some goods with credit if doing so reduces per unit transaction costs. In the presence of economies of scale in credit purchases, those individuals with higher levels of consumption face lower transaction costs. As a result, the welfare cost of inflation may be substantially higher for low income individuals relative to their high income counterparts. In this regard, inflation serves as a non-linear tax on consumption within our framework.

To conclude, we mention our plans for future research. We plan to study the implications of alternative transaction technologies for the demand of money at low interest rates. This is an interesting issue, for the welfare costs of inflation hinge on the interest elasticity of money demand and little is known about the magnitude of this elasticity at low inflation rates. As raised by Mulligan and Sala-i-Martin (2000), the practice of fitting money demand curves to aggregate data and extrapolating the fitted curve to low inflation rates does not seem appropriate.

Second, we also plan to investigate the impact of variable inflation rates in a framework with costly credit transactions. Economists have long argued that the variability of inflation should increase the burden of inflation substantially. To date, the available studies of variable inflation rates have not confirmed this conjecture. Finally, our findings suggest that the study of the distributional impact of high inflation rates is a question that deserves attention.

[19] Even though in our computational experiments we found, for all calibrations of the transactions technology, that inflation leads to an increase in the concentration of wealth, there is no theoretical presumption for this result. In other words, theory does not imply that the random variable $(1+\tau'(c_t))/(1+\tau'(c_{t+1}))$ will be systematically higher for Type-H individuals than for Type-L individuals.

Appendix A

A.1. Computation of the model with uninsurable idiosyncratic risk

Computation of steady-state equilibrium involves solving two equations in two unknowns: we search for capital to labor ratios in the goods sector and a labor income tax rate so that, after aggregating individuals' decisions, we obtain that the goods market clears and that the government budget constraint is balanced. We formulate the individual decision problem as a two stage problem. In the first stage, individuals decide how much wealth to accumulate. In the second stage, they solve the portfolio problem of allocating wealth between capital and money. The advantage of this approach is that the intertemporal decision problem will only have one endogenous state variable (wealth, instead of capital *and* money).

The state variables in the first stage problem are given by wealth, ϖ, the current productivity shock, z_1, and the last period productivity shock, z_0. Individuals also take as given two portfolio functions, g_a and g_m, that give the current capital and money holdings as a function of the current wealth and past productivity shock; that is, $a = g_a(\varpi, z_0)$, and $m = g_m(\varpi, z_0)$. Solving the first stage problem we obtain a wealth accumulation policy of the form $\varpi' = g_\varpi(\varpi, z_0, z_1)$:

$$v(\varpi, z_0, z_1; g_a, g_m) = \max_{c,s,\varpi'}\left\{u(c) + \beta\sum_{z_2} \Pi(z_1, z_2)v(\varpi', z_1, z_2; g_a, g_m)\right\},$$

$$c(1-s)\leqslant m,$$

$$c + q\int_0^s \gamma(c, i)\,\mathrm{d}i + \varpi' = (1+r)a + m + wz_1,$$

where $a = g_a(\varpi, z_0)$, $m = g_m(\varpi, z_0)$. In the portfolio problem, we consider an individual who has decided to accumulate wealth ϖ for the next period and faces a relative price between tomorrow's real money holdings and capital holdings of $1+\pi$. The individual uses information about the current realization of the productivity shock (z_0) and the wealth accumulation g_ϖ in order to forecast next period consumption. The state variables are thus (ϖ, z_0). The optimal portfolio of capital and money is denoted by g_a and g_m, respectively, and is obtained from the following problem:

$$(g_a(\varpi, z_0; g_\varpi), g_m(\varpi, z_0; g_\varpi)) = \arg\max_{a,m}\ \beta U(a, m; \varpi, z_0, g_\varpi) \quad \text{(A.1)}$$

$$\text{s.t.}\quad (1+\pi)m + a = \varpi, \quad \text{(A.2)}$$

where the indirect utility function U is defined as

$$U(a, m; \varpi, z_0, g_\varpi) \equiv \max_{\{c(z_1), s(z_1)\}} \sum_{z_1} \Pi(z_0, z_1)u(c(z_1)) \quad \text{(A.3)}$$

$$\text{s.t.}\quad c(z_1)(1 - s(z_1))\leqslant m, \quad \text{(A.4)}$$

$$c(z_1) + q\int_0^{s(z_1)} \gamma(c(z_1), i)\,\mathrm{d}i + \varpi'(z_1) = (1+r)a + m + wz_1, \tag{A.5}$$

where

$$\varpi'(z_1) = g_{\varpi}(\varpi, z_0, z_1), \quad \text{for all } z_1 \in Z. \tag{A.6}$$

Notice that a and m are non-contingent on z_1. The maximization problem defining U can be subdivided in many independent subproblems: one static optimization problem for each value of z_1. These problems are quite easy to solve: if the CIA constraint binds, we solve two equations in two unknowns (find $c(z_1)$ and $s(z_1)$ so that the CIA and the budget constraint are satisfied); if the CIA constraint does not bind, we set $s(z_1) = 0$ and obtain $c(z_1)$ from the budget constraint. Furthermore, it can be shown that the CIA binds if $wz_1 + a(1+r) - \varpi'(z_1) \geqslant 0$, and it does not bind otherwise.

