Abstract

The level of profits that can be sustained in repeated price game is higher when consumers are long-lived and firms can intertemporally bundle their output. With short-lived consumers, it is well known that any price and profit, up to the monopoly profit, can be sustained in a subgame perfect Nash equilibrium (SPNE) as long as the number of firms does not exceed \( \frac{1}{\delta} \), or equivalently, the discount factor is greater than \( \frac{n-1}{n} \), and that marginal cost pricing is the unique SPNE otherwise. We show that when firms face long-lived, repeat-purchase consumers and are free to offer intertemporal bundles, strictly positive profits can be supported for any number of firms and for any strictly positive discount factor. One equilibrium strategy that increases profits is to temporally segment the market with staggered long-term contracts and exploit the fact that consumers anticipate future price cuts in response to current deviations. A second, potentially more profitable, equilibrium strategy is to sell unstaggered multi-period contracts and simultaneously offer single-period discounts in the period in which the multi-period contracts are being renewed. The single-period contracts are not purchased in equilibrium, but they limit firms’ incentives to cut price.

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1 Introduction

Intertemporal bundling is a common practice. Products, such as newspapers and magazines, and services, such as cellular telephone and DSL internet access, are often sold using multi-period service agreements enabling firms to easily bundle their goods and services across time. This practice, which we call intertemporal bundling, occurs when firms’ contract offers require customers to commit to multi-period service agreements.\footnote{Not all multi-period service agreements are examples of intertemporal bundling. In particular, some multi-period service agreements leave the consumer free to change service providers at anytime, and instead only restrict the firm, either by limiting its ability to increase price or to terminate service.} We show that such service agreements may facilitate tacit collusion. That is, if consumers are small and firms are free to intertemporally bundle, then firms competing with one another will be able to sustain higher profits when consumers are long-lived.

In stark contrast to the traditional result on tacit collusion with short-lived consumers, we show that when consumers are long-lived and intertemporal bundling is feasible, firms can sustain supranormal profits for any number of firms and any discount rate. We think this finding is counterintuitive in two respects. First, it implies that long-lived consumers are worse off than short-lived consumers. Indeed, long-lived consumers would be better off if they could collectively commit to act like short-lived consumers. Second, since intertemporal bundling seems to increase the time between offers, one might think that it effectively lowers the discount factor and makes tacit collusion more difficult to sustain. However this intuition is misleading. Intertemporal bundling does not restrict the time between offers, only the time between consumers’ purchase decisions. In particular, punishments are just as swift and consumers realize this when making their purchase decisions.

To better understand the later point, and more generally, to better understand the role of intertemporal bundling in our model, it helps to consider what happens when firms are restricted to offer only long-term contracts. Suppose, for example, firms are restricted to offer twelve-month intertemporal bundles that begin and end on January 1st even though it is feasible to sell just once a month. This clearly makes tacit collusion more difficult since it increases the gains to business stealing.

Now suppose that firms are exogenously required to offer twelve-month contracts, but those contracts can begin on the 1st of any month. In this case, firms’ ability
to tacitly collude is unchanged. Consumers fully anticipate that a price war will begin starting one month after a deviation occurs, so the largest profit a deviator can capture is the entire market profit for just one month. If the deviator attempts to capture more surplus from consumers, the consumers will delay their purchase for a month and purchase from the rival firms at marginal cost. In other words, the gains to business stealing, and the difficulty of sustaining tacit collusion, are proportional to the time between offers, *not the length of the contracts.*

In fact, intertemporal bundling makes it even easier to tacitly collude, not harder. One way to see this is to realize that firms can temporally segment the market. When the market is temporally segmented, the ability to capture the entire market through a price deviation is removed. For example, suppose firms offer twelve-month contracts, and for some exogenous reason consumers’ contract renewals are staggered. Then each month only a twelfth of the consumers are looking to sign a new contract. A deviating firm can increase its short run profits by lowering its price, but it can only capture one twelfth of the market, and it can only capture the surplus in that market for one-month. However, the loss associated with retaliatory price cuts is forgone future profits in all twelve segments, so tacit collusion is easier to sustain.

Formally, we show that intertemporal bundling facilitates higher profits even when consumers are not exogenously staggered and even when the duration of firms’ contracts is chosen at the time the contracts are offered. Because we assume all consumers are present from the start of time, when firms offer staggered long-term contracts in equilibrium, they must initially offer consumers incentives to induce them to subsequently stagger their purchases. Despite the short-run costs of these incentives, they facilitate higher profits than can be sustained when firms face short-lived consumers (or firms are barred from using multi-period contracts).

We also consider a second class of equilibria in which firms can tacitly collude even more easily. In these equilibria, firms offer a discount on the first month’s service in every year. The same discount is offered through both an annual contract as well as a spot contract that each firm simultaneously offers. Since these two contracts are equally attractive, the consumers are willing to sign exclusively the annual contracts. And, because consumers all sign the annual contract at the beginning of each year, competition in the remaining eleven months is avoided. The existence of rival firms’ spot contracts limits any firm’s incentive to deviate because consumers always have the option to respond to a deviation by signing a rival’s spot contract and then waiting for the price war to begin. In fact, a deep enough discount on the first month’s service can eliminate any incentive to deviate.
This second class of equilibria can always sustain the same profit levels as the first and for some discount factors and numbers of firms it can sustain even higher profits. However, the first class of equilibria is of independent applied interest because it more closely resembles the observed behavior of firms.

The paper is organized as follows. We begin with a brief review of the related literature. We then describe the model and review the set of SPNE when firms face short-lived consumers (or are restricted to one-period contracts). The next two sections analyze two specific classes of SPNE with long-lived consumers and intertemporal bundling. In Section 6 we prove that any equilibrium profit level that is sustainable in a SPNE can be sustained using a SPNE in our second class. That is, there are no symmetric equilibria which yield higher profits. Section 7 considers some basic extensions and generalizations of our model. In particular, we analyze forward contracts and show that forward contracts can also be used to achieve the set of profits sustainable using intertemporal bundling. Section 8 concludes.

2 Literature

Our paper is related to the literature on product bundling and to the literature on multimarket contact. The traditional multi-product bundling literature suggests, by analogy, potential rationales for intertemporal bundling. For example, if consumers’ valuations are negatively correlated across time, firms with market power can reduce the deadweight loss by intertemporal bundling (see Adams and Yellen, 1976). Also, intertemporal bundling might be a mechanism for leveraging market power into the future (see Whinston, 1990, etc.).

In a competitive setting, Cabral and Villas-Boas (2005) show that bundling of unrelated products can lead to more homogeneous tastes and thus more intense competition among firms. Based on this result, the authors suggest that intertemporal bundling may intensify competition. Our conclusions stand in sharp contract with theirs.

A few papers have looked more specifically at intertemporal bundling. In the be-

\footnote{As in Whinston (1990), bundling in our model is valuable because of its impact on the behavior of rivals. However, the strategic reason for bundling in Whinston’s paper is to deter entry while the strategic reason in our paper is to soften price competition among existing firms.}
havioral economics literature, Loewenstein, Donoghue, and Rabin (2003) considered a model of projection bias. Consumers possessing “projection bias” tend to rely too much on their current tastes to estimate their future preferences. They conjectured that a firm may use intertemporal bundling to take advantage of consumers with projection bias who currently place a high value on its product. DellaVigna and Malmendier (2004) analyze firms’ optimal contract design when customers are time inconsistent and partially naive about it. The authors show that when a product brings consumers immediate benefit and postponed cost, firms tend to specify a high per usage fee in a subscription contract and when a product brings consumers immediate cost and postponed benefit, firms tend to set a per usage fee below the marginal cost. In our analysis, consumers are fully rational and their rational expectations play a crucial role in helping firms achieve higher profit through tacit collusion.\(^3\)\(^4\)

Our paper is also related to the literature on multimarket contact. If markets are exogenously temporally separate, then it follows from Bernheim and Whinston’s (1990) analysis of multi-market contact that tacit collusion is easier to sustain. For example, if one market is open only in January, another market is open only in February, a third only in March, and so on, then tacit collusion is easier to sustain if firms face the same rivals in all twelve markets then if they face independent firms. Multimarket contact facilitates tacit collusion only because it is impossible to simultaneously steal business in all twelve markets, but retaliation can occur in all twelve markets.

Our work is related because we consider equilibria of our repeated price game in which consumers renew their multi-period service contracts at different times. However, we have no exogenously distinct markets in our model. If consumers renew their contracts at different times, it is because they choose to do so. Nevertheless,

\(^3\)Perhaps the most common given justification for intertemporal bundling is that there are fixed costs of establishing service. While these costs undoubtedly exist in many environments, it isn’t immediately apparent why they must be recouped with service agreements rather than installation fees.

