ECMB36 LECTURE NOTES

DISCOUNTING AND NET PRESENT VALUE

Townley, Chapters 2 & 3
• Many private and public decisions can have important consequences that extend overtime.

• Assume discount rate is given, will figure out how to determine discount rate later in the course.
Simple Example

• Consider a 1-period problem, that is today and tomorrow, today is time 0 and tomorrow is time 1
• Can buy a parcel of land for $10 million.
• Can sell a parcel of land for $11 million
• This is illustrated on the next slide, with benefits above the time axis and costs below the time axis
• Question:
  – Should you buy the land?
  – Suppose the government can take the money it was planning to use to buy the land and buy some T-bills that pay 5%? Is buying the land a good investment?

• There are 3 ways to answer this and each produces the same answer
Future Value Analysis

- Compares what government would receive in the future if it invests in the project and what it will receive if it invests money in the T-bills.
- If buy $10,000,000 in T-bills, you will have $10,500,000 in a year.
- This is the future value of the T-bills, because it reflects what the government will have if it buys the T-bills.
- The government can compare future value of T-bills with future value of land to decide if it purchases land.
Future Value Analysis

• Since future value of land is greater, it should purchase the land.

• In general the future value of the land will be $FV = X(1 + i)$, where $X$ is the amount and $i$ is the annual interest rate.

• For more than 1 time period, say $n$ years, the future value can be written as $FV = X(1 + i)^n$. 
Present Value Analysis

• Present value analysis compares the amount of money the government must invest in T-bills today in order to have the same amount in a year that it will have if it buys the land with the amount it will invest in T-bills now if it doesn’t buy the land.

• The present of the land is the amount the state must invest in T-bills that yield 5% in order to obtain the value of the land in a year (11,000,000).
• This is computed as $X(1+0.05)=11,000,000$
which can be rewritten as
$X = \frac{11,000,000}{1+0.05} = 10,476,190$
Illustrating Present Value Analysis

\[ PV(B) = 10,476,190 \]

\[ PV(C) = 10,000,000 \]

\[ NPV = 476,190 \]
• Present value of amount that will be received in the future will be \( PV = \frac{X}{(1+i)} \)

• For \( n \) periods, the formula will be \( PV = \frac{X}{(1+i)^n} \)

• Note the term \( \frac{1}{(1+i)^n} \) equals the present value of $1 received in \( n \) years when the interest rate is \( i \) is sometimes called the discount factor
• Suppose the government wants to undertake a restructuring in 3 years that will cost $100,000; if the interest rate is 6% what is the present value of the $100,000 (i.e., what does the government have to set aside today to have $100,000 with a 6% interest rate)

\[ PV = \frac{\$100,000}{(1+0.06)^3} = \frac{\$100,000}{1.19102} = \$83962 \]

• The discount factor in this example \[\frac{1}{(1+0.06)^3} = 0.8396\]
• If a project yields benefits in many different periods we can compute the present value of the whole stream by adding up the present values of the benefits received in each period.

• If $B_t$ benefits received in period $t$, for $t = 0, 1, 2, \ldots, n$ then the present value of the stream of benefits, denoted $PV(B)$ is:

$$PV(B) = \frac{B_0}{(1+i)^0} + \frac{B_1}{(1+i)^1} + \cdots + \frac{B_{n-1}}{(1+i)^{n-1}} + \frac{B_n}{(1+i)^n} = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t}$$
• Similarly, if $C_t$ costs are incurred in period $t$, for $t = 0, 1, 2, \ldots, n$ then the present value of the costs, denoted $PV(C)$ is: 

$$PV(C) = \frac{C_0}{(1+i)^0} + \frac{C_1}{(1+i)^1} + \cdots + \frac{C_{n-1}}{(1+i)^{n-1}} + \frac{C_n}{(1+i)^n} = \sum_{t=0}^{n} \frac{C_t}{(1+i)^t}$$
• Comparing projects across time can be a little tricky, consider the following example:
  – Project I: yields a benefit of $10,500 in 4 years
  – Project II: yields a benefit of $5,500 in 4 years and $5,400 in 5 years

