Herding and Contrarian Behavior in Financial Markets

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Market Turmoil in the Autumn of 2008: from end September to mid-November I

- On 8 days the Dow changed by more than 5% in absolute terms.
  - Since WWII there have been only 16 other days with changes > 5%.
  - On 6 days the Dow declined, but on 2 days it increased by more than 10%.
    (Since WWII this has happened only once, on the day after the 1987 crash.)

- Intra-day fluctuations were even more pronounced: 14 days when max and min prices levels were more than 10% apart (7 with max(t)-min(t-1) ≥ 10%, 7 times with min(t)-max(t-1) ≤ -10%).
Market Turmoil in the Autumn of 2008: from end September to mid-November II

- Such extreme price fluctuations are possible only if there are dramatic changes in behaviour (from buying to selling or the reverse).
- Such behaviour and the resulting price volatility is often claimed to be inconsistent with rational traders and informationally efficient prices.
- Commentators often attribute dramatic swings to investors’ animal instincts.
- However, appealing to animal spirits or irrationality is not necessary. We argue that it can be the result of fully rational social learning where agents change their beliefs and behavior as a result of observing the action of others.
Rational Herding in a Nutshell
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Herding (following others) may of course be the result of irrationality; however, here we concentrate on traders rationally following others.
Rational Herding in a Nutshell
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he knows what he’s doing!
A crude herding example I

- A single asset world: Two values, \( V \in \{0, 1\} \), \( \Pr(V = 1) = \Pr(V = 0) \).
- Traders arrive in sequence, observe what predecessors did, and can buy or sell.
- Each receive privates (c.i.i.d.) informative signal \( S_h \) or \( S_\ell \):
  \[
  \Pr(S_h|V = 1) = \Pr(S_\ell|V = 0) = q > 1/2.
  \]
- If asset price \( p_t \) is always a constant then analogous to BHW (1992) model with herding cascade possible: after a finite number of buys (sales) all will do the same.
A crude herding example II

- Suppose asset price is efficient:
  \[ p_t = E(V|H^t) = \text{expected value given all public info} \]. Since

  \[ E(V|H^t, S_h) > E(V|H^t) > E(V|H^t, S_\ell), \forall H^t, \]

  it follows that \( p_t \) is always between the private expectations.

- **Consequence:** \( S_h \) types always buy and \( S_\ell \) types always sell; hence **herding (or social learning) not possible!**
A refined herding example I

- **Situation:** the competitor of financial institution FI has recently failed.

- **Three states of the world, \( V_1 < V_2 < V_3 \):**
  - \( V_1 \): FI will also fail because it has deals with the failed bank that will not be honored and/or that the business model of FI is as bad as that of the failed bank;
  - \( V_2 \): FI's situation is entirely unrelated to the bank and the latter's collapse will not affect FI.
  - \( V_3 \): FI may benefit greatly from the bank’s collapse as it is able to attract the failed bank’s customers and most capable employees.

- **Now consider two announcements:**
  - A *good news* public announcement \( G \) that rules out the worst state, \( \Pr(V_1|G) = 0 \).
  - A *bad news* public announcement \( B \) that rules out the best state, \( \Pr(V_3|B) = 0 \).
A refined herding example II

- ⇒ After each announcement, we are left with a 2-state world.
- In 2-state models, an investor has a higher (lower) expectation than the market if and only if his private information is more (less) favourable towards the better state than towards the worse state.
  - $E[V|G] \leq E[V|S, G] \iff Pr(S|V_2) \leq Pr(S|V_3)$
  - $E[V|S, B] \leq E[V|B] \iff Pr(S|V_2) \leq Pr(S|V_1)$.
  - E.g.: after good news $G$, an investor buys (sells) if he thinks, relative to the market, that it is more (less) likely that FI will thrive than being unaffected.
A refined herding example III

