Monotonic timing games

War of Attrition

Pre-emption Game

More predecessors helps
⇒ Network externality.

More predecessors hurts
⇒ Congestion.

There there many situations where there are both rank-network externality and rank-congestion simultaneously!

More generally, why not ...

Hill-shaped rank rewards

U-shaped rank rewards

Being a middle rank is optimal.
• Caller Number Five
• High-Tech Market Entry

Being either the very first or the very last is optimal.
• Gold-Rush

Rank-Congestion together with Rank-Network Externalities

- Industrial organization: High Tech market entry — market creation vs. competition, product marketing vs. brand marketing.
- Investment: More information vs. ‘potential’.
- Herding combined with strategic complementarities — e.g. restaurant herding with congestion.
- Parliamentary voting.
- Two-tier tender offers in mergers and acquisitions.
**Example: Caller Number Five**

- Delay costly – opportunity costs. Delay initially beneficial if others stop.
  ⇒ Initially trapped in a War of Attrition.
  ∴ play mixed strategy.
- Latest point to call: Probability that 4 have called is maximal
  → expected rank reward is maximal.
- Problem: Everyone else will also play with an atom!
  ∴ Have to share rewards.
  ∴ pre-empt the optimal ‘solo-moment’.
- Iterating the argument, atom happens when expected atomic rewards from a concluding atom are maximal:
  → expected average rank reward from playing in an atom is maximal.

**A New Class of Games - Players**

- $N + 1 \geq 3$ identical players.
- Actions: At any point in time stop or don’t stop. Can stop only once (irrevocable). Continuous time on $[0, \infty)$.
- Information sets: Unobservable actions ⇒ only one information set.
  Extensions:
  - Observable actions.
  - Partial information revelation.

**A New Class of Games - Strategies**

- Strategies: The stopping time $t \geq 0$.
  Mixed strategy: measures the probability that a player has stopped. Right-continuous, non-decreasing
  ⇒ cdf $G(t)$.
  We assume that strategies that have convex support including 0.
  ⇒ strategies cannot depend explicitly on calendar time other than current influence on $G(t)$
  convex support ⇒ no coordination on exogenous events (sunspots).

**A New Class of Games — Payoffs**

- Rank Rewards: Payment $v(k)$ depends on the ordinal stopping rank $k$.
  Caller Number 5: $v = (0, \ldots, 0, \text{prize}, 0, \ldots, 0)$.
- Simultaneous stopping: Share rewards.
  $k$ have already stopped, $j + 1$ stop simultaneously
  $$A(k, j) = (v(k + 1) + \ldots + v(k + j + 1))/(j + 1).$$
  Caller Number 5: $A(2, 5) = \text{prize}/6$.
- Exogenous delay costs: Continuous discounting at interest rate $r$ and unit time delay costs $c(t)$.
  Caller Number 5: $c(t) = t$.
- Extension: Time Rewards: Multiplicatively so $v(k) \cdot \gamma(t)$
**Equilibrium Analysis**

- We are looking only for the *symmetric* equilibria.
- If players have stopped with probability \( g \in [0, 1] \), chance that \( k \) have stopped and \( N - k \) have not:
  \[
  \binom{N}{k} g^k (1 - g)^{N-k}
  \]
- Expected stopping rewards are thus
  \[
  \phi(g) := \sum_{k=0}^{N} \binom{N}{k} g^k (1 - g)^{N-k} v(k + 1)
  \]
  reward to being \( k + 1 \)-st

**Simultaneous Entry**

- What happens when people enter simultaneously in an atom of size \( h - g \)?
- Ranks \( k + 1, \ldots, k + j \) stop simultaneously with probability
  \[
  \frac{N!}{k!j!(N-k-j)!} g^k (h-g)^j (1-h)^{N-k-j}.
  \]
- Expected payoffs are then
  \[
  \Lambda(g, h) := \sum_{k=0}^{N} \sum_{j=0}^{N-k} \frac{N!}{k!j!(N-k-j)!} g^k (h-g)^j (1-h)^{N-k-j} A(k, j).
  \]
  reward of ranks \( k + 1 \) to \( k + j + 1 \) stopping together