Summarizing, the algorithm to solve the individual's problem consists in the following steps:

1. Guess v and g_{ϖ}.
2. Compute g_a and g_m as indicated in Eqs. (1)–(4), (A.1) and (A.2).
3. Compute new guesses $T_v v$ and $T_g g_{\varpi}$ for v and g_{ϖ} from the functional equations defining these functions. We evaluate the distance between two functions as follows: $d(T_v v, v) = \|T_v v - v\|$, $\|\cdot\|$ denotes the sup norm. If $\max\{d(T_v v, v), d(T_g g, g_{\varpi})\} < \varepsilon$, the optimization problem is solved; otherwise, set $v = T_v v$ and $g_{\varpi} = T_g g_{\varpi}$ and go to (2) until convergence is achieved.

With the decision rules at hand, we perform the aggregation step by simulating all the variables of interest for a sample of 4000 individuals for 2000 model periods.

A.2. *Computation of transitional dynamics: model with no uninsurable idiosyncratic risk*

We assume that at date $t = 1$ the money supply growth rate changes to a new level g and that the economy reaches a new steady-state equilibrium at date T. Below we show that computing the transition path involves solving a system of $3nT - 2$ unknowns and equal number of equations, where n is the number of individual types. We use a Newton-based routine to find a numerical solution to this system of equations. Since the computational burden of computing long transitions with heterogeneous agents can be quite high, we follow Mercenier and Michel (1994) in using a time aggregation method which reduces such a burden significantly. This method is briefly explained below after developing the equilibrium conditions conforming the $3nT - 2$ equations.

A.2.1. *Equilibrium as a system of $3nT - 2$ equations*

We express nominal variables as a fraction of the aggregate money stock. We thus define $m_t^j = \hat{m}_t^j / M_t$ and $p_t = \hat{p}_t / M_t$, for all t and j, where $\hat{m}_t^j$ denotes nominal money holdings of individual j, and $\hat{p}_t$ the price level. Then, the maximization problem of a type j individual can be written as

$$\max_{\{c_t^j, s_t^j, a_{t+1}^j, m_t^j\}_{t=0}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t)$$

subject to

$$c_t^j(1 - s_t^j) \leqslant \frac{m_{t-1}^j}{p_t(1+g)},$$

$$c_t^j + a_{t+1}^j K_{t+1} + \frac{m_t^j}{p_t} + \hat{w}_t \Gamma(s_t^j) = w_t z^j + (1 + r_t) a_t^j K_t + \frac{m_{t-1}^j}{p_t(1+g)},$$

$$a_1^j \text{ and } m_0^j \text{ given,}$$

where $\Gamma(s_t^j) = \int_0^{s_t^j} \gamma(c_t^j, i)\,\mathrm{d}i$ denotes the amount of transaction services purchased at date t, a_t^j denotes capital holdings normalized by the aggregate per capita capital stock K_t, and g is the money growth rate. We associate the Lagrange multipliers $\beta^t \lambda_t^j$ and $\beta^t \phi_t^j$ to the date-t CIA and budget constraints, respectively. The FOC imply

$$(c_t^j)^{-\sigma} = \lambda_t^j + \phi_t^j(1 - s_t^j),$$

$$\phi_t^j c_t^j \leqslant \lambda_t^j \hat{w}_t \gamma(s_t^j), \quad \text{with} \ = \ \text{if} \ s_t^j > 0,$$

$$(1 + r_{t+1}) \lambda_{t+1}^j \beta = \lambda_t^j,$$

$$\beta(\lambda_{t+1}^j + \phi_{t+1}^j) = \lambda_t^j \frac{p_{t+1}}{p_t}(1+g),$$

$$c_t^j(1 - s_t^j) \leqslant \frac{m_{t-1}}{p_t(1+g)} \quad \text{with} \ = \ \text{if} \ \phi_t^j > 0.$$