- **Key observation:**
  1. Investor buys after $G$ and sells after $B \iff \Pr(S|V_3) > \Pr(S|V_2)$ and $\Pr(S|V_1) > \Pr(S|V_2)$.
     - Interpretation: herds in the sense that he acts like a momentum trader, buying with rising and selling with falling prices.
     - Structure of private information: *U shape*.
  2. Investor sells after $G$ and buys after $B \iff \Pr(S|V_3) < \Pr(S|V_2)$ and $\Pr(S|V_1) < \Pr(S|V_2)$.
     - Interpretation: trades contrary to the general movement of prices.
     - Structure of private information: *Hill shape*.
A refined herding example IV

- Some comments
  - \( B \) and \( G \) are degenerate — but idea applies generally.
  - Herding arise endogenously because of trading histories which generates effects similar to \( B \) and \( G \).
  - Key task: generate/show existence of the “right” histories
Our contribution and main points

1. Rational herding can emerge in models with a richer state space.
2. We identify necessary and sufficient conditions on traders’ private information that allow for
   - herding and
   - contrarian behavior
   in an informationally efficient financial market.
3. The conditions themselves are intuitively appealing and lend meaning to the emergence of herding.
4. Herd behavior has exactly the features that one expects
   - during herding prices can move substantially
   - herding can induce lower liquidity and higher price volatility than if there were no herding
Definition of Herding I

- Literature on informational herding deals with “observational learning”, i.e. models in which agents observe actions of predecessors.

- “Herding” has several aspects:
  - learning from others
  - change of behavior + following others
  - “mass-uniform” behavior

- Literature developed from settings without a price mechanism.

- Our general framework: “continuous” trading in a Glosten-Milgrom style model ⇒ prices move only if trades reveal information.
Definition of Herding II

- Imagine all traders act alike (= mass-uniform)
  - all informed take same action
  - trades reveal no new information
  - prices won’t move
- Such behavior can’t explain excess volatility or the like.
- We use a behavior-based definition that captures the social learning aspect of herding.
- Herding ⇒ one signal-type changes behavior.
- Prevalence/Importance? Depends on how many people have the herding signal. (I'll come back to this later.)
Formal Definition of Herding and Contrarianism

1. A trader rationally engages in *herd-buying* after a history of trade $H^t$ if and only if
   - he would sell at the initial history $H^1$,
   - he buys at history $H^t$,
   - prices at $H^t$ are higher than at $H^1$.

2. A trader rationally engages in *contrarian buying* after a history of trade $H^t$ if and only if
   - he would sell at the initial history $H^1$,
   - he buys at history $H^t$,
   - prices at $H^t$ are lower than at $H^1$. 
Other Issues

- **Herd behavior** = history induced switching in the direction of the crowd ≈ *momentum trading*.
- **Contrarianism** = history induced switching against the direction of the crowd ≈ *mean reversion trading*
- (Also: contrarian behavior is natural counterpart of herding.)
- There is a large empirical literature that studies contrarian and momentum investment strategies and their profitability. (Contrarian: Jegadeesh (1990), Lehmann (1990), Bondt and Thaler (1985); Momentum: Jegadeesh and Titman (1993), Jegadeesh and Titman (2001).)
The simplest possible model that allows for herding and contrarianism is one with 3 states. For this we show:

1. **Herding**

   - *Necessity*: Herding can occur only if the signal likelihood function (LF) for some signal is **U-shaped** in liquidation values. This implies that the signal is more likely to be the result of extreme values/states;

   - *Sufficiency*: Herding occurs if the LF for some signal is **U-shaped** and the proportion of informed insiders is not too large — enough noise (not always needed).
2. Contrarianism

- *Necessity*: Contrarian behaviour can occur only if the LF for some signal is **Hill-shaped** in liquidation values: extreme values generate this signal with smaller probability relative to middle values.