\(^4\)DeGraba and Mohammed (1999) studied how a capacity-constrained firm can induce a buying frenzy by first selling only a bundle of multiple products and then only later selling products individually. In this frenzy, consumers who place high values on one product but low values on the other are forced to purchase a bundle of both products in the first period, because they anticipate that their preferred product will be rationed in the second period. Rationing is a self-fulfilling prophecy when consumers respond to such expectation by rushing to purchase the bundle. While the authors describe this tactic as “intertemporal mixed bundling” consumers in their model do not make repeat purchases. The seller is changing his multi-product bundling strategy over time, not bundling consumption across different points of time.
one way to interpret this class of equilibria is that each firm’s incentive to deviate is weakened because it knows that if it steals other firms’ business in one “segment” it will be punished in every “segment”.

Finally, there are other papers in which consumers’ anticipation of retaliatory price wars effects firms ability to sustain high profits. First, Gul (1987) (see also Ausubel and Deneckere, 1987, and Dutta, Matros, and Weibull, 2003) considers a model of durable-goods, oligopoly pricing and shows that the Coase conjecture does not hold with more than one firm because firms find it easier to tacitly collude as the time between offers shrinks. Consumers who rationally anticipate a price war upon seeing a deviation will have an incentive to wait for the price war to begin, so a deviator will only be able to capture the time value of the consumers’ surplus. Durable goods can be thought of as a sequence of non-durable goods that have been intertemporally bundled. In this sense, Gul considers a model in which consumers are purchasing a service that is exogenously intertemporally bundled. In contrast, we allow firms to chose whether or not to intertemporally bundle their product.

Second, Liski and Montero (forthcoming) show that tacit collusion is easier to sustain when firms can offer forward contracts. They assume that there are multiple contracting periods associated with every transaction period and consider an equilibrium in which consumers sign contracts in an early period. If firms react to a deviation by cutting price, no firm has an incentive to deviate because forward looking consumers will wait and purchase in a later contracting period at an even lower price. And since much of the demand was already served in the early contracting period, there is also little incentive to deviate in a later period. We consider the role of forward contracts in our model in Section 6 of our paper.

3 The Model

We consider a standard infinitely repeated oligopoly price game. However, unlike the standard model though, we explicitly assume that consumers are long-lived.

There are $n$ infinitely-lived firms selling a homogeneous and perfectly divisible product. Each firm has zero unit costs.

There are a finite number, $m$, infinitely-lived, homogeneous consumers. Aggregate demand in each period is normalized to one. Each consumer derives valuation
for up to $\phi = 1/m$ units of the good in each period of their lives and derives valuation 0 from their outside option. Disposal of the good at any time is costless.\textsuperscript{5}

Firms and consumers have a common strictly positive discount factor, $\delta \in (0, 1)$.

Each period, firms simultaneously announce a menu of contracts, which can vary in both price and length, and then all of the consumers choose either a contract from among the firms’ menus of offers, or their outside option. The outside option for consumers who do not have a prior contract is consuming their default outside option with valuation 0 for one period. The outside option for consumers who have a prior contract is to continue consuming from their existing contract (which they are free to dispose of at any time).

Firms’ one-period contract offers are denoted simply by their price, $p_1$. Firms’ multi-period contracts are denoted by a price-duration pairs $\{P, k\}$ where $P$ is the present discounted value of the stream of per-period payments specified in the contract and $k$ is the contract length.\textsuperscript{6}

The exposition of the paper is made easier when we suppose that multi-period contracts require a stream of payments as opposed to a single lump sum payment up front (in particular this makes it easier to compare a multi-period contract to a series of single-period contracts). Since contracts are binding and both firms and consumers share a common discount factor, any two $k$-period contracts whose streams of payments have the same present discounted value are equivalent. So we will typically refer to a multi-period contract, $\{P, k\}$, as a contract requiring a stream of $k$ identical payments, which we will write as $\{p_s\}_{s=1}^k$, where the present discount value of this stream is

$$P = \sum_{s=1}^{k} \delta^{s-1} p_s.$$  

\textsuperscript{5}For simplicity we allow individual consumers to arbitrarily divide their purchases across firms and across contracts within firms. As a consequence we do not need to assume that $m$ is divisible by $n$ or $nk$ or consider the case where contract shares are different due to indivisibilities. However, this assumption can be easily relaxed.

\textsuperscript{6}Note that implicit in this specification is the assumption that firms do not offer forward contracts. However, in section 6 of the paper we show that the equilibria we construct are robust to a relaxation of this assumption.
Benchmark: SPNE without intertemporal bundling

An important benchmark is the case in which intertemporal bundling is not feasible. If the firms are restricted to make one-period offers, or equivalently consumers live only one period, the SPNE of the game are well-known (see, for example, Tirole, 1988).

In this case, the timing is as follows: Each period, firms simultaneously announce their prices for a single period of service, and then consumers simultaneously choose among all of the firms’ offers and their outside option.

Claim 1 When intertemporal bundling is not feasible, then i) if \( n \leq \frac{1}{1-\delta} \) then any level of profit between zero and the monopoly profit is sustainable in a symmetric SPNE, and ii) if \( n > \frac{1}{1-\delta} \), the unique sub-game perfect Nash equilibrium outcome is zero profits (marginal cost pricing).

4 Staggered Contract Equilibria: SPNE with Intertemporal Bundling I

In this section, and the next, we consider equilibria of the game in which firms use intertemporal bundling to sustain higher profits than are feasible using only spot contracts. First, we consider a class of equilibria which we call Staggered Contract Equilibria. In this class of equilibria firms induce consumers to accept staggered contracts so that only a small fraction of consumers are at the end of the contractual obligation in any given period.

The set of Staggered Contract Equilibria are all SPNE in which strategies are of the following form:

Firms’ equilibrium path actions: In Period 1, the firms offer a one-period contract at a price \( p_1 \), a two-period contract \((p_1 + \delta p, 2)\), and an \( n \)-period contract \((p_1 + \sum_{s=2}^{n} \delta^{s-1} p, n)\) for all \( n \) less than or equal to \( k \). In Period 2 and beyond the firms offer just the \( k \)-period contract \( \left(\sum_{s=1}^{k} \delta^{s-1} p, k\right) \).
Consumers’ equilibrium path actions: In Period 1, consumers’ aggregate purchases are divided equally among each of the \( nk \) different contracts offers \( k \) offers per firm and \( n \) firms. Individual consumers may sign just one contract, or may divide their purchases across contracts within or between firms. However, if \( m \) is not divisible by \( nk \), then at least some consumers must sign multiple contracts. In Period 2 and beyond, consumers renew their contracts by signing \( k \)-period contracts with the same firm or firms that they signed contracts with in period 1. So the aggregate demand in every period is \( 1/nk \) for each firm’s contract.

Off-the-equilibrium-path actions following a firm deviation: If any firm (or firms) deviates from these equilibrium strategies, then starting in the subsequent period every firm offers one-period contracts at a price equal to marginal cost forever. Following such a deviation consumers maximize their consumer surplus given current prices and the expectation of all future prices being equal to marginal cost.

Off-the-equilibrium-path actions following a consumer deviation: If any consumer deviates from these equilibrium strategies, firms continue to offer their equilibrium path contracts in all the periods that follow, and, in addition, offer contracts designed to return the deviating consumers to their equilibrium path strategies. Specifically, if in period \( ak + t' \) a consumer is available who failed to sign his or her equilibrium path contract with firm \( i \) in some earlier period \( bk + t \), where \( t \neq t' \), then firm \( i \) offers an additional contract at the same period price as its other contracts and with a duration chosen so that the contract expires in period \( (a + 1)k + t \). The available consumer accepts this new contract and all other consumers accept the equilibrium path contract offers.

