Chapter 6: Correcting Market Distortions: Shadow Prices Wages & Discount Rates
- Observed market prices sometimes reflect true cost to society. In some circumstances they don't because there are distortions which prevent market prices from conveying true economic values.

- When this occurs have to correct observed price to calculate the shadow price.

- Types of distortions include taxes, subsidies & other forms of gov’t intervention.

- In competitive markets $D$ represents marginal benefits to society and supply curve social costs. Social costs are equal to private costs. Likewise private benefits equal social benefits.

- Suppose have a market for good but price observed for the good includes a per unit tax, here price consumers pay is not the price the firms keeps.

$$T \text{ – is the tax}$$

$$P_c = P_f + T \quad P_c \text{ – price gross of tax}$$

$$P_f \text{ – price net of tax}$$

- Suppose look at good and there’s a project that requires the good as an input.
- Demand for the good increases
- leads to new equilibrium at point C
- Output increases from $X_e$ to $X_y$
- Price firms retain increases from $P_f$ to $P_f'$
- Price consumers pay increases from $P_c$ to $P_c'$

- Non-project demand for the firm falls from $X_e$ to $X_c$
- Gov’t requirement of $X_G$ comes from two sources
  $X_f - X_e$ – units of new supply
  $X_e - X_c$ – units of displaced demand

- If market weren’t distorted by the tax, there would not be a program because consumers marginal benefit would equal the firms marginal costs, this not the case here because of tax
- Tax has driven a wedge between consumers’ and firms’ valuation of this input.
- What the shadow price does is take a weighted average of the opportunity costs of the two sources of the gov’t’s input requirement.

Eg. Suppose the gov’t needs $X_G$ units of $X$ to complete the project, can calculate $P_G$ the shadow price as either:

$$P_G = P_f \left( \frac{X_f - X_g}{X_G} \right) + P_c \left( \frac{X_s - X_g}{X_g} \right)$$

OR

$$P_G = P_f \left( \frac{X_f - X_g}{X_G} \right) - P_c \left( \frac{X_s - X_g}{X_g} \right)$$

$P_f$ – net of tax
$P_c$ – price gross of tax ($P_c = P_f + T$)

Can rewrite $P_G$ in terms of elasticity’s as:

$$P_G = P_f \cdot \frac{E_S - E_D}{E_S - E_D} \cdot P_c$$

$E_S$ – elasticity of supply curve
$E_D$ – elasticity of demand curve

- Shadow price $P_G$ will depend critically on elasticities, elasticities will determine how big increases are in new demand as well as how big is displaced demand.

- Elasticity determines the slope of D & S curves.

→ $D_1$ is flatter than $D_2$
→ $D_1$ is more elastic than $D_2$

Some special cases.
a.)

\[ E_S = 0 \rightarrow P_G = P_c \text{ gross of tax price} \]

- E.g. – limited supply of inputs
- Heavy duty cranes

b.)

\[ E_S \rightarrow \infty \rightarrow P_G = P_f \text{ net – of – the tax price} \]

- Idle capacity in input market

c.)

\[ E_D = -\infty \rightarrow P_G = P_c \text{ gross of tax price} \]

- For local inputs

d.)

\[ E_D = \infty \rightarrow P_G = P_f \text{ net – of – tax price} \]

- input a necessity

**Distortionary Subsidies**
SHADOW WAGE RATES

- governments sometimes use discretionary fiscal policy to create jobs.

- if labour markets functioned perfectly market wages would reflect true opportunity cost of hiring labour for a project

- However, markets usually don’t function perfectly because of distortions so have to calculate the shadow price of labour. i.e. the shadow wage rate.

- Can draw labour for public projects from three sources
  i) those employed elsewhere in the economy
  ii) the voluntary unemployed
  iii) the involuntary unemployed

Discuss each of these in turn:

- i.) Those employed elsewhere in the economy:
  - When draw someone who already has a job into a public job, the value of the output they produced is forgone.
  - Measure by their marginal product of labour i.e. market wage
  - Recall firm is a profit maximizer or cost minimizer so they expand output as long MPL exceeds the wage rate.
  - Denote market wage by w₁

- ii.) Voluntary unemployed:
  - i.e. retired people or people in school
  - opportunity cost of these people is the minimum amount required to induce them to work
  - measure their opportunity cost by the wage the project pays them w₂

- iii.) Involuntary unemployed
  - people willing to work at the market wage but unable to find a job
  - can argue that this opportunity cost is zero
  - However can also argue that market wage is not sufficiently large enough to induce them to work [should work for the market wage rate X]
  - Denote this wage by w₃
- Since don’t really know what this wage is you have to make some assumptions about its values
- One assumption that is used is that the value is at least 20% greater than the market wage.