- *Sufficiency*: Contrarian behaviour can occur if the LF for some signal is **Hill-shaped** and the proportion of informed insiders is not too large — enough noise (not always needed).
Interpretation 1

Conditions on signals that allow herding/contrarianism have intuitive appeal (⇒ relevance and applicability):

1. Herding

- People believe that extreme outcomes occurred with higher probability than moderate ones: *fat-tailed* posterior distribution.
- For posteriors, signal recipient shifts weight away from the centre to extreme values faster than an agent who receives only the public information (market maker).
- This makes the recipient’s expectation more volatile and prone to changing behaviour and following the crowd even if prices reflect all public information.
Interpretation II

2. Contrarian

- Contrarian behavior occurs if traders’ signals generate thin-tailed distribution relative to the prior, indicating that moderate states are more likely than extreme states.
- The signal recipient shifts weight to the center ⇒ expectation more stable.
- Thus, he may act against the general movement of prices (contrarian).

3. We also show that no social learning (no change of behaviour) if the signal likelihood function is neither U nor Hill (is monotonic)
Technical Aside

• Literature on asymmetric information often uses MLRP signals.

• → Restrictive! But: Does it prohibit social learning?

• No! Our conclusions are general and valid with or without MLRP:
  • Both U shape and Hill shape are consistent with MLRP and therefore, both herding and contrarian can occur with MLRP

• Moreover: results sharper and proofs significantly simpler with the well behaved MLRP ⇒ here concentrate on MLRP.

• Characterisation robust for more than 3 signals/states (incl. continuum).
Related Literature

- Closest: Avery-Zemsky (AER 1998)
  - known for their “negative” result.
  - Argue that no herding with efficient prices unless signals are “non-monotonic” and risk “multi-dimensional” (investors have a finer information structure than the market).
  - present an appealing example with herding — alas very little price movements during herding episode.

- Survey literature interpretation (e.g. Brunnermeier (2000), Bikhchandani-Sumil (2000), Chamley (2004))
  1. With “monotonic” signals, herding is impossible.
  2. For herding one needs “multidimensionality” of risk.
  3. Herding does not involve violent price movements except in the most unlikely environment.
  4. Herding only with very special signals ⇒ not so relevant for explaining behaviour/outcomes in efficient financial market.
• Conformism and informational cascades caused by important, additional elements (e.g. transaction costs, reputation concerns):

• Overweighing of public information ⇒ Public signals can have a larger influence on stock price fluctuations than warranted by their information content.
Glosten-Milgrom Sequential Trade

- Three liquidation values: $V \in \{V_1, V_2, V_3\} \equiv \{0, \mathcal{V}, 2\mathcal{V}\}$. Symmetric prior, $\Pr(V_1) = \Pr(V_3)$.
- Risk neutral traders arrive in exogenous sequence.
- Trader can be
  1. Noise (probability $1 - \mu$): trade for reasons outside the model (liquidity, diversification, inventory rebalancing).
  2. Informed (probability $\mu$): receives one of three c.i.i.d. signals $S \in \{S_1, S_2, S_3\}$ (not necessarily ordered) that have distribution or likelihood function

| $\Pr(S|V)$ | $V_1$ | $V_2$ | $V_3$ |
|-----------|------|------|------|
| $S_1$     |      |      |      |
| $S_2$     |      |      |      |
| $S_3$     |      |      |      |
Properties of the Signal Likelihood Function

1. Increasing:

\[ \Pr(S|V_1) < \Pr(S|V_2) < \Pr(S|V_3). \]

2. Decreasing:

\[ \Pr(S|V_1) > \Pr(S|V_2) > \Pr(S|V_3). \]

3. U-shape:

\[ \Pr(S|V_1) > \Pr(S|V_2), \Pr(S|V_3) > \Pr(S|V_2). \]

4. Hill-shape:

\[ \Pr(S|V_1) < \Pr(S|V_2), \Pr(S|V_3) < \Pr(S|V_2). \]
More on Signals

- $\Pr(S|V_1) < \Pr(S|V_3) \rightarrow$ positive bias.
- $\Pr(S|V_1) > \Pr(S|V_3) \rightarrow$ negative bias.
- U shape and negative bias $\rightarrow$ nU shape etc.
- **Informative Private Signals/no cascade condition.**
  
  At each history $H^t$

  
  there exists $S \in \mathcal{S}$ such that $E[V|H^t, S] \neq E[V|H^t]$  \hspace{1cm} (1)
Glosten-Milgrom Sequential Trade (cont’d)