We now characterize the set of Staggered Contract Equilibria and the conditions under which they exist. To do this we need to show that firms have no incentive to deviate from their equilibrium strategies in period 1, that firms have no incentive to deviate in period 2 or beyond, and that consumers have no incentive to deviate. In the limiting case when \( m \) goes to infinity, it is easy to see that consumers have no incentive to deviate since their individual behavior has no affect on firms’ profits. Below we focus on this case and ignore deviations by consumers. In the proof of Proposition 1 we analyze the full model.
**Period 1**

The firms’ profit on the equilibrium path is

\[ \frac{1}{n} \left[ p_1 + \delta \frac{p}{1 - \delta} \right]. \tag{1} \]

Since consumers anticipate marginal cost pricing from period 2 onwards after observing a deviation, a deviating offer will only be accepted if it is more attractive to the consumer than signing another firm’s one-period contract at the price \( p_1 \) and waiting for the price war to follow. Therefore, the profit a firm can earn from undercutting with a one-period contract (or any other contract) in period 1, is at most \( p_1 \). So no deviation is profitable in period 1 if

\[ \frac{1}{n} \left[ p_1 + \delta \frac{p}{1 - \delta} \right] \geq p_1, \tag{2} \]

or equivalently,

\[ p_1 \leq \frac{1}{n - 1} \delta \frac{p}{1 - \delta}. \tag{3} \]

**Period 2 and beyond**

Each firm’s profit on the equilibrium path in any period after period 1 is

\[ \frac{1}{n} \frac{p}{1 - \delta}. \tag{4} \]

If a firm deviates, consumers can respond either by accepting the offer, by postponing their consumption for one period, or by signing a rival’s \( k \)-period contract. It follows that if \( p \sum_{j=1}^{k} \delta^{j-1} > V \), then the most profitable deviation for a firm is a one-period contract at a price just below \( V \). Any offer that tries to extract more surplus from the consumer will be rejected because the consumer would prefer to wait one period. However, if \( p \sum_{j=1}^{k} \delta^{j-1} < V \), then the most profitable deviation for a firm is a \( k \)-period contract at a slightly lower price than the equilibrium one. In this case, if
the firm tries to extract more surplus from the consumer the consumer will sign a rival’s $k$-period contract.

So the profit associated with the optimal deviation is $\min\{\frac{V}{k}, \frac{p}{k} \sum_{j=1}^{k} \delta^{j-1}\}$. On the other hand, if a firm deviates, it gives up its equilibrium profit it earns from sales to the current cohort of consumers,

$$\frac{1}{nk} \frac{V}{1-\delta},$$

as well as the equilibrium profit it earns from renewals of the other $k-1$ cohorts of consumers,

$$\sum_{j=1}^{k-1} \frac{1}{nk} \frac{\delta^{j}V}{1-\delta}.$$

So no deviation is profitable as long as

$$\min\{\frac{V}{k}, \frac{p}{k} \sum_{j=1}^{k} \delta^{j-1}\} \leq \frac{1}{nk} \frac{p}{1-\delta} + \sum_{j=1}^{k-1} \frac{1}{nk} \frac{\delta^{j}p}{1-\delta}. \quad (5)$$

If $p \sum_{j=1}^{k} \delta^{j-1} < V$, or $p \in [0, \frac{V}{k \sum_{j=1}^{k} \delta^{j-1}})$, (5) becomes

$$\frac{p}{k} + \frac{p}{k} \sum_{j=1}^{k-1} \delta^{j} \leq \frac{1}{nk} \frac{p}{1-\delta} + \sum_{j=1}^{k-1} \frac{1}{nk} \frac{\delta^{j}p}{1-\delta},$$

which is satisfied if and only if $n \leq 1/ (1-\delta)$. If $p \sum_{j=1}^{k} \delta^{j-1} \geq V$, or $p \in [\frac{V}{k \sum_{j=1}^{k} \delta^{j-1}}, V]$, (5) becomes

$$\frac{V}{k} \leq \frac{1}{nk} \frac{p}{1-\delta} + \sum_{j=1}^{k-1} \frac{1}{nk} \frac{\delta^{j}p}{1-\delta}. \quad (6)$$

Since

$$\lim_{k \to \infty} \sum_{j=1}^{k-1} \frac{1}{n} \frac{\delta^{j}p}{1-\delta} = \frac{\delta p}{n(1-\delta)^2},$$

there exists a $k$ sufficiently large such that (6) is satisfied if and only if

$$V < \frac{p}{n(1-\delta)} + \frac{\delta p}{n(1-\delta)^2}.$$
or
\[
\frac{nV}{p} < \frac{1}{(1-\delta)^2}.
\]

So any second period (and beyond) price in \([0, V]\) can be supported in a SCE when \(n \leq \frac{1}{(1-\delta)}\), any second period price in \([n(1-\delta)^2 V, V]\) can be supported in a SCE when \(n \in \left[\frac{1}{1-\delta}, \frac{1}{(1-\delta)^2}\right]\), and no SCE exists when \(n \geq \frac{1}{(1-\delta)^2}\). In particular, the price \(p = V\) can be supported in a SCE as long as
\[
n < \frac{1}{(1-\delta)^2}.
\] (7)

**Equilibrium Profits**

Next we derive the range of profits sustainable in a *Staggered Contract Equilibrium* when \(n \in \left[\frac{1}{1-\delta}, \frac{1}{(1-\delta)^2}\right]\) (because for lower \(n\) any profit is sustainable with spot contracts and for higher \(n\) no SCE exists).

The highest firm profit sustainable in a *Staggered Contract Equilibrium* is equal to
\[
\frac{1}{n} \left[ p_1 + \frac{\delta}{1-\delta} V \right].
\] (8)

The equilibrium with the highest profit is the one in which \(p = V\) and \(p_1\) is the highest possible price. Since \(p_1\) must satisfy (3) and \(p_1 \leq V\), the upper bound on the firm’s equilibrium profit is
\[
\frac{1}{n-1} \frac{\delta}{1-\delta} V
\] (9)
or
\[
\frac{1}{n \frac{1}{1-\delta}} V,
\] (10)

whichever is smaller. Note that \(k\) affects whether or not the equilibrium is sustainable but not the maximal profit level when it is sustainable.

The lowest firm profit sustainable in a *Staggered Contract Equilibrium* is equal to \(\delta (1-\delta) V\). Recall, that the per-period price from period 2 onwards is bounded from below by \(n(1-\delta)^2 V\). So, since the lowest possible \(p_1\) is zero, the firm’s profit in a SCE must be at least \(\delta (1-\delta) V\).
The set of *Staggered Contract Equilibria* (SCE) are characterized in the following proposition.

**Proposition 1** For $\phi$ sufficiently small a Staggered Contract Equilibrium exists if and only if $n < \frac{1}{(1-\delta)^2}$. The range of industry profit levels that can be supported in a SCE is

$$[0, \frac{V}{1-\delta}] \quad \text{if } n < \frac{1}{1-\delta},$$

$$(n\delta (1-\delta)V, \frac{n\delta}{n-1(1-\delta)} V) \quad \text{if } n \in \left[\frac{1}{(1-\delta^2)}, \frac{1}{(1-\delta)^2}\right).$$

$$\text{(11)}$$

Proof: See Appendix 1. □

The intuition that using staggered contracts facilitates tacit collusion is best illustrated by considering the case in which $p = V$. First, as long as $p_1$ is sufficiently low, no firm has an incentive to deviate in the first period. However, the big advantage of staggered contracts is that they relax the incentive constraint in subsequent periods. Since customers correctly anticipate the price will fall to zero in the following period, the deviation profit in any subsequent period is no larger than $V$ per customer, or $V/k$ in total (since only $1/k$ customers are available in each period). Part of the profit foregone by the deviator is all of the future profit from the firm’s share of these $1/k$ consumers future renewals. This tradeoff ($V/k$ now versus $V/kn$ forever) within the current cohort of customers is the same as the tradeoff that a deviating firm must consider when there is no intertemporal bundling. However, when contracts are staggered, the deviating firm also foregoes all the renewal profits associated with the other $(k-1)/k$ consumers who are currently locked in to existing contracts. As the contract length $k$ increases, each cohort of customers will constitute a smaller fraction of the population of customers, and the benefit of deviation will decline both in absolute terms and relative to the cost of deviation. Hence, intertemporal bundling through staggered subscription contracts weakens firms’ incentives to deviate from a collusive outcome.
Discussion

Note that for $n = 2$, the incentive constraint is $\delta > 1 - 1/\sqrt{2} \approx .29$. In other words, intertemporal bundling using staggered contracts helps a duopoly sustain profitable tacit collusion when the discount factor lies between $.29$ and $.5$.

Comparing Claim 1 and Proposition 1 we can see that intertemporal bundling using staggered contracts helps the industry sustain tacit collusion when the number of firms is between $\frac{1}{1-\delta}$ and $\frac{1}{(1-\delta)^2}$. To get an idea of the power of staggering contracts, suppose, for example, that $\delta = 0.9 + \varepsilon$ for $\varepsilon > 0$ arbitrarily close to zero. In this case, when intertemporal bundling is not feasible, tacit collusion can be sustained in an industry with up to ten firms. But when firms can offer staggered contracts, tacit collusion can be sustained among a hundred firms.

Figure 1 graphically illustrates how staggered contracts help expand the set of equilibrium profit:

**Figure 1** Comparison of sets of equilibrium profits

In Figure 1, $\Omega_{NI}$ denotes the set of equilibrium profits that are sustainable, as a function of the number of firms, when intertemporal bundling is not feasible; and

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7 Note that $n \in \mathbb{N}$, but for ease of illustration Figure 1 treats $n$ as any positive real number.
\( \Omega_{NI} \cup \Omega_{SCE} \) denotes the set of equilibrium profits that are sustainable, as a function of the number of firms, when firms tacitly collude using staggered subscription contracts. Finally, \( \Omega_{NI} \cup \Omega_{SCE} \cup \Omega_{IDE} \) denotes the set of equilibrium profits that are sustainable, as a function of the number of firms, when firms tacitly collude using any contract. We show this, and explore more general contracts, in the next two sections.