**First Order Condition for ‘Smooth’ Play.**

- Payoff = discounted rewards minus costs \( e^{-rt} [\phi(G(t)) - c(t)] \).
- Mixed strategy \( \rightarrow \) same expected payoff on the support.
- Assume \( \dot{G} \) exists: payoffs constant:
  \[
  \text{expected reward at } t = \text{discounted expected reward at } t + dt + \text{flow cost}
  \]
  \[
  \Leftrightarrow 0 = -\dot{c} - r[\phi(g) - c] + \dot{G} \cdot \phi'(g)
  \]
  Interpretation of this differential equation
  \[
  \text{Exogenous cost of delay} = \text{Change in mixed strategy} \times \text{Strategic benefit of delay}
  \]

**Atomic Rewards**

- In an atom from \( g \) to \( h \): Averages \( \Lambda(g, h) \) apply.
- Use \( \Phi(g) = \int_0^g \phi(x)dx \) for the anti-derivative of \( \phi \).
- How does \( \Lambda(g, h) \) relate to expected flow rewards \( \phi \)?
  \[
  \text{Lemma 1:}
  \]
  \[
  \frac{\Phi(h) - \Phi(g)}{h-g} = \Lambda(g, h) \quad \text{or} \quad \frac{\phi(h) - \Lambda(g, h)}{h-g} = \frac{\partial}{\partial h} \Lambda(g, h).
  \]
  Interpretation: An Analogy to Average vs. Marginal Revenue:
  \[
  \frac{MR - AR}{q} = AR' \quad \frac{\text{Marginal Rank Rewards} - \text{Average Rank Rewards}}{\text{Atomic Jump-Size}} = \text{Marginal Average Rank Rewards.}
  \]
What is an Equilibrium?

Cdf $G : [0, \infty) \to [0, 1]$ is a symmetric stationary NE if

- (E1) The support of $G$ is convex starting at 0.
- (E2) Payoffs are constant on the support when $G(t) < 1$.
- (E3) If $G(t) > G(t-)$ then
  
  $$\phi(G(t-)) = \Lambda(G(t-), G(t)) \geq \phi(G(t)) \text{, “=” with } G(t) < 1.$$  

- If there is atomic entry at $t$ then atomic entry must pay as much as smooth entry just before the atom and after the atom;
- if the atom is terminal, immediate entry must pay more than delayed entry (thereby earning last rank payoff).

Characterizing an Equilibrium

- We want the equilibrium to be induced by a $C^2$ function. $\Gamma : [0, 1] \to \mathbb{R}_+.$
- $\Gamma$ will be strongly related to $\phi$ in the sense that $\Gamma$ often coincides with $\Phi$.
- Then $\Gamma$ induces an equilibrium if
  - when $\Gamma = \Phi$ it solves the differential equation properly
  - when $\Gamma \neq \Phi$ between $g$ and $h$ there is atomic entry of size $h - g$.

Potential Functions

- The matching functions are potential functions in the following sense:
  - (P1) $\Gamma(0) = 0$, $\Gamma(1) = \Phi(1)$, $\Gamma'(1) \geq \Phi'(1)$.
  - (P2) $\Gamma$ is: monotonically increasing, convex, cont. diff.
  - (P3) at any $\xi \in (0, 1)$, either $\Gamma'(\xi) = \Phi(\xi)$ or $\Gamma$ is linear around $\xi$.
- Lemma 2:
  - Any potential fct. $\Gamma$ induces a unique equilibrium.
  - Any equilibrium is induced by a unique potential fct. $\Gamma$.

Graphically: Potential Functions

- $\phi$ concave and $\Gamma$ convex $\Rightarrow$ $\Gamma$ linear!
Proof-Sketch of the Equivalence Lemma

- \( \Gamma \rightarrow \) unique equilibrium.
  - By (P3) \([0,1]\) partitions into sub-intervals where (alternating)
    - either \( \Gamma(g) = \Phi(g) \)
    - or \( \Gamma \) linear.
  - When \( \Gamma \) shifts from smooth to linear or back, then derivative of smooth at transition equals slope of linear line (as in picture):
    - linear slope: \( (\Gamma(g) - \Gamma(h))/(h-g) = \Lambda(h, g) \)
    - smooth slopes: \( \Gamma'(g) = \Phi'(g) = \phi(g) \) and \( \Gamma'(h) = \Phi'(h) = \phi(h) \)
    - Both equal \( \Rightarrow (E2) \)
  - If linear portion for \( h = 1 \), by (P3) \( \Gamma'(1) \geq \Phi'(1) = \phi(1) \)
    \( \Rightarrow (E3) \)

Proof-Sketch of the Equivalence Lemma

- \( \Gamma \rightarrow \) unique equilibrium (cont'd)
  - If at \( g: \Gamma(g) = \Phi(g) \)
    - (E3) and (E1) are (almost) clear.
  - For (E2), observe \( \Gamma' = \Phi' \) so ‘the’ ODE admits constant a payoff solution that is also unique subject to correct initial condition as \( \phi' > 0 \).