Combining these conditions in an appropriate way, we obtain (notice that the budget constraint and the transversality conditions on money and capital holdings are also necessary conditions for optimization)

$$\lambda_t^j = (c_t^j)^{-\sigma} - \frac{(c_t^j)^{-\sigma} \hat{w}_t \gamma(s_t^j)}{c_t^j + (1 - s_t^j) \hat{w}_t \gamma(s_t^j)}(1 - s_t^j),$$

$$\phi_t^j = \frac{(c_t^j)^{-\sigma} \hat{w}_t \gamma(s_t^j)}{c_t^j + (1 - s_t^j) \hat{w}_t \gamma(s_t^j)},$$

$$s_t^j = \max\left\{1 - \frac{m_{t-1}^j}{c_t^j p_t (1+g)}, 0\right\},$$

and we set $s_t^j = 0$ and $\lambda_t^j = (c_t^j)^{-\sigma}$ when $\phi_t^j = 0$.

The government budget constraint at date t is given by $G = \tau^k \hat{r}_t K_t + \tau_t^l \hat{w}_t L + g/(1+g)p_t$, where L denotes the (constant) aggregate labor supply. We denote by $k_t = K_t/_{L_t^g}$ the capital to labor ratio in the goods production sector.

From the firms' FOC we know that the date t before tax rental prices of capital and labor services are determined by k_t. We thus express $\hat{r}_t = \hat{r}(k_t)$ and $\hat{w}_t = \hat{w}(k_t)$ for all t and some functions $\hat{r}$ and $\hat{w}$ which are independent of time. Notice that s_t^j can be expressed as a function $s^j(c_t^j, p_t, m_{t-1}^j)$. Then, the market clearing condition for labor implies that $L = L^g_t + \sum_{j=1}^n \mu_j \Gamma(s^j(c^j_t, p_t, m^j_{t-1}))$, where μ_j denotes the fraction of individuals of type j. Using the government budget constraint and the market clearing condition for labor services, we can express labor income taxes as $\tau^l_t = \tau^l(c_t, p_t, m_{t-1}, k_t)$ for some function τ^l, where we have also made use of the fact that $K_t = k_t L^g_t$, and of the compact notation $c_t = (c^1_t, \ldots, c^n_t)$ and $m_{t-1} = (m^1_{t-1}, \ldots, m^n_{t-1})$. As a result, after tax factor income prices can be written as $w_t = w(c_t, p_t, m_{t-1}, k_t)$ and $r_t = r(c_t, p_t, m_{t-1}, k_t)$. Then, it is also true that $\lambda^j_t = \lambda^j(c_t, p_t, m_{t-1}, k_t)$ and $\phi^j_t = \phi^j(c_t, p_t, m_{t-1}, k_t)$.

Since by assumption, the economy reaches the steady-state in period T, the unknowns are given by nT values of consumption $\{c^j_1, \ldots, c^j_T\}_{j=1}^n$, $T-1$ capital to labor ratios $\{k_1, \ldots, k_{T-1}\}$ (notice that at date T the pre-tax return on capital, and thus the capital to labor ratio, is pinned down by the rate of time preference), $(T-1)(n-1)$ values of normalized capital holdings $\{a^j_2, \ldots, a^j_T\}_{j=2}^n$ (notice that the date 1 shares are given and that $a^1_t = 1 - \sum_{j=2}^n \mu_j a^j_t$ for all t), $(T-1)(n-1)$ money shares $\{m^j_1, \ldots, m^j_{T-1}\}_{j=2}^n$ (notice that m^j_0 are given by the initial steady-state and that $m^1_t = 1 - \sum_{j=2}^n \mu_j m^j_t$ for all t), and $T-1$ values for the inverse of real aggregate per capita money holdings $\{p_1, \ldots, p_{T-1}\}$ (it can be shown that $p_T = p_{T-1}$). Thus, the unknowns add up to $3nT - 2n$.

The equations are given by $n(T-1) + n - 1$ budget constraints of the form

$$c^j_t + a^j_{t+1} k_{t+1} L^g(c_{t+1}, p_{t+1}, m_t) + \frac{m^j_t}{p_t} + \hat{w}(k_t)\Gamma(s^j(c^j_t, p_t, m^j_{t-1}))$$
$$= w_t z^j + (1 + r_t) a^j_t k_t L^g(c_t, p_t, m_{t-1}) + \frac{m^j_{t-1}}{p_t(1+g)},$$

$n(T-1)$ euler equations regarding capital accumulation decisions

$$\lambda^j(c_t, p_t, m_{t-1}, k_t) = \beta(1 + r(c_{t+1}, p_{t+1}, m_t, k_{t+1}))\lambda^j(c_{t+1}, p_{t+1}, m_t, k_{t+1}),$$