• Which project is better?
  – Need to compute the present value of each
• PV(Project I): \( \frac{10500}{(1.08)^4} = \$7718 \)

• PV(Project II): \( \frac{5500}{(1.08)^4} + \frac{5400}{(1.08)^5} = \$4043 + \$3675 = \$7718 \)

• The present value of the two projects are the same.
Illustrating the previous example

For Project I:
- **Benefits ($)**
- **Costs ($)**
- **Present Value (PV(I)) = 7,718**
- Costs at 1 year: 3,675
- Costs at 4 years: 10,500
- **Time (years)**: 0, 1, 2, 3, 4, 5

For Project II:
- **Benefits ($)**
- **Costs ($)**
- **Present Value (PV(II)) = 7,718**
- Benefits at 4 years: 5,500
- Costs at 5 years: 5,400
- **Time (years)**: 0, 1, 2, 3, 4, 5
Net Present Value Analysis

• This method calculates the present value of all the benefits and costs of a project and sums them to obtain the net present value of the project.

• For example, land example:
  – PV(B)=10,476,190
  – PV(C)=10,000,000 (Initial Purchase Price)
  – NPV=PV(B)-PV(C) =10,479,190-10,000,000=479,190
Illustrating Present Value Analysis

\[ PV(B) = 10,476,190 \]
\[ PV(C) = 10,000,000 \]
\[ NPV = 476,190 \]
Another Example Applying NPV

• Suppose you are about to retire. You are also married. You are entitled to receive a pension while you are alive and when you die your spouse will receive a survivor benefit, which is less than your pension.

• You need to pick a payment option, which determine the size of your pension and the size of the survivor benefit.

• The options are on the next slide.
These are your payment options

Your Pension Payment Options

<table>
<thead>
<tr>
<th>Monthly Payments as of 07–01–2015</th>
<th>Payable to you</th>
<th>Payable to Your Spouse Upon Your Death</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ 60% Joint and Survivor Pension</td>
<td>$10,171.39</td>
<td>$6,102.83</td>
</tr>
<tr>
<td>Total Monthly Payment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>□ 70% Joint and Survivor Pension</td>
<td>$9,876.93</td>
<td>$6,913.85</td>
</tr>
<tr>
<td>Total Monthly Payment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>□ 80% Joint and Survivor Pension</td>
<td>$9,598.94</td>
<td>$7,679.15</td>
</tr>
<tr>
<td>Total Monthly Payment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>□ 90% Joint and Survivor Pension</td>
<td>$9,336.32</td>
<td>$8,402.69</td>
</tr>
<tr>
<td>Total Monthly Payment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>□ 100% Joint and Survivor Pension</td>
<td>$9,087.53</td>
<td>$9,087.53</td>
</tr>
<tr>
<td>Total Monthly Payment</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Which Payment Option is Best?

• To evaluate the options need to make some assumptions because you need inputs into NPV
  – Assume discount rate is 3%
  – Scenario 1: Assume that you will live 16 years and your spouse dies 5 years after you die
  – Scenario 2: Assume that you live another 10 years and your spouse dies 11 years after you die
Assuming Scenario #1

\[ \sum_{t=0}^{16} \frac{(12+10171)}{(1+0.03)^t} + \sum_{t=17}^{21} \frac{(12+6102.83)}{(1+0.03)^t} = 1.8642 \times 10^6 \]

\[ \sum_{t=0}^{16} \frac{(12+9876.93)}{(1+0.03)^t} + \sum_{t=17}^{21} \frac{(12+6913.85)}{(1+0.03)^t} = 1.8441 \times 10^6 \]

\[ \sum_{t=0}^{16} \frac{(12+9598.94)}{(1+0.03)^t} + \sum_{t=17}^{21} \frac{(12+7679.15)}{(1+0.03)^t} = 1.8251 \times 10^6 \]

\[ \sum_{t=0}^{16} \frac{(12+9336.32)}{(1+0.03)^t} + \sum_{t=17}^{21} \frac{(12+8402.69)}{(1+0.03)^t} = 1.8071 \times 10^6 \]