Two Illustrative Examples

- Equilibrium \( G(t) \rightarrow \) unique \( \Gamma \):
  - Whenever \( G(t) \) continuous, set \( \Gamma = \Phi \).
  - When \( G \) jumps, \( \Gamma \) is linear fct defined by points \((g, \Phi(g))\) and \((h, \Phi(h))\)
    - Slope: \( (\Phi(h) - \Phi(g))/(h-g) \)
    - Equals \( \Lambda(g, h) \rightarrow \) equals \( \phi(g) \) and \( \phi(h) \).
      \( \Rightarrow (P2) \& (P3) \)
  - (P1) clear when \( G(t) \) continuous for \( t \rightarrow 0,1 \).
    When atom at 0 or 1, then also clear after closely examining atom-slopes.

- \( N + 1 = 3 \)
- \( v = (0, 1, 0) \)
- \( \phi(g) = 2g(1-g) \)
- \( \Phi(g) = g^2(1 - 2g/3) \)
- \( N + 1 = 3 \)
- \( v = (1, 0, 1) \)
- \( \phi(g) = g^2 + (1 - g)^2 \)
- \( \Phi(g) = 1/3g^3 - 1/3(1 - g)^3 \)
Potential function $\Gamma_1$
- For $g \leq 1/4$, $\Gamma_1(g) = \Phi(g)$.
- For $g > 1/4$, $\Gamma_1(g)$ is linear $\Gamma_1(g) = 3g/8 - 1/24$.
  → a tangent on $\Phi$ at $g = 1/4$ which goes through point $(1, \Phi(1))$.

Potential function $\Gamma_2$
- For all $g$, $\Gamma_2(g) = g/3$.
- Assume $c(t) = t$ and $r = 0$. Solving the ODE for smooth entry (relevant for $\Gamma_1$)
  
  $0 = -1 + \dot{G}\phi'(G(t)) \iff 0 = -1 + \dot{G}(2 - 4G(t))$.

Problem: Initially delay is costly both strategically and exogenously.
Solution: Atomic entry!
But: How large? → It cannot be a unit atom – players get 2/3 in expectation, deviating yields 1.
Same reasoning as above: Time zero-atom equates $\phi(g)$ and $\Lambda(0, g)$.

Three Examples: U-Shape
- $v = (1, 0, 1)$, $c(t) = t$
  ⇒ (‘Marginal”) Flow payoff $\phi(g) = 1 - 2g(1 - g)$.
- Smooth entry equilibrium: Solve (FOC)
  
  $0 = -1 - \dot{G}(2 - 4G(t)) \iff G(t) = 1/2 + 1/2\sqrt{1/4 + 2t}$.

Problem: Initially delay is costly both strategically and exogenously.
Solution: Atomic entry!
Existence

Theorem 1:
A mixed strategy equilibrium exists and ends in finite time.

Proof:
The convex hull of \( \Phi \) (the largest convex function smaller than \( \Phi \)) exists and it is a potential function.
No matter what the strategy, at some point in time
\[
\max_g \Phi(g) - c(t) < 0
\rightarrow \text{delaying further is strictly dominated}
\rightarrow \text{game must end in finite time.}
\]

Expected Rewards Preserve the Shape of Rank Rewards

Lemma 3 [Variation Diminishing Property of expected flow rewards]
- Let the slope of rank rewards \( v \) change sign \( m \) times.
  Then the slope of expected rewards \( \phi \) changes sign at most \( m \) times.
- The number of sign variations in \( \phi \) is smaller by a multiple of 2 (incl. 0).
- The signs of the first slopes of \( v \) and \( \phi \) coincide as do the signs of the last slopes of \( v \) and \( \phi \).