As is seen in Figure 1, when \( n \in (\frac{1}{1-\delta}, \frac{1}{(1-\delta)^2}) \), only industry profit levels above \( n\delta (1 - \delta) V \) are sustainable in a SCE. An SCE with lower profits (and lower \( p \)) cannot be sustained because firms will strictly prefer to offer a slightly lower priced \( k \)-period contract. Because the candidate equilibrium price is so low, consumers will accept this contract rather than wait for the price war to begin.

**Generalizations**

Our equilibrium explicitly exploits the fact that consumers are unable to trigger a price war by coordinating their purchases. If every consumer accepted the same contract in the Period 1, then tacit collusion would be no easier. The failure to coordinate is a free rider problem. Consumers don’t expect others consumers to bear the cost, so they are unwilling to bear the cost.

However, our game is dynamic and it is plausible to imagine that consumers will bear the cost if they expect others to bear this cost in the future. Most importantly, while deviations by consumers after Period 1 are costly, once they have deviated consumers are indifferent among the firms’ contract offers, yet all consumers expect that any consumer who has deviated will accept a contract undoing their past deviations. While this makes coordination by consumers impossible, it appears to rely on consumers choosing weakly dominated strategies off the equilibrium path. However it is easy to generalize firms’ strategies in our equilibrium so that consumers who deviate must bear a positive cost not only when they first deviate, but every period thereafter until a price war begins. In this case, it is no longer as plausible that consumers might be able to coordinate their actions through sequential deviations.

There are two ways to generalize firm strategies. First, we can assume that firms can ration their sales. In particular, if firms limit their sales to \( 1/nk \) units of each contract both on and off the equilibrium path, than any consumer who deviates faces a strictly positive expected cost in each period they renew their contract until they
rejoin their equilibrium cohort. This cost is born by other consumers who purchase in the same period as well. Second, we can assume that firms raise their per period price in response to any deviation. We explore these generalizations in Appendix 2.

In the next section, we consider an alternative tactic firms may adopt to intertemporally bundle consumption. As it turns out, with such tactic, for any discount factor, profitable tacit collusion may be sustainable among arbitrarily large number of firms.

5 Intermittent Discount Equilibria: SPNE with Intertemporal Bundling II

In this section, we consider a class of equilibria which we call Intermittent Discount Equilibria. In this class of equilibria, firms offer both long term contracts, and in the period in which the long term contracts are signed, a one-period discount contract. Though consumers don’t ever choose the discount contract on the equilibrium path, its presence reduces the rival firms’ incentives to steal market share and hence makes it easier to tacitly collude.

The set of Intermittent Discount Equilibria (IDE) are all the SPNE in which strategies are of the following form:

**Firms’ equilibrium path actions:** In Period 1, the firms offer a one-period contract at a price $p_1$, and a $k$-period contract $\left( p_1 + \sum_{i=2}^{k} p_i, k \right)$. In periods $k + 1$, $2k + 1$, $3k + 1$, etc. the firms offer the same contracts as in Period 1. In periods 2 through $k$, and every future period except periods $k + 1$, $2k + 1$, $3k + 1$, etc, firms offer just a one-period contracts at a price $p$.

**Consumers’ equilibrium path actions:** In periods 1, $k + 1$, $2k + 1$, $3k + 1$, etc. consumers’ aggregate purchases are divided equally among firm’s $k$-period contracts. Individual consumers may sign just one contract or divide their purchases across firms. However, if $m$ is not divisible by $n$, then at least some consumers must sign multiple contracts.

**Off-the-equilibrium path actions following a deviation by a firm:** If any firm deviates from its equilibrium path strategies, then all firms revert to one-period,
marginal-cost contracts forever. Consumers choose their purchases to maximize the present value of their consumer surplus given firm behavior.

**Off-the-equilibrium path actions following a deviation by a consumer:** If any consumer deviates from his or her equilibrium path strategies, firms and other consumers continue to adopt their equilibrium path strategies. Any consumer who has not signed a $k$-period contract, signs one when it is available and purchases a one-period contract at the lowest available price when it is not. If two or more firms tie for the lowest available price, then consumers divide their purchases equally among each of these firms.

We now characterize the set of *Intermittent Discount Equilibria* and show that they exist for all $\delta$ and $n$.

 Consumers’ strategies are optimal as long as no consumer’s individual purchase decisions affect firms’ behavior. This is true because each consumer’s surplus is the same whether they sign the $k$-period or a series of one-period contracts. Importantly, as long as firm behavior is unchanged, consumers cannot influence other consumers’ purchases by deviating. This is because consumers anticipate that any deviations will be undone when they next have the opportunity to change their behavior. This implies that we only need to consider the optimality of firms’ strategies under the assumption that at most one consumer deviates.

So demonstrating the existence of an IDE requires showing that firms’ strategies are also optimal, *even if a single consumer deviates*. We first look at firms’ behavior in the first period of their $k$-period contracts, as well as at their behavior in the remaining periods.

**Periods $1, k+1, 2k+1, 3k+1$, etc.**

Since firms offer a discount of $(p - p_1)$ once every $k$ periods, a firm’s equilibrium profit in periods $1, k+1, 2k+1, 3k+1$, etc. (that is, in period $ak+1$ where $a$ is any non-negative integer) is

$$\pi_{ak+1} = \frac{1}{n} \sum_{i=1}^{\infty} \delta^{i-1} p - \frac{1}{n} \sum_{i=1}^{\infty} \delta^{(i-1)k} (p - p_1).$$

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If any firm deviates in these periods, consumers have the option to purchase from a non-deviating firm at a price of $p_1$ in the current period and at a price of 0 in every subsequent period, because the deviation will start a price war. So the highest profit a deviating firm can earn is $p_1$. This means the firm prefers its equilibrium strategy as long as
\[
\frac{1}{n} \sum_{i=1}^{\infty} \delta^{-i-1} p - \frac{1}{n} \sum_{i=1}^{\infty} \delta^{(i-1)k} (p - p_1) \geq p_1. \tag{12}
\]
This is clearly satisfied for $p_1 = 0$ (and for some $p_1 > 0$ sufficiently close to zero).

Also, it is clear that $\pi_1$ is increasing in $k$ and as $k$ approaches infinity $\pi_1$ approaches
\[
\frac{1}{n} \sum_{i=2}^{\infty} \delta^{-i-1} p + \frac{1}{n} p_1, \tag{13}
\]
or
\[
\delta \frac{p}{1 - \delta} + \frac{1}{n} p_1. \tag{14}
\]
Therefore, there exists a $k$ sufficiently large such that (12) is satisfied if and only if
\[
p_1 < \frac{1}{n - 1} \frac{\delta}{1 - \delta}. \tag{15}
\]

All other periods

In these periods firms’ actions have no impact on profits as long as consumers play their equilibrium strategies, so it is easy to see that their actions maximize profits. If a consumer deviates in the period $ak+1$ and begins purchasing one-period contracts, the firms continue to prefer to offer the equilibrium one-period price $p$ rather than offering a lower price as long as
\[
\frac{1}{n} \sum_{i=1}^{k-t} \phi \delta^{i-1} p + \delta^{k-t+1} \pi_1 \geq \phi p, \forall t = 1, \ldots, k - 1. \tag{16}
\]
Clearly, if this set of conditions holds for $t = 1$, then it holds for all $t = 1, 2, \ldots, k - 1$ because the gain from a deviation remains the same, but the present value of the cost increases as it draws nearer. So all of the conditions hold as long as
\[
\frac{1}{n} \sum_{i=1}^{k-1} \delta^{i-1} \phi p + \delta^{k-1} \pi_1 \geq \phi p. \tag{17}
\]
This condition clearly holds as long as \( \phi \) is sufficiently small. Indeed, (17) implicitly defines an upper bound, \( k^*(\phi) \), on the contract length for all \( \phi \). It clearly follows that as \( \phi \to 0 \), \( k^*(\phi) \to \infty \).