\[ \sum_{t=0}^{16} \frac{(12+9087)}{(1+0.03)^t} + \sum_{t=17}^{21} \frac{(12+9807.53)}{(1+0.03)^t} = 1.8146 \times 10^6 \]
Assuming Scenario #2

\[
\sum_{t=0}^{10} \frac{12 + 10171}{(1+0.05)^t} + \sum_{t=11}^{21} \frac{12 + 6102.83}{(1+0.05)^t} = 1.4380 \times 10^6
\]

\[
\sum_{t=0}^{10} \frac{12 + 9876.93}{(1+0.05)^t} + \sum_{t=11}^{21} \frac{12 + 6913.85}{(1+0.05)^t} = 1.4568 \times 10^6
\]

\[
\sum_{t=0}^{10} \frac{12 + 9598.94}{(1+0.05)^t} + \sum_{t=11}^{21} \frac{12 + 7679.15}{(1+0.05)^t} = 1.4745 \times 10^6
\]

\[
\sum_{t=0}^{10} \frac{12 + 9336.32}{(1+0.05)^t} + \sum_{t=11}^{21} \frac{12 + 8402.69}{(1+0.05)^t} = 1.4913 \times 10^6
\]

\[
\sum_{t=0}^{10} \frac{12 + 9087.53}{(1+0.05)^t} + \sum_{t=11}^{21} \frac{12 + 9087.53}{(1+0.05)^t} = 1.5072 \times 10^6
\]
Comparing Scenario 1 & 2

• If you live a long time and your spouse dies a few years after you, payment option #1, 60% Joint and Survivor Pension, is always best

• If you die soon and your spouse dies a long time after you, payment option #5, 100% Joint and Survivor Pension, is the best option
• The Net Present Value (NPV) of a project is difference between of Benefits and of Costs: NPV = PV(B) - PV(C)
  – If NPV > 0 then PV(B) > PV(C) and should undertake project.
  – If NPV < 0 then PV(B) < PV(C) and should not undertake project.

• With multiple periods, NPV can be written as

$$ NPV = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+i)^t} = \sum_{t=0}^{n} \frac{(B_t-C_t)}{(1+i)^t} $$
Library Example for NPV

• Suppose the U of T library is interested in purchasing a new information system that will give users access to a number of online databases for five years. The benefits of the system are said to be $100,000 per annum (cost savings to library and benefits to users). The system costs $325,000 to purchase and setup and $20,000 to maintain. After five years the system will be inadequate for U of T’s needs and will be dismantled and sold to either York or Ryerson for $20,000. Assume the discount rate is 7%.
Diagram Illustrating Library Example

Benefits ($)

85,558
76,290
81,630
87,344
93,458

$PV(B) = 424,280$

Costs ($)

325,000
18,691
17,469
16,326
15,258
14,260

$PV(C) = 407,004$

$NPV = 17,276$
# Table Presenting Benefits and Costs

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
<th>Annual Benefits</th>
<th>Annual Costs</th>
<th>Annual Net Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Purchase and install</td>
<td>0</td>
<td>325,000</td>
<td>−325,000</td>
</tr>
<tr>
<td>1</td>
<td>Annual benefits and costs</td>
<td>100,000</td>
<td>20,000</td>
<td>80,000</td>
</tr>
<tr>
<td>2</td>
<td>Annual benefits and costs</td>
<td>100,000</td>
<td>20,000</td>
<td>80,000</td>
</tr>
<tr>
<td>3</td>
<td>Annual benefits and costs</td>
<td>100,000</td>
<td>20,000</td>
<td>80,000</td>
</tr>
<tr>
<td>4</td>
<td>Annual benefits and costs</td>
<td>100,000</td>
<td>20,000</td>
<td>80,000</td>
</tr>
<tr>
<td>5</td>
<td>Annual benefits and costs</td>
<td>100,000</td>
<td>20,000</td>
<td>80,000</td>
</tr>
<tr>
<td>5</td>
<td>Liquidation</td>
<td>20,000</td>
<td>0</td>
<td>20,000</td>
</tr>
<tr>
<td></td>
<td><strong>PV</strong></td>
<td><strong>$424,280</strong></td>
<td><strong>$407,004</strong></td>
<td><strong>$17,276</strong></td>
</tr>
</tbody>
</table>
Computing the NPV for the Library Example