Shape Preservation: Idea of the Proof

Proof-Idea: Summing by parts,
\[
\phi'(g) = \sum_{k=1}^{N} \binom{N}{k} \left[ k g^{k-1} (1-g)^{N-k} [v(k+1) - v(k)] \right].
\]
- \([v(k+1) - v(k)]\) changes sign \( m \) times.
- Scale \( \phi' \) with \( g/(1-g)^N \)
\[
\frac{g}{(1-g)^N} \phi'(g) = \sum_{k=1}^{N} \binom{N}{k} k [v(k+1) - v(k)] \left( \frac{g}{1-g} \right)^k.
\]
- Define \( a_k := \binom{N}{k} k [v(k+1) - v(k)] \) and \( z := g/(1-g) \).

Shape Preservation: Idea of the Proof (cont’d)

- Define \( a_k := \binom{N}{k} k [v(k+1) - v(k)] \) and \( z := g/(1-g) \).
\[
\frac{g}{(1-g)^N} \phi'(g) = \sum_{k=1}^{N} a_k z^k =: P(z).
\]
- By Descartes’ Rule of Sign, the number of sign changes in \( P(z) \) is at most the number of sign changes in coefficients \( a_0, \ldots, a_N \).
- Slopes at beginning and ends coincide as \( \phi'(0) = v(2) - v(1) \), and \( \phi'(1) = v(N+1) - v(N) \). □
Phases

We distinguish three phases of play
- If $\dot{G}(t+) > 0$ exists and $\phi'(G(t+)) > 0$ on some $(t, \bar{t})$
  ⇒ War of Attrition phase.
- If $\dot{G}(t+) > 0$ exists and $\phi'(G(t+)) < 0$ on some $(t, \bar{t})$
  ⇒ slow Pre-emption Game phase.
- If $G$ jumps at $t$ as $G(t) > G(t-)$
  ⇒ Pre-emptive Atom phase

Behavioral Results I: Phase Transitions

Theorem 2:

(a) Equilibrium consists only of War of Attrition or pre-emptive atoms but not slow Pre-emption Games.
(b) There are at most as many phase-transitions as slope sign changes in $v(k+1) - v(k)$.
(c) If $\phi$ has $m$ alternating slope-signs then there are at most $m - 1$ many phase-transitions and the maximum number is attained if the convex hull of $\Phi$ touches every convex portion of $\Phi$.

Proof of Theorem 2: (a)

- Expected payoffs are constant along the support.
- Delay costly $\rightarrow \Gamma'$ must be increasing.
- If ever $\phi < 0$, then players must stop at once.
- Consecutive atoms can be merged.

Proof of Theorem 2: (b)

- Follows from the Variation Diminishing Lemma.
Proof of Theorem 2: (c)

Sufficiency:
- Phase transition $\equiv \Gamma$ switches between locally linear and locally strictly convex.
- $\Gamma$ changes slope only when $\Gamma = \Phi$.
- Polynomial $\Phi$ is strictly convex at most as often as $\Gamma$.
- $\rightarrow$ equality iff $\Gamma$ touches every convex portion of $\Phi$.
- convex hull is potential function $\Rightarrow$ sufficiency.

Necessity:
- Suppose $\Gamma$ touches every convex portion of $\Phi$.
- $\Gamma$ contains every supporting tangent between consecutive portions.
- The unique such potential function is the convex hull. $\square$

Example: Zick-Zack

Example: Zick-Zack
- 4 players
- $v = (0, \psi, 0, 1)$, with $\psi > 0$

\begin{align*}
\phi(g) & = 3g(1-g)^2 \cdot \psi + g^3 \cdot 1 \\
\phi'(g) & = 3\psi(2g-1)^2 + 3(1-\psi)g^2, \\
\Phi(g) & = \left(1/4 \left(1 - (1-g)^4\right) - g(1-g)^3\right) \cdot \psi + g^4/4.
\end{align*}

Example: Zick-Zack - Analysis

Example: Zick-Zack
- $\psi < 1$: $\phi$ is monotonic $\Rightarrow$ no phase change
  - Shows that strict inequality w.r.t. phase transitions is possible.
  - One cannot gauge # of phase transitions from $v$ alone.
- $\psi > 1$: $\phi$ has max and min $\Rightarrow$ at least one and up to two phase changes.
- $\psi \leq (5 + \sqrt{33})/4 \rightarrow$ convex hull touches both convex portions in $\Phi$.
- $\psi > (5 + \sqrt{33})/4 \rightarrow$ convex hull touches only the first convex portion in $\Phi$. 