- Let $\alpha_1, \alpha_2, \alpha_3$ denote the proportions of the type of labour hired e.g. Types (i) to (iii) then shadow wage rate for the project is given by:

$$W_s = \alpha_1 W_1 + \alpha_2 W_2 + \alpha_3 W_3$$

Where $\alpha_1 + \alpha_2 + \alpha_3 = 1$

- The above is one crude way of evaluating the shadow wage, other more sophisticated methods:

I) Hughes formula for the shadow wage

$$W_s = P_i \cdot W_i + (1 - P_i) \cdot V \cdot W_i$$

$W_i$ – wage rate paid by workers on the project

$U$ – Unemployment rate in region

$U_{\text{min}}$ – historical minimum of unemployment rate in regions

$U_{\text{max}}$ – historical maximum of unemployment rate in regions

$V$ – Proportion of wage paid that represents the opportunity cost of hiring involuntary unemployed labour

$P_i$ – probability of hiring a person who is already employed.

$1 - P_i$ – probability of hiring someone who is involuntary unemployed.

$P_i$ is computed as

$$P_i = 1 - 0.5 \left[ \sin \left( \frac{\pi}{2} \left( \frac{U - U_{\text{min}}}{U_{\text{max}} - U_{\text{min}}} \right) \right) \right] + 1$$

Where $\sin (\cdot)$ is evaluated in radians.

- How is $V$ computed? We can assume different values for it and see how estimate of shadow wage differs
- For example, Pinfold proposes the following estimate of $V$:

$$V = W \left( 1 - t_w \right) - UI \left( 1 - t_{UI} \right)$$
W – wage rate
UI – UI benefits (computed as a percentage of wages
t_w – tax rate on wages
t_{ui} tax rate on UI benefits

Choosing a Discount Rate

\[
\text{NPV} = \sum_{t=0}^{T} \frac{(B_t - C_t)}{(1 + r)^t}
\]

r – is the discount rate
the NPV depends just as critically on the value of \( r \), i.e. the discount rate, as \( B_t \) and \( C_t \).

- a smaller discount rate will lead to larger values of the NPV, large values of the
discount rate lead to smaller values of the NPV

- A discount rate of zero means that society weights future benefits as much as costs
today (Assume costs exceed benefits early on)

- will discuss some of the methods for picking a discount rate.

Marginal Rate of Time Preference

- problem of $100 today versus $100 tomorrow

- most people have a preference to consume sooner rather than latter; this is
referred to as time preference

- The rate at which individuals make these time preference is called the individuals
marginal rate of time preference (MRTP)

Example: Income year 1: $5000 Income year 2: $9000.
Suppose you have the choice of $1000 today or $1200 tomorrow. Suppose you
pick to have the $1000 today, then you have MRTP of 20 percent

Suppose max U \( (C_1,C_2) \)

Subj to \( C_1 + C_2 = T \)
\[ i = \text{interest rate, } T \text{ – PV of Income for two year} \]
Absolute value of slope of the indifference curve measures the rate at which individuals are indifferent between substituting current consumption for future consumption - called MRS between consumption this year and consumption next year.

\[(1 + p) - \text{MRS}\]
but \(p\) – marginal rate of time preference

To find equilibrium need to find point where indifference curves and budget constraint are tangent to each other.

- if you can freely borrow then you will shift consumption from the future to the present until the MRTP falls to the interest rate you must pay
  
  if \(i > \text{MRTP}\) then save and reduce consumption
  if \(i < \text{MRTP}\) then borrow and increase consumption

In perfect capital market MRTP = market interest rate.

ii) Investment demand
- look at firms making investment decisions
- perfect capital markets
- firm has a variety of investment projects to select from which have different rates of return associated with them.
- Can plot them as follows

Savings schedule
- supply of funds for investment is provided by individual saving
- if rate of interest > rate of time preference then save
- represented by Aggregate savings
Interest Rate determination

In the real world, markets aren’t perfect and there are also distortions, e.g. Taxes, risk, gov’t borrowing, and this drives wedges between market and social outcomes, end up with under investment.

**Marginal Rate of Return on Private Investment**

- D₀; S₀ – Investment demand and supply of funds represented in absence of taxes
- Suppose there are corporate taxes as well as personal taxes
- Shifts supply and demand curves back (D₁ & S₁) (reduces investment)

- Given taxes, market clearing rate would be i - savers receive i before paying taxes
  - Borrowers pay i to get funds
- marginal return on investment before taxes is r_x (opportunity cost of forgone investment)
- marginal return on savings after taxes would be p_z (gap between I and p_z represent taxes paid on savings
- If gov’t implements a project, demand for funds increase
- private sector savings increase by ΔC
Arnold Harberger using this framework suggests the following estimate of the social discount rate.

\[ S = a r_z + (1-a) p_z \]

\[ a = \frac{\Delta I}{\Delta I + \Delta C} \quad (1-a) = \frac{\Delta C}{\Delta I + \Delta C} \]

- Empirical evidence suggests that savings is not responsive to interest rates => $S$ curve is vertical
- \( \Delta C \approx 0 \) which suggests that \( a \approx 1 \) and \( (1-a) \approx 0 \)
  \[
  \rightarrow \text{suggests that } r_z \text{ is a good approximation to social discount rate}
  \]

- Another approximation to social discount rate would be \( p_z \), people argue in favour of \( p_z \) as an approximation to social discount rate because social discount rate should be rate at which individuals should be willing to postpone a small amount of consumption for future consumption.