- **Market maker** (MM) sets bid-ask price.
  - He *sells* at the ask.
  - He *buys* at the bid.
- World is perfectly competitive → MM makes zero expected profits and sets prices

\[
ask^t = E[V|H^t, \text{a buy at time } t \text{ at ask}^t], \\
bid^t = E[V|H^t, \text{a sale at time } t \text{ at bid}^t].
\]

- Call public expectation, \(E[V|H^t]\) the *average price*, \(\text{bid}^t < E[V|H^t] < \text{ask}^t\).
- \(\text{ask}^t - \text{bid}^t\) increases in \(\mu\) (probability that a trader is informed).
Useful first insights I

For buy herding:

- sell at $H_1$: $E[V|S, H^1] - E[V|H^1] < 0$.
- buy at $H_t$: $E[V|S, H^t] - E[V|H^t] > 0$.
Lemma

For any signal $S$, $E[V|S]$ is less than $E[V]$ if and only if $S$ has a negative bias.

$\Rightarrow$ Must have $\Pr(S|V_1) > \Pr(S|V_3)$.

Lemma

If $E[V|H^t] > E[V]$ then $q_3^t > q_1^t$.

$\Rightarrow$ Must have $q_3^t > q_1^t$. 

Useful first insights II
Useful first insights III

Lemma

For any $S$, $t$ and $H^t$, $E[V|S, H^t] - E[V|H^t]$ has the same sign as

$$q_2q_3[Pr(S|V_3) - Pr(S|V_2)] + q_1q_2[Pr(S|V_2) - Pr(S|V_1)] + 2q_1q_3[Pr(S|V_3) - Pr(S|V_1)]$$

$\Rightarrow$ For switch from $E[V|S] < E[V]$ to $E[V|S, H^t] > E[V|H^t]$ cannot have increasing or decreasing signal.
Proposition (Necessary Conditions)

(a) Type $S$ buy herds only if $S$ is nU shaped and sell herds only if $S$ is pU shaped.

(b) Type $S$ acts as a buy contrarian only if $S$ is nHill shaped and acts as a sell contrarian only if $S$ is pHill shaped.

- Suppose $S$ buys herds $\Rightarrow$ can’t be increasing/decreasing.
- Suppose $S$ is nHill shaped: LF puts less weight on the tails of and more on the center relative to the public belief.
- Shift from the tails towards the center is more for value $V_3$ than for $V_1$ because of the negative bias.
- Buy herding $\Rightarrow q_{1t}^t < q_{3t}^t$.
- $\Rightarrow$ Redistribution of probability ensures that $S$’s expectation is less than public expectation.
- Hence an nHill shaped $S$ cannot be buying $\Rightarrow S$ must be nU.
Sufficiency: Insight Number One I

- $S$ is nHill or nU $\Rightarrow$ buy herding/ buy contrarianism candidates.
- To illustrate the mechanism, consider two extreme outcomes:
  1. The occurrence of state $V_1$ can be ruled out.
  2. The occurrence of state $V_3$ can be ruled out.
- How will nU and nHill types trade?
Sufficiency: Insight Number One II

• Suppose there is a history so that, relative to $V_3$ and $V_2$, state $V_1$ does not occur $\Rightarrow q_1 \approx 0 \Rightarrow$

$$q_2q_3[\Pr(S|V_3) - \Pr(S|V_2)] + q_1q_2[\Pr(S|V_2) - \Pr(S|V_1)] + 2q_1q_3[\Pr(S|V_3) - \Pr(S|V_1)] \
\approx q_2q_3[\Pr(S|V_3) - \Pr(S|V_2)]$$

• $\Rightarrow$ for all practical purposes, two state world $(V_2, V_3)$.

1. Hill shaped type has a signal favouring the “lowest state” $(V_2)$.
   $\Rightarrow \Pr(S|V_3) - \Pr(S|V_2) < 0$
   $\Rightarrow$ Should sell $\Rightarrow$ no contrarianism or herding!