$n(T-1)$ euler equation regarding money holdings decisions

$$\beta(\lambda^j(c_{t+1}, p_{t+1}, m_t, k_{t+1}) + \phi^j(c_{t+1}, p_{t+1}, m_t, k_{t+1}))$$
$$= \lambda^j(c_t, p_t, m_{t-1}, k_t)\,\frac{p_{t+1}}{p_t}(1+g),$$

and one initial boundary condition involving initial aggregate capital holdings $K_1 = k_1 L^g(c_1, p_1, m_0)$. It can be verified that the number of equations add up to $3nT - 2n$.

A.2.2. Time aggregation

Following Mercenier and Michel (1994), we pick an increasing sequence of dates $\{t_n\}_{n=0}^T$, not necessarily uniformly spaced, and with $t_1 = 1, t_0 = 0$ (that is, t_1 and t_0 correspond to calendar time 1 and 0, respectively). These dates define T intervals of time of length $\Delta_n = t_{n+1} - t_n$. We define the sequence $\alpha_{n+1} = \alpha_n/(1 + (1/\beta - 1)\Delta_{n+1})$ for all $n \geqslant 0$, $\alpha_0 = 1$. The individual's problem is now formulated as

$$\max_{\{c^j_t, s^j_t, a^j_{t+1}, m^j_g\}_{t=0}^\infty} \sum_{n=1}^{T-1} \alpha_n \Delta_n u(c_{t_n}) + \beta \frac{\alpha_{T-1}}{1-\beta} u(c_{t_T})$$

subject to

$$c^j_{t_n}(1 - s^j_{t_n})\Delta_n \leqslant \frac{m^j_{t_{n-1}}}{p_{t_n}(1+g)}\Delta_n,$$

$$\begin{aligned} &a^j_{t_{n+1}}K_{t_{n+1}} - a^j_{t_n}K_{t_n} + \frac{m^j_{t_n}}{p_{t_n}} - \frac{m^j_{t_{n-1}}}{p_{t_{n-1}}} \\ &\quad = \Delta_n\left(w_{t_n}z^j + r_{t_n}a^j_{t_n}K_{t_n} - c^j_{t_n} - \hat{w}_{t_n}\Gamma(s^j_{t_n}) - \frac{m^j_{t_{n-1}}}{p_{t_{n-1}}} + \frac{m^j_{t_{n-1}}}{p_{t_n}(1+g)}\right), \end{aligned}$$

$$c_{t_T} = (1/\beta - 1)a^j_{t_T}K_{t_T} + w_{t_T}z^j - \hat{w}_{t_T}\Gamma(s^j_{t_T}) - \frac{m^j_{t_{T-1}}}{p_{t_{T-1}}} + \frac{m^j_{t_{T-1}}}{p_{t_T}(1+g)},$$

a^j_1 and m^j_0 given.

The equilibrium conditions are almost identical to the ones derived in the previous subsection. The number of equations and unknowns is given by $3nT - 2$. The unknown variables are the same as in the algorithm with no time aggregation (the only change is that now we index variables by t_n instead of t). The expressions λ^j, ϕ^j, and s^j that give the equilibrium values of $\lambda^j_{t_n}, \phi^j_{t_n}$, and $s^j_{t_n}$ as functions of $(c_{t_n}, p_{t_n}, m_{t_n-1}, k_{t_n})$ are identical to the ones defined in the previous section. The Euler equation for capital is now given by

$$\begin{aligned} &\alpha_{t_n}\lambda^j(c_{t_n}, p_{t_n}, m_{t_n-1}, k_{t_n}) \\ &\quad = \alpha_{t_{n+1}}(1 + \Delta_{n+1}r(c_{t_{n+1}}, p_{t_{n+1}}, m_{t_{n+1}}, k_{t_{n+1}}))\lambda^j(c_{t_{n+1}}, p_{t_{n+1}}, m_{t_n}, k_{t_{n+1}}), \end{aligned}$$

and the Euler equation for money now satisfies

$$\begin{aligned} &\alpha_{t_{n+1}}\Bigg[\phi^j(c_{t_{n+1}}; p_{t_{n+1}}; m_{t_n}; k_{t_{n+1}})\frac{\Delta_{n+1}}{(1+g)\,p_{t_{n+1}}} \\ &\quad + \lambda^j(c_{t_{n+1}}, p_{t_{n+1}}, m_{t_n}, k_{t_{n+1}})\left(\frac{1}{p_{t_n}} - \frac{\Delta_{n+1}}{p_{t_n}} + \frac{\Delta_{n+1}}{(1+g)p_{t_{n+1}}}\right)\Bigg] \\ &\quad = \lambda^j(c_{t_n}, p_{t_n}, m_{t_{n-1}}, k_{t_n})\frac{p_{t_{n+1}}}{p_{t_n}}(1+g). \end{aligned}$$

Notice that the Euler equations are identical to the ones of the previous section if $\Delta_{n+1} = 1$ for all n (no time aggregation).