### Equilibrium Profits

Clearly the equilibrium profits are highest if \( p = V \). In this case, the equilibrium profits are

\[
\pi_1 = \frac{1}{n} \sum_{i=1}^{\infty} \delta^{i-1} V - \frac{1}{n} \sum_{i=1}^{\infty} \delta^{(i-1)k}(V - p_1),
\]

(18)

So the highest sustainable equilibrium profits are defined by (18) evaluated at \( k^*(\phi) \). As \( \phi \to 0 \) and \( k^*(\phi) \to \infty \) this profit level approaches

\[
\frac{1}{n} p_1 + \frac{1}{n} \frac{\delta V}{1 - \delta}.
\]

(19)

From (15), we know that the equilibrium profit is the highest when \( p_1 \) approaches \( \frac{1}{n-1} \delta \frac{V}{1-\delta} \). In this case, each firm’s profit approaches \( \frac{1}{n-1} \delta \frac{V}{1-\delta} \) and the industry profit approaches \( \frac{n}{n-1} \delta \frac{V}{1-\delta} \).

So we have the following result:

**Proposition 2** For all \( \delta \) and \( n \), there exist \( \phi \) sufficiently small such that an Intermittent Discount Equilibrium exists that sustains any industry profit level in the interval \( [0, \pi_{\text{max}}] \) for all

\[
\pi_{\text{max}} < \min\left\{ \frac{V}{1 - \delta}, \frac{n - 1}{n - 1 - \delta} \right\}.
\]

(20)

Proposition 2 establishes that Intermittent Discount Equilibria are able to sustain tacit collusion in some circumstances in which Staggered Contract Equilibria cannot. In Figure 1, the set \( \Omega_{NI} \cup \Omega_{SCE} \cup \Omega_{IDE} \) depicts the equilibrium profit levels that are supported by Intermittent Discount Equilibria. The set \( \Omega_{IDE} \) represents profit levels that can be supported by an Intermittent Discount Equilibrium but not by a Staggered Contract Equilibrium.
Most strikingly, we see that tacit collusion is sustainable among any number of firms with any discount factor and the highest sustainable industry profit is uniformly bounded away from zero. Indeed, the upper bound on industry profits is at least \( \frac{\delta V}{1 - \delta} \) which is strictly positive. So, regardless of the market structure, for \( \phi \) sufficiently small, the industry can sustain a profit level arbitrarily close to \( \delta V/ (1 - \delta) \), which is the monopolist profit just with one period of delay.

In an Intermittent Discount Equilibrium, firms do not offer multi-period contracts every period. When these contracts are offered, firms induce all consumers to purchase long-term contracts at the end of which the firms offer multi-period contracts again. So there is no incentive to deviate in the intervening periods. Whenever multi-period contracts are offered, firms simultaneously offer one-period contracts with the same discount. Although the one-period discounted contracts are never accepted, they serve an important function by setting a bound on the deviation profit. A deviating firm can only capture one period of demand because consumers always have the option of signing any other firm’s one-period discounted contract and then waiting for the price war to begin. When the discount on the one-period contract is set arbitrarily low, the deviation profit becomes arbitrarily small. That explains why tacit collusion can be sustained among arbitrary number of firms with any discount factors.

The price of the multi-period contract makes consumers indifferent between accepting the one-period contract and accepting the multi-period contract, and the incentive constraint is relaxed by lowering the price of the one period contract. This means that the collusive profit can be increased by increasing the length of the multi-period contract and reducing the frequency with which one-period discounts are offered.

The Importance of Recurring Discounts

When there are finitely many consumers it is clear that there is an upper bound on the equilibrium contract length. While this constraint vanishes as the number of consumers goes to \( \infty \), this implies that infinitely long contracts are not plausible. While increasing contract length increases the upper bound on the profit level sustainable in an Intermittent Discount Equilibrium, sellers cannot benefit from offering contracts with infinite length. The equilibrium relies on firms having an incentive to maintain a one-period price of \( V \) if any consumer deviates and accepts a one-period discount.
offer. However, this is reasonable only if the loss from a future price war is greater than the gain from serving the deviating consumer. But the cost of future price wars goes to zero as \( k \) goes to infinity, so the maximum size of a consumer for which tacit collusion is sustainable goes to zero as \( k \) goes to infinity.

Suppose sellers offer contracts of infinite length and one-period contracts in the first period and only one-period contracts thereafter if any consumer signs the one-period contract in the first period. Then under some conditions consumers are able to trigger a price war in the second period by signing the one-period contract in the first period and reentering the market in the second. The deviating customer will be the only source of business starting from the second period. Suppose firms tacitly collude to sell one-period contracts at the price \( V \), which is easiest to sustain if the consumer assigns the same probability of purchasing from each firm, namely, \( 1/n \) (or divides her purchase equally among the \( n \) firms). Then in each period, a firm’s expected profit is

\[
\frac{1}{n} \phi V
\]

If any firm undercuts the others, it will be able to earn an immediate profit of \( \phi V \) which is the customer’s one-period valuation, but firms will revert to marginal cost pricing afterward. Therefore, a firm will be tempted to cut its price and steal the customer’s business if

\[
\phi V > \frac{1}{n} \phi V
\]

or

\[
n > \frac{1}{1 - \delta}.
\]

In other words, although the intermittent discounts can be arbitrarily far apart if \( \phi \) is sufficiently small, they must be infinitely repeated. Without repeatedly offering these discounts, firms cannot do better than offering only one-period contracts.

As long as the contracts are of finite length, and the discount recurs each time one contract ends and another begins, there will be a strictly positive cost associated with a firm’s deviation.
6 General Results

The following Proposition establishes that the above class of SPNE achieves the highest feasible profit sustainable in any equilibrium.

**Proposition 3** The highest industry profit that can be supported in a symmetric SPNE is \( \min \left\{ \frac{V}{1-\delta}, \frac{n}{n-1} \frac{V}{1-\delta} \right\} \), or more precisely \( \frac{V}{1-\delta} \) if \( n \leq \frac{1}{1-\delta} \) and \( \frac{n}{n-1} \delta \frac{V}{1-\delta} \) if \( n > \frac{1}{1-\delta} \).

**Proof.** See Appendix 1. \( \square \)

It is not surprising that the profit level in an *Intermittent Discount Equilibrium* is able, or almost able, to achieve the theoretical highest sustainable profit. In an *Intermittent Discount Equilibrium* when \( k \) is arbitrarily large, the industry captures all the consumer surplus starting from period 2 on. But, if the industry charges a first-period price \( p_1 \), the deviation profit is necessarily at least \( p_1 \) in any equilibrium. Since this is the deviation profit in an *Intermittent Discount Equilibrium*, no other equilibrium support tacit collusion more effectively.

7 Extensions

Heterogeneous Consumers

An important assumption to generalize is the assumption that consumers are homogeneous. When consumers are heterogeneous, tacit collusion is still easier if consumers are long-lived. However, the range of \( \delta \) and \( n \) for which tacit collusion can be sustained is quite sensitive to the distribution of consumer valuations, the contracts used by the firms, and the price level. For that reason we illustrate the impact of consumer heterogeneity using a particular example.

Suppose valuations are distributed uniformly on the interval \([0, 1]\). In this case the monopoly price is \( 1/2 \). Consider an *Intermittent Discount Equilibrium* in which firms offer two-period contracts, \((0 + \delta p, 2)\), and offer one period contracts at price 0
at the same time and one-period contracts at price $p$ in all other periods. At these prices, every consumer with valuation $p$ or above purchases the two-period contract and every other consumer purchases the one-period contract when the price is zero and does not consume in the other periods.

So the equilibrium profit is

$$\frac{1}{n}\delta p(1-p)\frac{1}{1-\delta^2}$$

(21)

because in odd periods the instantaneous profit is zero, and in even periods the instantaneous profit is $\frac{1}{n}\delta p(1-p)$. In even periods a deviator can increase its current profit by $\frac{p^2}{4}$ by selling at a price of $\frac{p}{2}$ to the remaining consumers, but the deviator will suffer a loss of long run profit equal to

$$\frac{1}{n}\delta^2 p(1-p)\frac{1}{1-\delta^2}$$

(22)

So tacit collusion is sustainable as long as

$$\frac{1}{n}\delta^2 p(1-p)\frac{1}{1-\delta^2} \geq \frac{p^2}{4}$$

(23)

$$\Leftrightarrow \quad n \leq 4 \frac{\delta^2 (1-p)}{1-\delta^2 p}$$

So it is clear that tacit collusion can be supported with multiperiod contracts when $n > \frac{1}{1-\delta}$ even though it is infeasible with spot contracts. Moreover, even with heterogenous consumers, tacit collusion is sustainable for all $\delta$ and for all $n$ as long as $p$ is sufficiently small.

**Forward Contracts**

In this subsection, we show that forward contracts are another instrument firms can use to facilitate collusion. Using forward contracts they can support the same set of payoffs as with *Intermittent Discount Equilibrium*. We define a *Forward Contract Equilibrium* (FCE) as a SPNE of the following form:

**Firms’ equilibrium path actions:** In Period 1, the firms offer a one-period contract at a price $p_1$, and a one period ahead forward contract $p$. In Period 2 and beyond firms offer a one-period contract at a price $p_2 \geq p_1$ and the one period ahead forward contract $p$. 