2. U shaped type has a signal favouring the “highest state” $(V_3)$.
   $\Rightarrow \Pr(S|V_3) - \Pr(S|V_2) > 0$
   $\Rightarrow$ Should buy $\Rightarrow$ Herd buying!
Sufficiency: Insight Number One III

- Suppose there is a history so that, relative to \(V_1\) and \(V_2\), state \(V_3\) does not occur \(\Rightarrow q_3 \approx 0 \Rightarrow\)

\[
q_2q_3[\Pr(S|V_3) - \Pr(S|V_2)] + q_1q_2[\Pr(S|V_2) - \Pr(S|V_1)] + 2q_1q_3[\Pr(S|V_3) - \Pr(S|V_1)]
\approx q_1q_2[\Pr(S|V_2) - \Pr(S|V_1)]
\]

- \(\Rightarrow\) for all practical purposes, two state world \((V_1, V_2)\).
  1. Hill shaped type has a signal favouring the “highest state” \((V_2)\).
    \(\Rightarrow \Pr(S|V_2) - \Pr(S|V_1) > 0\)
    \(\Rightarrow\) Should buy \(\Rightarrow\) **Buy Contrarian!**
  2. U shaped type has a signal favouring the “lowest state” \((V_1)\).
    \(\Rightarrow \Pr(S|V_2) - \Pr(S|V_1) < 0\)
    \(\Rightarrow\) Should sell \(\Rightarrow\) **no contrarianism or herding!**
Sufficiency: Insight Number Two

- The formal model uses bid and ask prices with $\text{ask}^t > \mathbb{E}[V|H^t] > \text{bid}^t$.
- Herding/contrarianism requires type $S$ to trade.
- Need to ensure that bid-ask spread is “tight enough” so that $S$ trades.
- Specifically, $\mu$ must be small enough at two points:
  1. At $H^1$ so that $\mathbb{E}[V|S, H^1] < \text{bid}^1$.
  2. At $H_t$ when $\mathbb{E}[V|S, H^t] > \mathbb{E}[V|H_t]$ must also have $\mathbb{E}[V|S, H^t] > \text{ask}^t$.
- Intuition: As $\mu \searrow 0$, $\text{bid}^t \nearrow \mathbb{E}[V|H^t]$ and $\text{ask}^t \searrow \mathbb{E}[V|H^t]$.
Lemma (Possibility of Herding and Contrarian Behavior)

1. Suppose that signal $S$ is nU shaped. Then there exist $\mu^i$ and $\mu^s_{bh} \in (0, 1]$ such that $S$ buy herds if $\mu < \mu_{bh} \equiv \min\{\mu^i, \mu^s_{bh}\}$ and if the following holds: For any $\epsilon > 0$ there exists a history $H^t$ such that $q^t_1/q^t_l < \epsilon$ for all $l = 2, 3$.

2. Suppose signal $S$ is nHill shaped. Then there exist $\mu^i$ and $\mu^s_{bc} \in (0, 1]$ such that $S$ acts as a buy contrarian if $\mu < \mu_{bc} \equiv \min\{\mu^i, \mu^s_{bc}\}$ and if the following holds: For any $\epsilon > 0$ there exists a history $H^t$ such that $q^t_3/q^t_l < \epsilon$ for all $l = 1, 2$. 
1. Finding the right noise level

- Initial noise level $\mu^i$ is unique.
- But: $\mu_{bh}$ and $\mu_{bc}$ may not be unique. Depends on which other signal type trades with the herding/contrarian candidate.
- For some parametric configurations, no restriction on $\mu$ is needed (namely, if the herding/contrarian candidate is the only type to trade).
Causes for Pain and Suffering II

2. Is there a history with “$q_1$ sufficiently small relative to $q_2$ and $q_3$”?  
   - For some signal structures: no problem (sufficiently many more buys than sales will do the trick); example: MLRP signals  
   - Other signal structures: very involved (2-stage histories needed).  
   - But bottom line: it can be done (terms and conditions apply).  
   - Easiest case: There exists a trade such that both $q_1/q_3$ and $q_1/q_2$ or both $q_3/q_1$ and $q_3/q_2$ decrease uniformly (independent of time).  
   - Example: for some $\epsilon$

   $$\beta_t^i > \beta_t^j + \epsilon \text{ for any } i > j \text{ and any } t.$$  
   - Holds with MLRP distributions (coming up).
Proposition (Sufficient Conditions)

(a) Let $S$ be nU shaped. Assume also that if one other signal is pU shaped then the third signal has a non negative bias. Then there exists $\mu_{bh} \in (0, 1]$ such that $S$ buy herds if $\mu < \mu_{bh}$.

