• Cost Benefit Analysis is concerned with answering the question “Will a project make a society better off?”

• To answer this question, we first need to consider social benefits and costs of a project.
  – How are those benefits / costs measured?

• Consider the utility function $U(x_1, ..., x_N)$. A household has a preference ordering over $x_1, ..., x_N$ goods and services.
Definition: Pareto Improvement

• A project will be a Pareto improvement if at least one household is made better off and no household worse off.
Pareto Superior and Pareto Inferior

• **Definition: Pareto Superior**
If a project is Pareto improving, then the project is Pareto superior to the status quo.

• **Definition: Pareto Inferior**
If a project is *not* Pareto improving then the project is Pareto inferior to the status quo.
Problems with Pareto Outcomes

Drawback of Pareto outcomes, hard to implement as well as hard to measure welfare of society.

• For example:
  – Suppose there are $k$ persons in society.
  – $K-1$ persons are in favour of a project.
  – 1 person is opposed since it makes him/her worse off
– Pareto improvement criteria implies do not proceed with project

• Also, how do we measure utility?
  – Utility functions are ordinal, i.e. they rank things, so this is fine for one person, but how do we aggregate rankings over all persons (e.g., does every get the same weight, do some people count more than others?)

• Need to find alternatives that are easier to implement.
Willingness-to-Pay Measures

• Transform utility into something that is easier to measure

• Two measures:
  – Compensating Variation
  – Equivalent Variation
Compensating Variation

• Answer to following question “what is the maximum amount you are willing to pay for this project?”
  – Reply to this sort of question, a positive dollar amount, is the compensating variation.
• Note that this is a proxy for the change in utility, not actual change in utility.
• For negative outcomes use reword question, “what is the minimum amount you are willing to accept in order to induce you to tolerate the project?”, the answer is expressed in dollars, but should be treated as a negative amount.
Equivalent Variation

• Compensating variation questions are framed in terms of a new project being undertaken, equivalent variation questions are interested in what happens if a project is foregone.
• For positive benefits (record answer as positive)
  – “What is the minimum amount you would be willing to accept to forego a project?”
• For negative benefits (record answer as negative)
  – “What is the maximum amount you are willing to pay in order to prevent the project from being undertaken?”
Advantages of Compensating and Equivalent Variation

• Since measured in dollars, can aggregate them.

• For example,
  • \( CV = CV_1 + \ldots + CV_N \)
  • \( EV = EV_1 + \ldots + EV_N \)

• Each household or individual in the sums above is given equal weight.
• If the CV is positive or zero for most households and positive for at least one household, then have a Pareto improvement.
• If some households have negative CVs and others are positive. If the sum of the CVs is positive the gains exceed the losses, so it is positive to compensate those that lose from undertaking a project and still be better off.
• The preceding is called a potential Pareto Improvement, and project passes compensation test. Note that this is hypothetical not actual compensation so compensation may not occur if project proceeds.

• Note CV >0 need not imply that a project will increase social welfare, unless you compensate the individuals that are made worse off; recall that CV is the sum of individual CV_i terms, some of which could be negative.
• In absence of direct compensation, the government can use tax or transfer policies to redistribute income (assumes marginal utility of income is equal).

• An alternative compensation test is based on the equivalent variation, \( EV = EV_1 + \ldots + EV_N \).
• If EV>0 then the project is a potential Pareto improvement

• EV>0 means that the minimum amounts of those in favour of a project are willing to pay exceeds maximum amounts of those opposed to project are willing to present.
  – This means you could compensate those who are opposed to a project
The Double Compensation Test

• If $CV > 0$ and $EV > 0$ then the project passes a double test and it should proceed.
• If $CV < 0$ and $EV < 0$ then the project should be rejected.
• If $CV > 0$ and $EV < 0$, or $CV < 0$ and $EV > 0$ then you have an indeterminate outcome.
  – The above is called the Scitovsky Paradox (or Reversal).
Scitovsky solution to the paradox (i.e., conflicting answers from the equivalent and compensation variation) is to use double tests:

- If CV > 0 and EV > 0 – proceed with project
- If CV < 0 and EV < 0 – do not proceed with project
- If either CV or EV < 0 – the compensation tests do not offer definitive guidance.
Getting Information on CV and EV

• There are a few approaches you could take.
• First, you can run a survey and ask respondents the CV and EV questions and use their answers.
• There are some problems with this approach:
  – Not everyone might respond to the survey.
  – Can also get reporting errors, e.g., people can answer questions depending on how they are worded, i.e., a negative wording might get a different answer than something phrased positively. This needs to be accounted for when designing the survey.
Getting Information on CV and EV

• An alternative approach is to use markets to infer this information
• Suppose the initial price is $P_1$ and $X_1$ units are consumed
• The most consumers will be willing to pay for these units is area under demand curve $A + B + D$. The amount they will eventually pay is $P_1 \times X_1$ or $B + D$; the consumer surplus will be different between the two, i.e. $A$. 
• If the price falls to $P_2$, consumers will be better off because the consumers surplus increases by B+C.

• Can use the change in consumers surplus, $\Delta CS$, to approximate or bound the CV and EV.

• The following relationships are known and depend on the income elasticity of demand:
  – If you have an normal good, i.e., a good with an income elasticity greater than 0 then $EV > \Delta CS > CV$
  – If you have an inferior good, i.e., a good with an income elasticity less than 0 then $EV < \Delta CS < CV$
  – If the income elasticity is zero, then $EV = \Delta CS = CV$
• If the demand for a project is independent of income, i.e., the income elasticity is zero, then the change in consumers surplus that results from a price change will exactly measure the CV and EV

• In practice, the change in consumers surplus $\Delta CS$ will be a good approximation to EV and CV when the income elasticity of a good is close to zero
Measuring benefits

• Consider 3 goods, X, Y and Z
• Y is a substitute for X
• Z is a complement for X
Suppose the price of X falls, this will have an effect on the markets for Y and Z; the demands for Y and Z will shift.
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<td>X</td>
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• Since income hasn’t changed any change in expenditure on X must be offset by changes in expenditures on Y and Z; this implies that the Initial Expenditure = New Expenditure

• B+D+F+G+H=D+E+F+H+J

• Simplifying, you get B=E-G+J

• In economics, there are two tricks that you can use you simplify expressions, you can either add 0 or multiply by 1 in a clever way
• Adding 0 in a clever way, i.e., add C to both sides of B=E-G+J, you get:

• B+C=C+E-G+J

• This can be interpreted as follows
  – B+C is the change in the consumers surplus induced by the price change
  – C+E is the total amount consumers are willing to pay for extra units of $X_2 - X_1$, this price increase allows them to consume; i.e., the total increase in benefit from consuming more X
- G+J is the loss from consuming less Y and J is the increase in benefit from consuming more Z

• So you can measure the change in benefits by using the change in consumers surplus (net benefits) or by looking at all the effected markets and calculating the total change in benefits

• Can double check measures of welfare change with two alternative measures based on change in benefits described by the demand curves
• To get the welfare change need to also consider the resource costs, then the Welfare change will be
  – Change in Welfare = Change in Benefits – Change in Resource Costs
• Change in resource costs will be measured by supply curve, which is sum of individual firms MC curves
• This assumes that there are no distortions in the market so that the MC of the firm is equal to the MC to society;
  – When market is distorted we say that there is a wedge between private and social costs, i.e., an externality or a tax
  – Easiest to illustrate with flat MC,
• In the distorted market, i.e., the market for good k, $MC \neq Market\ Price$, this might occur because of an externality or some sort of tax, this is sometimes called a “wedge”

• If this is the case then you need to measure the benefits in market k, with $MC_k$ (social cost) instead of $P_k$ (private cost)