Possible Equilibria in Zick-Zack

![Graphs showing possible equilibria]

Behavioral Results II: Phase Truncations and Atom Inflation

- If pre-emptive atom subsumes some portion of adjacent interval where \( \phi \) increases and a war is played \( \Rightarrow \) atom is inflated and the war truncated.
- Truncation in cdf-space implies truncation in time (as seen in Caller Number Five).
- **Theorem 3:** Pre-emptive atoms are inflated and wars of attrition truncated.
  
**Proof:**
- Linear portions of \( \Gamma \) join distinct convex portions of \( \Phi \).
- If connection is not directly at inflection points of \( \Phi \) (i.e. where \( \phi' = 0 \) so that \( \phi \) changes slope), there is *perforce* inflation/truncation.
- But it cannot join consecutive inflection points as this means slicing through \( \Phi \).

Behavioral Results III: Number of Equilibria

- Let \( \mathcal{E}_m \) := set of symmetric stationary Nash equilibria.
- **Theorem 4:** Assume \( \phi \) has exactly \( m \) alternating slope-signs.
  - If \( \phi \) slopes up at \( g = 0 \), then \( |\mathcal{E}_{2k}|, |\mathcal{E}_{2k+1}| \leq 2^k \);
  - if \( \phi \) slopes up at \( g = 0 \), then \( |\mathcal{E}_{2k-1}|, |\mathcal{E}_{2k}| \leq 2^{k-1} \).

Proof of Theorem 4

- Let \( \mathcal{J}_m \) is the set of up-slopes of \( \phi \) followed by down-slopes.
- Then number of equilibria = \( 2|\mathcal{J}_m| \):
  - equilibrium \( \Rightarrow \) unique set of common tangents on pairs of strictly convex portions of \( \Phi \)
  - \( \Rightarrow \) unique set of up-slopes (= convex portions) played.
  - \( \Rightarrow \) a 1-1 map from equilibria \( \mathcal{E}_m \) to sets \( \mathcal{J}_m \).
  - Set of all combinations of up-slope pairs has cardinality of a power set.
Example for Number of Equilibria: Zick-Zack

- For $\psi \leq 1$, $\phi(g)$ has only one slope-sign.
- For $\psi > 1$, $\phi(g)$ has only three slope-signs.
  - $\phi'(0) > 0 \Rightarrow \text{Number of equilibria } |E_{2.1+1}| = 2^1 = 2$.
- Candidate inducing potential functions:
  - linear then smooth.
  - smooth then linear.
  - smooth linear smooth.

Possible Equilibria in Zick-Zack

- For $\psi \leq 1$,
  - $\phi'(0) > 0 \Rightarrow \text{Number of equilibria } |E_{2.1+1}| = 2^1 = 2$.
- Candidate inducing potential functions:
  - linear then smooth.
  - smooth then linear.
  - smooth linear smooth.

Equilibrium Payoffs

- Focus of timing games: hold-up loss/payoff-burn/rent-dissipation.
- When $r = 0$ and $c(t) = t$ then
  - $\text{rent-dissipation} = \text{elapse time}$.
- Questions:
  - What is the ex-ante expected value of the game?
  - For a given equilibrium, how long will the game last?
- Benchmark War of Attrition:
  - all rents are dissipated,
  - $\text{expected payoff} = \text{time-zero payoff } v(1)$,
  - $\text{length} = \text{highest payoff} - \text{smallest payoff}$,
  - $v(N+1) - v(1)$.

Theorem: Equilibrium Payoffs

- **Theorem 5:**
  - $(a)$ For given potential function $\Gamma$
    - The ex-ante expected payoff is $\Gamma'(0)$.
    - the maximum elapse time is $\Gamma'(1) - \Gamma'(0)$.
  - $(b)$ The equilibrium induced by the convex hull
    - has global minimum ex-ante expected payoff and
    - the global maximum elapse time.
Proof of Theorem 5, Part (a)

Proof of (a):
- mixed strategy $\Rightarrow$ constant payoff along the support of play.
- expected payoff of the game = time zero payoff $= \Gamma'(0)$.
- Theorem 1: the game ends in finite time.
- length of play depends on the payoffs dissipated.
- expected rank-payoffs increase along the support of play
  $\Rightarrow$ largest rank-payoff when the game ends $= \Gamma'(1)$.