(b) Let $S$ be pU shaped. Assume also that if one other signal is nU shaped then the third signal has a non positive bias. Then there exists $\mu_{sh} \in (0, 1]$ such that $S$ sell herds if $\mu < \mu_{sh}$.

(c) Let $S$ be nHill shaped. Assume also that if one other signal is pHill shaped then the third signal has non positive bias. Then there exists $\mu_{bc} \in (0, 1]$ such that $S$ is a buy contrarian if $\mu < \mu_{bc}$.

(d) Let $S$ be pHill shaped. Assume also that if one other signal is nHill shaped then the third signal has a non negative bias. Then there exists $\mu_{sc} \in (0, 1]$ such that $S$ is a sell contrarian if $\mu < \mu_{sc}$. 
All things considered, the general result is

**Theorem (Herding and Contrarianism Characterisation Result)**

(a) **Herding.** (i) Necessity: If type $S$ herds, then $S$ is U shaped with a non-zero bias. (ii) Sufficiency: If there is a U shaped type with a non-zero bias, there exists $\mu_h \in (0, 1]$ such that when $\mu < \mu_h$ some informed type herds.

(b) **Contrarianism.** (i) Necessity: If type $S$ acts as a contrarian, then $S$ is Hill shaped with a non-zero bias. (ii) Sufficiency: If there is a Hill shaped type with a non-zero bias, there exists $\mu_c \in (0, 1]$ such that when $\mu < \mu_c$ some informed type acts as a contrarian.
The literature on asymmetric information often assumes that the information structure satisfies the Monotone Likelihood Ratio Property (MLRP).

MLRP: for \( S_l < S_h \) and \( V_l < V_h \),

\[
\text{Pr}(S_h|V_l)\text{Pr}(S_l|V_h) < \text{Pr}(S_h|V_h)\text{Pr}(S_l|V_l)
\]

**Theorem (Herding and Contrarianism with MLRP)**

Assume signals are ordered, \( S_1 < S_2 < S_3 \), and that the signal structure satisfies the MLRP. Then the following holds:

(a) Let \( S_2 \) be \( nU \). Then there exist \( \mu_{bh} \in (0, 1] \) such that \( S \) buy herds if and only if \( \mu < \mu_{bh} \).

(b) Let \( S_2 \) be \( nHill \). Then there exist \( \mu_{bc} \in (0, 1] \) such that \( S \) acts as a buy contrarian if and only if \( \mu < \mu_{bc} \).
Implications of social learning: Overview I

1. Prices can move substantially during herding and contrarian episodes even with MLRP.
   • Surprising! (Loose but wrong intuition: little information revealed during a herding episode).

2. To measure impact on *liquidity* and *price volatility* compare price paths:
   • when agents know public information (transparent market)
   • hypothetical economy in which herding/contrarian does not occur (e.g. agents are naïve and ignore the public information).

3. Result:
   • price rises and falls are more extreme in transparent market
   • Liquidity (measured by bid-ask spreads) declines when herding/contrarianism starts.
Implications of social learning: Overview II

- Implications for “market transparency”: (traders in hypothetical world do not use/have access to past public data ⇒ less transparent). Social learning causes transparent markets to be more volatile, less stable, and less liquid.