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Consumers’ equilibrium path actions: In Period 1, consumers split their purchases among the firms’ one period offers, and consumers all purchase forwards contracts. They split their purchases evenly among the firm’s forward contract offers. In Period 2 and beyond consumers all purchase forward contracts, and they split their purchases evenly among the $n$ firms.

Off-the-equilibrium-path actions following a firm deviation: If any firm (or firms) deviates from these equilibrium strategies, then starting in the subsequent period every firm offers one-period contracts at a price equal to marginal cost forever. Following such a deviation consumers maximize their consumer surplus given current prices and the expectation of all future prices being equal to marginal cost.

Off-the-equilibrium-path actions following a consumer deviation: If any consumer deviates from these equilibrium strategies, firms continue to offer their equilibrium path contracts in all the periods that follow. Firms and consumers expect deviating consumers to return to the equilibrium path in the subsequent period.

It is straightforward to prove the following proposition:

**Proposition 4** For all $\delta$ and $n$, there exist $\phi$ sufficiently small such that an Forward Contract Equilibrium exists that sustains any industry profit level in the interval $[0, \pi_{max}]$ for all

$$\pi_{max} < \min\{\frac{V}{1 - \delta}, \frac{n}{n - 1} \frac{\delta}{1 - \delta} V\}. \quad (24)$$

Profits don’t achieve the monopoly profit because there is a upper bound on the price in the first period. This exists because firms might otherwise be tempted to steal the entire market in period one. However, in every later period, because consumers purchase forward contracts, it is impossible for a deviator to increase market share at any non-zero price. This is because consumers expect a price war to begin and can buy on the spot market after the price wars begins rather than on the forward market today.

Liski and Montero (forthcoming) also show that forward contracts facilitate tacit collusion, though they consider a slightly different model. Specifically, they allow
two contracting periods for every transaction period, and they consider downward sloping demand. Adding an additional contracting period (even just in period one) eliminates the constraint on price in period one and allows firms to sustain the monopoly price for any number of firms and any discount factor. However, adding downward sloping demand (e.g., consumer heterogeneity) means that some demand is not served in equilibrium and that the deviation profit is always strictly positive. As a consequence neither Liski and Montero nor Proposition 4 report that the monopoly price is feasible, but this is for two different reasons.  

It is also important to note that introducing forward contracts does not change our analysis of intertemporal bundling. When firms are allowed to deviate by offering forward contracts, it is possible to simultaneously steal other firms’ businesses in market segments which open at different points of time. However, as long as consumers are forward looking, this does not threaten the stability of tacit collusion. This is because consumers can anticipate a price war, and they will not accept any deviation forward contract offer unless it has a zero price. As a result, the deviating firm cannot increase its profit by offering forward contracts.

Naïve Consumers

In this subsection, we study the case when consumers are naïve in the sense that they do not anticipate that a price war will follow after a firm offers a price cut. Consumer naïveté hampers firms’ ability to tacitly collude because when consumers are naïve, a deviating firm can offer all available consumers infinitely long contracts to capture all their future demand. Because of this, there does not exist an equilibrium in which tacit collusion is sustainable when firms use either one period contracts or infinitely long contracts. Similarly, when firms offer intermittent discounts, firms can steal all the consumer surplus by offering infinitely long contracts in periods in which both one-period and multiple-period contracts are offered. So Intermittent Discount Equilibria do not exist either. However, we show that tacit collusion can be sustained in Staggered Contract Equilibria, although only among smaller number of firms. Such comparison points out that temporally partitioning the market into multiple segments is particularly important when consumers are naïve because the other mechanisms that we know of all fail.

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8Green and Coq (2006) extended Liski and Montero’s analysis to allow forward contracts of multiple periods.
Proposition 5  If consumers are naïve about the consequence of a price cut, then in all equilibria in which firms offer only one-period contracts on the equilibrium path, the industry profit is necessarily zero. Similarly, in all equilibria in which firms offer only infinite-period contracts on the equilibrium path, the industry profit is necessarily zero. Also, Intermittent Discount Equilibria do not exist.

Proof. See Appendix 1. \(\square\)

Now, suppose firms offer contracts of different lengths, \(p_1, (p_1 + \sum_{s=2}^{\tau} \delta^{s-1}V, \tau), \tau = 2, ..., k\), in the first period to create the staggering of submarkets and starting from period two offer only \(k\)-period contracts \((\sum_{s=1}^{k} \delta^{s-1}V, k)\). Consider any period \(t \geq 2\). Since consumers are naïve, any deviating offer as good as \((\sum_{s=t+1}^{\infty} \delta^{s-1}V, \infty)\) will be accepted by consumers who currently do not have a contract. A consumer who has a contract expiring in \(t\) periods is willing to accept a deviating offer as good as \((\sum_{s=t+1}^{\infty} \delta^{s-1}V, \infty)\). The deviator chooses how many cohorts of consumers to attract by setting the deviation price and earns a deviation profit of

\[
\max_{i \in \{1, 2, ..., k\}} \frac{i \delta^{i-1}V}{k(1 - \delta)}
\]

(in addition to the profits from historical contracts). By following the equilibrium strategy as prescribed by a Staggered Contract Equilibrium, each firm earns (in addition to the profits from historical contracts)

\[
\sum_{j=0}^{k-1} \frac{1}{nk(1 - \delta)} \delta^{j}V
\]

Therefore, no deviation is profitable in period 2 if

\[
\max_{i \in \{1, 2, ..., k\}} \frac{i \delta^{i-1}V}{k(1 - \delta)} \leq \sum_{j=0}^{k-1} \frac{1}{nk(1 - \delta)} \delta^{j}V. \quad (25)
\]

Since

\[
\lim_{k \to \infty} \sum_{j=0}^{k-1} \frac{\delta^{j}}{1 - \delta} = \frac{1}{(1 - \delta)^{2}},
\]

there exists a \(k\) such that (25) is satisfied as long as

\[
\frac{n}{1 - \delta} \left( \max_{i \in \{1, 2, ..., k\}} i \delta^{i-1} \right) < \frac{1}{(1 - \delta)^{2}}.
\]
If $i$ was a real number, $\arg \max_{i \in \{1, 2, \ldots, k\}} i \delta^{i-1} = -1 / \ln \delta$. In other words, a sufficient condition for sustainable tacit collusion is

$$\frac{n}{1 - \delta} \left( \frac{-1}{\delta^{\frac{1}{\ln \delta} - 1}} \right) < \frac{1}{1 - \delta} + \frac{\delta}{(1 - \delta)^2}$$

or equivalently

$$n < \frac{-\delta^{\frac{1}{\ln \delta} + 1} \ln \delta}{1 - \delta} = -e \delta \ln \delta \frac{1}{1 - \delta}. \tag{26}$$

Since

$$\lim_{\delta \to 1} \frac{-e \delta \ln \delta}{1 - \delta} = \lim_{\delta \to 1} \frac{d}{d\delta} (-e \delta \ln \delta) = \lim_{\delta \to 1} \frac{d}{d\delta} (1 - \delta) = \lim_{\delta \to 1} e (\ln \delta + 1) = e,$$

it follows from (26) that tacit collusion can never be supported among more than two firms. However, one can show numerically that when $n = 2$, tacit collusion can be sustained for a wide range of discount factors, i.e., for all $\delta > 0.557$.

As long as tacit collusion starting from period 2 can be maintained, tacit collusion in period one can also be maintained with a low enough $p_1$. From our previous analysis, we know that the upper bound on $p_1$ is $\frac{1}{n-1} \delta V_{n-1}$. This leads to the following:

**Claim 2** If consumers are naïve about the consequence of a price cut, then tacit collusion is sustainable by firms offering staggered contracts if and only if $n < -e \delta \ln \delta$.

Claim 3 suggests that staggering contracts helps firms to partially restore the potential to tacitly collude that consumer naïveté takes away. Recall that when consumers are forward looking and firms do not intertemporally bundle, tacit collusion is sustainable if and only if $n \leq 1 / (1 - \delta)$. When consumers are naïve, however, without intertemporal bundling tacit collusion is never sustainable.

One observation worth mentioning is that although forward contracts facilitate tacit collusion, if consumers are naïve and at the same time firms can offer forward contracts, then there do not exist $n$ and $\delta$ such that tacit collusion can be sustained.
Under these assumptions, a deviating firm can offer infinitely long contracts starting in different periods to consumers who become available at different points of time to successfully capture all consumers’ future demands. As a result, there is no punishment enforceable on the deviating firm.