Welfare: The most efficient equilibrium

- The convex hull describes the least efficient equilibrium - but which is the most efficient one?
- \(\bar{\beta}\) easy answer.
  - If $\Phi(1) \geq \Phi'(1)$, then
    - $\Gamma(g) = g\Phi(1)$ is potential function
    - $\Rightarrow$ equilibrium = unit atom
    - most efficient and highest payoff
  - If $\Phi(1) < \Phi'(1)$ then
    - no clean-cut answer
    - highest $\neq$ most efficient (by example)

Observable Actions

- Unobserved actions:
  - most tractable, cleanest, most elegant.
  - indicative for observable actions.
  - foundation for observable actions analysis.
- To illustrate the latter point: break cardinal rule “one paper - one model” and briefly discuss observable actions.
Observable Actions

- Solve backwards: 1 player remaining → 2 players remaining → . . .
- \( w(k+1) \) is the expected SPE payoff from subgame when \( k \) have stopped.
- Complications:
  - In general, the analysis can get messy.
  - More atomic entry equilibria (BUT: many are unstable if a small fraction of players may have unsynchronized watches).
  - Thus focus on equilibria where
    - War of Attrition is played whenever possible → when \( w(k+2) > v(k+1) \)
    - Pre-emption Game is played whenever necessary → when \( w(k+2) \leq v(k+1) \)

Observable Actions: Results

- Assume: On down-slopes rank payoffs are more valuable than the overall average remaining payoff
  \[ v(k+1) < v(k) \implies v(k+1) > A(k, N-k). \]
- Theorem 6: Wars of Attrition are weakly truncated in ranks, truncated in time
  - Pre-emptive atoms are weakly inflated.
- Corollary: Number of phases restricted by number of slope-signs in ranks \( v(k) \).

Observable Actions

- Atom \( p < 1 \) must solve
  \[
  \text{expected continuation payoff} = \text{atomic payoff}
  \]
  \[
  \sum_{i=0}^{N-k} \binom{N-k}{i} p^i (1-p)^{N-k-i} w(k+1+i)
  \]
  \[
  \text{atomic payoff:}
  \sum_{i=0}^{N-k} \binom{N-k}{i} p^i (1-p)^{N-k-i} A(k,i).
  \]
- If no \( p < 1 \) exists, \( p = 1 \) is equilibrium as LHS>RHS always.

Observable Actions: Results

- Proof-Sketch:
  - Atomic payoff \( \Rightarrow w(k) < v(k) \).
  - Why smaller? - Players cannot expect more than remaining average \( A(k, N-k) \).
  - War of Attrition time-truncated as expected length shrinks to \( w(k+1) - v(k) < v(k+1) - v(k) \).
  - If \( w(k+1) < v(k) \) then rank-truncation.
- Thus: unobservable actions analysis (= easy) sets benchmark for observable actions equilibria.
Results with Hill- and U-shape — Applied Sequel

- **Comparative Statics:**
  1. Positive linear magnification:
     → same atom, longer War of Attrition.
  2. Monotone ratio domination ≡ “Shifting hill-tops and valley-bottoms”
     → smaller pre-emptive atom, longer War of Attrition

- **Adoption rates**
  → adoption density decreases monotonically away from pre-emptive atom

- **Private values**
  → qualitatively similar behavior ("purification")
  → “pooling at the top”

- **Announcements:** some information is revealed in the process (e.g. in Caller Number Five, the winner is announced).
  → qualitatively similar

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**Literature**

- Enormous literature on timing games:
  - War of Attrition: Evolutionary biology, R&D races, market exit, public good provision, investment, adoption of new technologies.
  - Preemption Game: market entry, duels, bubbles.
  → We breach the wall between the two strands of the literature.

- Two recent papers that go into the same direction as we:
  - Sahuguet (GEB, forthcoming): 3 player public goods provision (no ranks, but second level decision).

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**Summary**

- New class of timing games allowing the study of **ordinally** non-monotonic payoffs.

- This paper is the **tools** paper.
  An “**applied**” sequel deals with time-rewards, incomplete information, applications, comparative statics and adoption rates (testable implications).

- Advantage: Simple formulation (rank-rewards, potential functions).

- Many possible applications; economically relevant.

- Equilibria are **not** the direct sum of equilibria from monotonic games.

- Equilibrium behavior is empirically observable and interesting.