- Possible empirical implication: serial correlation of trading activities (buys beget more buys) as a measure of social learning should go hand in hand with an relative increase in price volatility and bid-ask-spreads.
Persistent Herding and Self-Defeating Contrarianism

Assume MLRP. Consider any history $H^r = (a^1, \ldots, a^{r-1})$ and suppose that $H^r$ is followed by $b \geq 0$ buys and $s \geq 0$ sales in some order; denote this history by $H^t = (a^1, \ldots, a^{r+b+s-1})$.

1. If there is buy herding by $S$ at $H^r$ then there exists an increasing function $\bar{s}(\cdot) > 1$ such that $S$ continues to buy herd at $H^t$ if and only if $s < \bar{s}(b)$.

2. If there is buy contrarianism by $S$ at $H^r$ then there exists an increasing function $\bar{b}(\cdot) > 1$ such that $S$ continues to act as a buy contrarianism at $H^t$ if and only if $b < \bar{b}(s)$. 
Proposition: Social Learning and the Price Range

Let signals obey MLRP.

1. Consider any history $H^r = (a^1, \ldots, a^{r-1})$ at which there is buy herding (contrarianism). Then for any $\epsilon > 0$, there exists history $H^t = (a^1, \ldots, a^{t-1})$ following $H^r$ such that there is buy herding (contrarianism) at every $H^\tau = (a^1, \ldots, a^{\tau-1})$, $r \leq \tau \leq t$, and the average price $E[V|H^{r+\tau}]$ exceeds $V_3 - \epsilon$ (is less than $V_1 + \epsilon$).

2. Let $\mu$ admit buy herding (contrarianism). Then for every $\epsilon > 0$ there exists a $\delta > 0$ such that if $\Pr(V_2) > 1 - \delta$ there is a history $H^t = (a^1, \ldots, a^{t-1})$ and a date $r < t$ such that (i) there is buy herding (contrarianism) at every $H^\tau = (a^1, \ldots, a^{\tau-1})$, $r \leq \tau \leq t$, (ii) $E[V|H^r] < V_2 + \epsilon$ ($E[V|H^r] > V_2 - \epsilon$) and (iii) $E[V|H^t] > V_3 - \epsilon$ ($E[V|H^t] > V_1 + \epsilon$).
Establishing a Benchmark for Volatility

• Three questions. Once herding/contrarian behaviour starts:
  1. Will liquidity, as measured by the difference between the bid and ask prices, be less with social learning than in the case with no social learning (no types changing behaviour)?
  2. Will further buys move prices more/less with social learning than without?
  3. Will sales move prices more/less with social learning than without?

• For the comparison, we compare to hypothetical economy in which
  1. $\forall t$ each informed trader takes the same action as that he would take at the initial history.
  2. MM sets prices taking into account that the informed behaves as above.
Proposition (The Impact of Herding and Contrarian Behavior on Volatility)

1. Once buy herding starts the average price in the transparent world is larger after any number of buys than in the opaque world.

2. Once buy herding starts the average price in the transparent world is lower for a certain number of sales than in the opaque world (while herding lasts).

3. An analogue of this holds for buy contrarianism.
Proposition (Formally: Relative Volatility)

Assume MLRP. Consider any finite history $H^r = (a^1, \ldots, a^{r-1})$ at which the priors in the two markets coincide: $q^r_i = q^r_{i,o}$ for $i = 1, 2, 3$. Suppose that $H^r$ is followed by $b \geq 0$ buys and $s \geq 0$ sales in some order; denote this history by $H^t = (a^1, \ldots, a^{r+b+s-1})$. Assume that there is buy herding at $H^\tau$, for every $\tau = r, \ldots, r+b+s$.

1. Suppose $s = 0$. Then $E[V|H^t] > E_o[V|H^t]$ for any $b > 0$.

2. Suppose $b = 0$. Then there exists $\bar{s} \geq 1$ such that $E[V|H^t] < E_o[V|H^t]$ for any $s \leq \bar{s}$.

3. For any $s$ there exists $\bar{b}$ such that $E[V|H^t] > E_o[V|H^t]$ for any $b > \bar{b}$.
Liquidity

Proposition (The Impact of Herding and Contrarian Behavior on Liquidity)

Consider any history $H^t$ at which type $S$ engages in buy herding or buy contrarianism.