In the numerical exercises reported in the paper we work with 110 artificial dates t_n, which allow us to compute transitions of 1,145 model periods of actual length (we have considered intervals of length 1, 2, 5, 20, 40, 50, with a number of intervals of each type given, respectively, by 40, 15, 15, 15, 15, 10).

References

Aiyagari, R., Braun, A., Eckstein, Z., 1998. Inflation, transaction services and welfare. Journal of Political Economy 106 (6), 1274–1301.

Al-Marhubi, F., 1997. A note on the link between income inequality and inflation. Economic Letters 55, 317–319.

Altig, D., Carlstrom, C., 1991. Inflation, personal taxes and real output: a dynamic analysis. Journal of Money, Credit and Banking 23 (3), 547–571.

Attanasio, O., Guiso, L., Japelli, T., 1998. The demand for money, financial innovation and the welfare cost of inflation: an analysis with household data. NBER Working Paper 6593.

Avery, R., Elliehausen, G., Kennickell, A., 1986. The use of transaction accounts by American families. Federal Reserve Bulletin 72 (2), 87–107.

Avery, R., Elliehausen, G., Kennickell, A., 1987. Changes in the use of transaction accounts and cash from 1984 to 1986. Federal Reserve Bulletin 73 (3), 179–195.

Cavalcanti, R., Erosa, A., 1999. Inflation, capital gains taxation and business turnover. Mimeo.

Chatterjee, S., Corbae, D., 1992. Endogenous market participation and the general equilibrium value of money. Journal of Political Economy 100 (3), 615–646.

Cole, H., Stockman, A., 1991. Specialization, transaction technologies and money growth. International Economic Review 33, 283–298.

Dotsey, M., Ireland, P., 1996. The welfare cost of inflation in general equilibrium. Journal of Monetary Economics 37, 29–47.

Easterly, W., Fischer, S., 2001. Inflation and the poor. Journal of Money, Credit and Banking 33 (2), 160–178.

Gillman, M., 1993. The welfare cost of inflation in cash-in-advance economy with costly credit. Journal of Monetary Economics 31, 97–115.

Imrohoroglu, A., 1992. The welfare cost of inflation with imperfect insurance. Journal of Economics Dynamics and Control 16, 79–91.

Imrohoroglu, A., Prescott, 1991. Evaluating the welfare effects of alternative monetary arrangements. Federal Reserve Bank of Minneapolis Quarterly Review 15 (3), 3–10.

Kennickell, A., Starr-McCluer, M., 1996. Household Saving and Portfolio Change: Evidence from the 1983–1989 SCF Panel. Finance and Economics Discussion Series 18, Division of Monetary Affairs, Federal Reserve Board, Washington, DC.

Kennickell, A., Starr-McCluer, M., Sunden, A., 1997. Family finances in the US: recent evidence from the survey of consumer finances. Federal Reserve Bulletin 83, 1–24.

Kessler, D., Wolff, E., 1991. A comparative analysis of household wealth patterns in France and the United States. Review of Income and Wealth 37 (3), 249–266.

Lucas, R., 2000. Inflation and welfare. Econometrica 68 (2), 247–274.

Mercenier, J., Michel, P., 1994. Discrete-time finite horizon approximation of infinite horizon optimization problems with steady-state invariance. Econometrica 62 (3), 635–656.

Mulligan, C., Sala-i-Martin, X., 2000. Extensive margins and the demand for money at low interest rates. Journal of Political Economy 108 (5), 961–991.

Prescott, E., 1987. A multiple means of payment model. In: Barnett, W.A., Singleton, K. (Eds.), New Approaches to Monetary Economics. Cambridge University Press, Cambridge.

Romer, C., Romer, D., 1998. Monetary policy and the well-being of the poor. NBER Working Paper 6793.

Schreft, S., 1992. Transaction costs and the use of cash and credit. Economic Theory 2, 283–294.

Sprenkle, C., 1993. The case of the missing currency. Journal of Economic Perspectives 7, 175–184.

Wolff, E., 1983. The size distribution of household disposable wealth in the united states. Review of Income and Wealth 29 (2), 125–146.