Finitely-lived Consumers

Here we discuss two natural ways to model finitely-lived consumers. The first is a model in which a new cohort of consumers arrive in each period in overlapping generations and these consumers each live \(l\) periods and then exit the market forever. The second is a model in which consumers arrive simultaneously every \(l\) periods. That is, each cohort of consumers lives \(l\) periods and then is replaced by a new cohort of consumers.

Model One

In this extension, consumers are exogenously partitioned into \(l\) submarkets each of which open at different points of time. For simplicity, suppose that no consumers are in the market at time 0. That is, the market grows each period from period 1 to period \(l\), at which time the market reaches steady state.

Consider an equilibrium in which firms simply offer \(l\)-period contracts \((\sum_{s=1}^{l} \delta^{s-1}V, l)\) every period. The incentive constraint every period is

\[
\frac{1}{n} \sum_{i=1}^{\infty} \sum_{s=1}^{l} \delta^{s-1}V \geq \frac{1}{l}V
\]

which, by analogy to (37) through (40), holds as long as

\[
n \leq \frac{1}{1-\delta} + \left[ \sum_{j=1}^{k-1} \frac{\delta^j}{1-\delta} \right].
\]

Therefore, Staggered Contract Equilibria exist for some \(n > \frac{1}{1-\delta}\).

In our main model, recurring discounts facilitate tacit collusion in an Intermittent Discount Equilibrium because in periods in which a discount is offered, all consumers
sign long-term contracts until the next time firms offer a discount and thus competition in all other periods can be avoided. When consumers arrive in overlapping generations, however, firms necessarily compete for new consumers in every period. Therefore, recurring discounts do not facilitate tacit collusion.

Model Two

Next we demonstrate that firms can tacitly collude more easily when consumers arrive simultaneously every $l$ periods and each live $l$ periods at which time a new cohort of consumers arrive. Here we propose a class of equilibria that we term as Modified Staggered Contract Equilibria.

The set of Modified Staggered Contract Equilibria are all SPNE in which strategies are of the following form:

**Firms’ equilibrium path actions:** In the first period of the life time of each generation of consumers, each firm simultaneously offers $l$ different subscription contracts of different lengths: $p_1 \leq V$ and $\left\{ (p_1 + \sum_{s=2}^{\tau} \delta^{s-1}V, \tau) \right\}_{\tau=2}^l$. In period $\tau \in \{2, 3, ..., l\}$ within each generation, each firm offers a $(l - \tau + 1)$-period contract $\left( \sum_{s=1}^{l-\tau+1} \delta^{s-1}V, l - \tau + 1 \right)$.

**Consumers’ equilibrium path actions:** In the first period of the lifetime of each generation of consumers, consumers’ aggregate purchases are divided equally among each of the $nk$ different contracts offers ($k$ offers per firm and $n$ firms). In periods $\tau \in \{2, 3, ..., l\}$ within each generation, consumers renew their contracts by signing $(k - \tau)$-period contracts with the same firm that they signed a contract with in the first period of their life.

**Off-the-equilibrium-path actions following a firm deviation:** If any firm (or firms) deviates from these equilibrium strategies, then starting in the subsequent period every firm offers one-period contracts at a price equal to marginal cost forever. Following such a deviation consumers maximize their consumer surplus given current prices and the expectation of all future prices being equal to marginal cost.

**Off-the-equilibrium-path actions following a consumer deviation:** If any consumer deviates from these equilibrium strategies, firms continue to offer their
equilibrium path contracts in all the periods that follow, and, in addition, offer contracts designed to return the deviating consumers to their equilibrium path strategies. Specifically, if in period \( t' \) a consumer is available who failed to sign his or her equilibrium path contract with firm \( i \) in some earlier period \( t \), then firm \( i \) offers an additional contract at the same per period price as its other contracts and with a duration chosen to match the duration of the equilibrium contract that was not signed. The available consumer accepts this new contract and all other consumers accept the equilibrium path contract offers.

The incentive constraint in periods \( 1, l + 1, ..., al + 1, ... \) is analogous to (12), with \( k \) replaced by \( l \):

\[
\pi_1 = \frac{1}{n} \sum_{i=1}^{\infty} \delta^{i-1} V - \frac{1}{n} \sum_{i=1}^{\infty} \delta^{(i-1)l} (V - p_1) \geq p_1. \quad (29)
\]

The incentive constraint in the \( t \)th period of the current generation of consumers’ lives is

\[
\sum_{j=0}^{l-t} \sum_{i=j}^{l-t} \delta^i \frac{V}{nl} + \frac{1}{n} \delta^{l-t+1} \pi_1 \geq \frac{V}{l}, \quad t \in \{2, 3, ..., l\}. \quad (30)
\]

We now show that tacit collusion can be supported for some \((\delta, n)\) pairs which do not satisfy \( n \leq 1/(1 - \delta) \).

**Proposition 6** Suppose a measure one of consumers arrive in periods \( 1, l + 1, ..., al + 1, ... \) and each generation of consumers live for \( l \) periods, where \( l \geq 2 \). Modified Staggered Contract Equilibria exist for all \((\delta, n)\) satisfying \( n \leq 1/(1 - \delta) \) and also for some \((\delta, n)\) such that \( n > 1/(1 - \delta) \).

**Proof.** See Appendix 1. \( \square \)

Although tacit collusion can be sustained when \( n > 1/(1 - \delta) \) as long as \( l \geq 2 \), it is obvious to see that as \( l \) increases, tacit collusion becomes easier to sustain.

It is also obvious that *Intermittent Discount Equilibria* exist when \( n > 1/(1 - \delta) \). In these equilibria, firms offer one-period discounted contracts \( p_1 \) and \( l \)-period contracts \((p_1 + \sum_{i=2}^{l} V, l)\) in periods 1, \( l + 1 \), \( 2l + 1 \), etc. In all other periods, firms
offer the one-period contract $V$; consumers of every generation all sign the $l$-period contracts when they arrived at the market. Tacit collusion is sustainable for all $n$ and for all $\delta$ as long as $p_1$ is small.

8 Summary and Discussion

In this paper, we have demonstrated how intertemporal bundling may help soften competition by facilitating tacit collusion. In one class of equilibria, by offering a menu of multi-period contracts of different lengths, firms break up the market into multiple segments each open at different points of time. Tacit collusion is easier to sustain because a deviating firm can steal business only in one market segment at one point of time but will be punished in every market segment.

In another class of equilibria, multi-period contracts are always offered with a discount on the first period’s consumption/service. Moreover, when these multi-period contracts are offered, each firm also offers a discounted one-period contracts. In equilibrium consumers only sign up for multi-period contracts so firms compete infrequently. Firms have little incentive to deviate because consumers will respond to a deviation by accepting other firms’ discounted one-period contract and then waiting for the price war to follow.

Our analysis crucially relies on the assumption that consumers are forward looking, being able to foresee a price war upon observing a deviation by any firm. When consumers are naïve about the relationship between deviation and price war, a deviating firm can capture large profit by offering life long contracts with a small discount, and this makes sustaining tacit collusion harder. In this case, among all the mechanisms we have studied in this paper, only by staggering multi-period contracts firms may be able to sustain tacit collusion.

We also analyze forward contracts (in a model closely related to Liski and Montero, forthcoming), but while forward contracts can accomplish the same thing as intertemporal bundling, it is also clear that intertemporal bundling is much more common than forward contracting. Empirically, there are more potential reasons for using subscription contracts than there are for using forward contracts (for example reducing transactions costs or increasing economies of scope). So it is not surprising that they are more common. While tacit collusion may be only one of these reasons,
it may be the case empirically that tacit collusion via subscription contracts occurs as a consequence of subscription contracts being adopted rather than the other way around.

Another important assumption we have made is that consumers are arbitrarily small and no group of consumers can coordinate to collectively deviate. If a large measure of consumers can collectively deviate, firms’ ability to tacitly collude will be affected. Suppose firms tacitly collude on a Staggered Contract Equilibrium. If a significant measure of available consumers collectively refuse to purchase for one period, the total number of available consumers in the following period will increase substantially and this may cause the incentive constraint to break down. Knowing that, consumers will coordinate the collective deviation to trigger a price war. Similarly, if firms tacitly collude on an Intermittent Discount Equilibrium, the deviation by a significant measure of consumers may also trigger a price war, as discussed in the end of section 4. In general, the larger is the measure of consumers who can collectively deviate, the shorter will be the multi-period contract firms can offer and the lesser will be the power of multi-period contracts in helping firms sustain tacit collusion.