1. **The ask price when the buy herding or buy contrarian candidate $S$ rationally buys exceeds the ask price when he chooses not to buy.**

2. **The bid price when the buy herding or buy contrarian candidate $S$ rationally buys is lower than the bid price when he chooses to sell.**
Herding starts and the rational price increases above the naïve price. The rational price drops below the naïve price.
More States, More Signals

- Results don’t hinge on 3-everything setting → extend to $N$ state world.
- Assume
  - values of the asset are on an equal grid,
    \[
    \{V_1, V_2, \ldots, V_N\} = \{0, V, 2V, \ldots, (N - 1)V\}
    \]
  - prior probability distribution is symmetric,
    \[
    \Pr(V_i) = \Pr(V_{N+1-i}).
    \]
- Buy herding/contrarian: candidate must sell at the initial history ($E[V|S] < E[V]$ is necessary).
- E.g. $E[V|S] < E[V]$ if signal $S$ is negatively biased in the sense that $\Pr(S|V_i) > \Pr(S|V_{N+1-i})$ for all $i < (N + 1)/2$. 
\( N \)-States: Results have same flavour I

- \( E[V|S,H^t] - E[V|H^t] \) has the same sign as

\[
\sum_{j=1}^{N-1} \sum_{i=1}^{N-j} (V_{i+j} - V_i) \cdot q_i q_{i+j} [\Pr(S|V_{i+j}) - \Pr(S|V_i)]
\]

**Lemma**

*With \( N \) states, a signal type \( S \) with an increasing or decreasing LF will never switch from buying to selling or vice versa.*
Lemma (Possibility of Herding and Contrarian Behavior)

(i) Let $S$ be negatively biased & $\Pr(S|V_{N-1}) < \Pr(S|V_N)$ & for all $\epsilon > 0$ exists $H^t$ such that $q_i^t/q_l^t < \epsilon$ for all $l = N - 1, N$ and $i < N - 1$. Then there exists a $\mu_{bh} \in (0, 1]$ such that $S$ buy herds if $\mu < \mu_{bh}$.

(ii) Let $S$ be negatively biased & $\Pr(S|V_1) < \Pr(S|V_2)$ & for all $\epsilon > 0$ exists $H^t$ such that $q_i^t/q_l^t < \epsilon$ for all $l = 1, 2$ and $i > 2$. Then there exists a $\mu_{bc} \in (0, 1]$ such that $S$ is a buy contrarian if $\mu < \mu_{bc}$.
Theorem (Herding and Contrarianism with $N$ States)

Assume that signals satisfy the MLRP and let signal $S$ be negatively biased.

(a) If $\Pr(S|V_{N-1}) < \Pr(S|V_N)$ then there exists $\mu_{bh} \in (0, 1]$ such that $S$ buy herds if $\mu < \mu_{bh}$.

(b) If $\Pr(S|V_1) < \Pr(S|V_2)$ there exists $\mu_{bh} \in (0, 1]$ such that $S$ is a buy contrarian if $\mu < \mu_{bh}$.
Is herding economically significant?

- Answer: Depends on the prevalence of herding types.

<table>
<thead>
<tr>
<th>$\text{Example 1}$</th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
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<tbody>
<tr>
<td>$S_D$</td>
<td>.01</td>
<td>.009</td>
<td>0</td>
</tr>
<tr>
<td>$S_U$</td>
<td>.99</td>
<td>.98</td>
<td>.985</td>
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<tr>
<td>$S_I$</td>
<td>0</td>
<td>.011</td>
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</table>

- Signal $S_U$ is the herding-U-shaped signal.
- At least 98% of informed traders switch behavior when herding begins.
- Herding may have large effect.
• Herding and contrarianism does not arise under all circumstances.

• But: the conditions under which it does arise have very intuitive interpretations. (e.g. when there is ‘great uncertainty’ (many U shaped signals) in markets.)

• Herding and contrarianism has noticeable effects on prices and liquidity.

• Herding is self-sustaining, contrarianism is self defeating.