- PV(B) = $424,280; PV(C) = $407,004; NPV = PV(B) – PV(C) = $424,280 - $407,004 = $17,276
  - Since NPV > 0 should purchase information system

- Alternative, and equivalent approach: \( NPV = \sum_{t=0}^{5} \frac{(B_t - C_t)}{(1+0.07)^t} \)

\[
\begin{align*}
\sum_{t=0}^{5} \frac{(B_t - C_t)}{(1+0.07)^t} &= \frac{(0-325000)}{(1+0.07)^0} + \frac{(100000-20000)}{(1+0.07)^1} + \\
&+ \frac{(100000-20000)}{(1+0.07)^2} + \frac{(100000+20000-20000)}{(1+0.07)^3} + \\
&+ \frac{(100000-20000)}{(1+0.07)^4} + \frac{(1+0.07)^5} = 17276
\end{align*}
\]
Some Special Cases of NPV

• All costs are incurred initially and benefits occur only in ensuing years $NPV = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t} - C_0$

• Some projects have an infinite life, replace $n$ with $\infty$, $NPV = \sum_{t=0}^{\infty} \frac{(B_t-C_t)}{(1+i)^t}$

• For an infinite life project another approach to evaluation might be to select a relatively short discounting period (useful life of project) and include a terminal (or horizon) value to reflect all subsequent benefits and costs (and try different values of terminal value) $NPV = \sum_{t=0}^{k} \frac{(B_t-C_t)}{(1+i)^t} + T(k)$
Estimating the Terminal Value

- Use information on costs and benefits to project the terminal value.
  - For example, if the useful life of a project is 35 years, you need to project the net benefits for the years after the 35\textsuperscript{th} year. Suppose net benefits are $1 million in the 35\textsuperscript{th} year and are expected to grow at 1.5 \% per year and the interest rate is 8\%, can project forward using the Gordon growth formula \( T(k) = \frac{net\text{benefit}}{i-g} \) \( T(k) = \frac{1000000}{0.08-0.015} = 15,384,615.38 \)
Estimating the Terminal Value

• The terminal value can also be estimated using the scrap or liquidation value. This method is appropriate when: 1) No other costs or benefits arise beyond the discounting period; 2) There is a well functioning market in which to value the asset.; and, 3) The market values reflect social values (i.e., no externalities).

• However, if you are demolishing something there could be the scrap value in what’s left over, e.g., some wood is quite valuable as are metals (e.g., steel, cooper, nickel)
Estimating the Terminal Value

• Use depreciated value of asset/project. This method estimates the (economic) depreciation of an asset based on empirical market studies of similar assets and then subtracts this amount from the initial value. (One never uses accounting depreciation in CBA).

• Estimate the terminal value using some arbitrary proportion of the initial construction cost.

• If there is a really long discounting period set the present value of subsequent net benefits to zero.
Internal Rate of Return Criteria

• The third way to evaluate a stream of benefits over time.
• The internal rate of return (IRR) of a project equals the discount rate at which the project’s NPV would equal 0. It indicates the effective annual rate of return on the project.
  – This is often used in the resource sector (mining and oil exploration) where the IRR for various projects are computed and the company picks the best projects to develop.
Internal Rate of Return Criteria

• The IRR is computed as follows

\[ (B_0 - C_0) + \frac{(B_1 - C_1)}{(1 + IRR)} + \cdots + \frac{(B_T - C_T)}{(1 + IRR)^T} = 0 \]

• You take the IRR from above and compare it to the social discount rate, if IRR > social discount rate then proceed with the project; if not, don’t.

  – Of course the above equation is very difficult to solve by “hand” even for a short time period, so it requires either someone writes a program or uses the function in Excel that computes it.
Concluding Remarks

- While there are three methods to deal with streams of benefits and costs over time, we’ll be focused on using the present value, the NPV in the rest of the course to determine whether or not to proceed with a project.