**How to estimate \( p_z \) and \( r_z \)**  
(computed using U.S. figures)

\( r_z \) – proxies for a rate of return on low risk private sector investments before taxes but after correcting for inflation

- start with nominal return on long term corporate bond  
  e.g. average monthly yield on Moody’s AAA bonds between 1947 and 1999 is 6.86%  
  (1) Figure out before return  
    - Corporate tax rates average about 35%  
      \[
      \frac{0.0686}{1 - 0.35} = 10.55\%
      \]
  (2) Adjust before tax return for inflation (i.e. real terms)  
    - between 1947 and 1999 inflation averaged 3.92%  
      \[
      \frac{1.1055 - 0.392}{1 + 0.0392} = 0.0638 \rightarrow 6.38\%
      \]
  (3) Adjust for bias in CPI  
    - [inflation tends to be underestimated by about 1%]  
  \[
  r_z \approx 7.38\%
  \]

**Calculating \( p_z \)**

- start with interest rate on gov’t bonds  
  e.g. 10 year U.S Gov’t bond average 6.77% between 1953 & 1999
- In U.S, personal tax rate averages 30% (In Canada about 40%)
- CPI averaged 3.98% between 1953 and 1999

(1) Figure out after tax yield
After tax yield: \(0.0677(1-0.3) = 4.74\%\)

(2) Adjust for inflation (i.e., real terms)
After inflation: \(\frac{0.0474 - 0.0398}{1 + 0.0398} = 0.73\%

(3) Adjust for bias in CPI
- if adjust for bias in CPI add 1%

→ 1.73% is social discount rate

**Criticisms of \(r_z\) approach**

- tends to produce large discount rate estimates
- using corporate bond, which may have a risk premium (e.g. firm may go bankrupt, investors want a higher return to cover this)

**Criticisms of \(p_z\)**

- discount rate too low, individuals may not properly account for the long run effects of infrastructure programs on future generations

**Weighted Social opportunity cost of Capital (WSOC)**

- Another proposal is to calculate the social discount rate in terms of the social opportunity cost of required resources with weighted based on the relative contributions of the different services of the resources.

- \(WSOC = ar_z + bi + (1-a-b)p_z\)

- \(a\) : proportion of project’s resources that displace private investment
- \(b\) : proportion of resources that are financed by borrowing from foreigners
- \(1-a-b\) : proportion if resources displacing domestic consumption
- \(i\) : Gov’t’s real long-term borrowing rate.

Since \(p_z < i < r_z\) => \(p_z < WSOC < r_z\)

Example: (Adjust gov’t bond for inflation) (need \(i\) to compute WSOC)
Previous example U.S. gov’t 10 year bond average 6.77% between 1953 and 1999

Adjust for inflation (3.98%)
\[
\frac{0.0677 - 0.0398}{1 + 0.0398} = 2.68\%
\]

+ 1% for CPI underestimates

= 3.7%

Estimates of \(a\) and \(b\) are harder to obtain

For Canada
\[
\begin{align*}
    a &= 0.75 \\
    b &= 0.20 \\
    1 - a - b &= 0.05
\end{align*}
\]

using other numbers we have

\[
\text{WSOC} = ar_z + bi + (1 - a - b) p_z
\]
\[
\begin{align*}
    &= 0.75 \times 0.0738 + 0.2 \times 0.037 + 0.05 \times 0.0173 \\
    &= 0.05535 + 0.00074 + 0.00087 \\
    &= 0.05696 \\
    \rightarrow &= 5.7\%
\end{align*}
\]

Suppose project is financed by taxes, then \(b = 0\)

The weight \(a\) should represent a proportion of taxes that reduce investment and \(1 - a - b\) should represent a proportion of taxes that reduce consumption.

- can estimate \(a\) with ratio: gross fixed investment to real GDP
E.g. in 1998 this was 16.8%

previous example

\[
\begin{align*}
    \text{WSOC} &= 0.168 \times 0.0738 + 0 \times 0.037 + 0.832 \times 0.0173 \\
    &= 0.0124 + 0 + 0.0144 \\
    &= 0.0268 \\
    \approx &= 2.7\%
\end{align*}
\]