The paper has some important empirical implications. First, firms’ margins are potentially sensitive to the expected lifetime of consumers and more generally to the feasibility of long-term contracts. If consumers are not long-lived, then firms will be unable to use long-term contracts. Similarly, firms’ margins are potentially sensitive to the stability of consumers demands over time. Absent predictable, stable demand, long-term contracts would be inefficient and unable to facilitate tacit collusion. However, as with other models of tacit collusion, the presence of multiple equilibria may be a serious limitation to empirical analysis. For example, while we show that the highest sustainable profit level is sensitive to consumer lifetime, it does not follow that the equilibrium profit level is necessarily higher when consumers lifetimes grow. As importantly, intertemporal bundling could alter the equilibrium payoffs through equilibrium selection even if the highest profit sustainable is unchanged.
Appendix 1

Proof of Proposition 1:

In the body of the paper we showed that firms have no incentive to deviate from their equilibrium strategies. It remains only to show that consumers have no incentive to deviate.

First, it is clear that consumers’ responses to any firm deviation are subgame perfect. Consumers maximize present value of consumer surplus given expected prices and cannot do anything to increase this surplus since they are already capturing all of the surplus.

Second, it is also clear that if firms and other consumers play their equilibrium strategies, consumers cannot be made better off by deviating.

The last step is to show that firms and consumers are still willing to play their equilibrium strategies if a consumer deviates. In any period in which fewer consumers are available (because of consumer deviations), firms’ incentive constraints are still satisfied. In any period in which either one or two additional consumers are available (because of consumer deviations), the incentive constraint (5) becomes

\[
\left(\frac{1}{k} + \phi\right) \min\{V, p \sum_{j=1}^{k} \delta^{j-1}\} \leq \frac{1}{nk} \left(\frac{p}{1-\delta} \right) + \sum_{j=1}^{k-1} \frac{1}{nk} \delta^j p, \quad (31)
\]

After any single consumer deviation, firms and consumers both expect that the deviation will be undone as soon as the deviating consumer is available to sign a new contract. Given these expectations, and assuming that (31) is satisfied, no individual consumer can affect any firm’s pricing decision. The inability of consumers to coordinate their actions implies that they are unable to impact firms’ pricing decisions and undermine tacit collusion.

Next, we identify the conditions under which firms can coordinate to make (31) hold when we allow \( \phi \) to be arbitrarily small. If \( p \sum_{j=1}^{k} \delta^{j-1} < V \), or \( p \in \)
\[ [0, \frac{V}{\sum_{j=1}^{\delta^{j-1}}}), (31) \text{ becomes} \]

\[
\left( \frac{1}{k} + \phi \right) p \sum_{j=1}^{k} \delta^{j-1} \leq \frac{1}{nk} \frac{p}{1 - \delta} + \sum_{j=1}^{k-1} \frac{1}{nk} \frac{\delta^j p}{1 - \delta}.
\]

This necessarily holds for sufficiently small \( \phi \) if and only if

\[
\frac{1}{k} \phi \sum_{j=1}^{k} \delta^{j-1} < \frac{1}{nk} \frac{p}{1 - \delta} + \sum_{j=1}^{k-1} \frac{1}{nk} \frac{\delta^j p}{1 - \delta}.
\]

which is equivalent to \( n < 1/(1 - \delta) \).

If \( p \sum_{j=1}^{k} \delta^{j-1} \geq V \), or \( p \in \left[ \frac{V}{\sum_{j=1}^{\delta^{j-1}}}, \frac{V}{\sum_{j=1}^{\delta^{j-1}}} \right] \), (31) becomes

\[
(1 + \phi k) V \leq \frac{1}{n} \frac{p}{1 - \delta} + \sum_{j=1}^{k-1} \frac{1}{n} \frac{\delta^j p}{1 - \delta},
\]

(32)

Therefore, for sufficiently small \( \phi \), there exists sufficiently large \( k \), yet \( \phi k \) is still sufficiently small, such that (32) holds if and only if

\[
V \leq \frac{p}{n(1 - \delta)} + \frac{1}{n(1 - \delta)^2}
\]

or

\[
\frac{V}{p} \leq \frac{1}{(1 - \delta)^2}.
\]

Summing up, if \( n < 1/(1 - \delta) \), then any \( p \in [0, \frac{V}{\sum_{j=1}^{\delta^{j-1}}} \) can be supported by a SCE with some \( k \) for sufficiently small \( \phi \), and if \( n \in \left[ \frac{1}{1 - \delta}, \frac{1}{(1 - \delta)^2} \right] \), then any \( p \in \left[ \frac{V}{\sum_{j=1}^{\delta^{j-1}}}, V \right] \) can be supported by a SCE with some \( k \) (sufficiently large) for sufficiently small \( \phi \). The statement of the proposition will immediately follow once the bound on \( p_1 \), namely,

\[
p_1 \leq \min \left\{ \frac{1}{n - 1} \frac{1}{1 - \delta}, V \right\}
\]

is considered. □
Proof of Proposition 4:

Consider any symmetric equilibrium that sustains the highest possible industry profit in the set of all SPNE. Let $\pi_1$ denote the present value of each firm’s equilibrium profit stream in this equilibrium beginning from period 1 onwards. Let $\pi_2$ denote the present value, in period 1, of each firm’s equilibrium profit stream in this highest profit equilibrium from period 2 onwards.

In period 1, each firm can either earn its equilibrium profit, $\pi_1$, or it can deviate. And, there always exists a deviation that yields a profit of $n(\pi_1 - \pi_2)$. To see this, first note that regardless of the deviator’s offer and consumers’ expectations about the future, the best the consumers can ever hope to capture in aggregate if they buy from the other firm is $\frac{V}{1-\delta} - \pi_1$. This is the surplus they would earn if subsequent pricing dissipated all of the firms’ future profits. So a deviator can capture the entire market by offering consumers an aggregate surplus less than or equal to $\frac{V}{1-\delta} - n(\pi_1 + \pi_2)$, which means the deviator can capture a profit of $n(\pi_1 - \pi_2)$.

This implies that the highest profits equilibrium must satisfy $\pi_1 \geq n(\pi_1 - \pi_2)$. If not, firm have an incentive to deviate. This in turn implies $\pi_1 \leq \frac{n-1}{n} \pi_2$. Since any symmetric equilibrium necessarily satisfies $\pi_2 \leq \delta \frac{V}{1-\delta}$, it follows that $\pi_1 \leq \frac{1}{n-1} \delta \frac{V}{1-\delta}$. So total industry profit cannot exceed $\frac{n}{n-1} \delta \frac{V}{1-\delta}$. □

Proof of Proposition 5:

First, consider the case that firms offer only one-period contracts. Note that marginal cost pricing using one-period contracts is an equilibrium, so equilibria in this set do in fact exist. Let $\Pi_t$ be the present discounted value of the industry profit at time $t \in \mathbb{N}$. At time $t$, the profit of the least profitable firm is no larger than $\Pi_t / n$. Since consumers are naïve about the consequence of a price cut, the least profitable firm can deviate by offering consumers a life-time contract providing them a consumer surplus arbitrarily slightly higher than they would receive in equilibrium. Therefore, the deviation profit is arbitrarily close to $\Pi_t$ which is obviously
larger than $\Pi_t/n$, for $n \geq 2$. Next, if firms offer only infinitely long contracts and consumers are naïve, then firms essentially are playing an one-shot game and clearly the unique equilibrium outcome is marginal cost pricing. The logic for the nonexistence of *Intermittent Discount Equilibria* is similar. $\square$

**Proof of Proposition 6:**

It is obvious that if $n \leq 1/(1 - \delta)$, by setting $p_1 = V$ so that $\pi_1 = V/(1 - \delta)$, both (29) and (30) are satisfied. However, (30) is also satisfied for some $\hat{\delta}$ such that $n > 1/(1 - \delta)$, i.e., $\hat{\delta} < (n - 1)/n$, for all $t \in \{2, 3, \ldots, l\}$. Focus on $n > 1/(1 - \delta)$. Since

$$
\frac{d\pi_1}{dp_1} = \frac{1}{n} \sum_{i=1}^{\infty} \delta^{(i-1)} l
= \frac{1}{n (1 - \delta^l)} < \frac{1}{n (1 - \delta)} < 1,
$$

(29) is satisfied more easily for $p_1 < V$, there exists $p_1 < V$ such that (29) holds for some $\tilde{\delta} = (n - 1)/n - \varepsilon$. By continuity, for $\varepsilon$ sufficiently small, (30) still holds for some $\delta' \in (\hat{\delta}, (n - 1)/n)$. Therefore, both (29) and (30) hold for $\delta'' = \max\{\tilde{\delta}, \delta'\} < (n - 1)/n$. $\square$
Appendix 2

Claim 3 For \( \phi \) sufficiently small there exists a Staggered Contract Equilibrium in which consumers do not play weakly dominated strategies if and only if \( n < \frac{1}{(1-\delta)^2} \). These SCEs support the same range of industry profit as stated in Proposition 1.

Proof. From our analysis in section 4, it is clear that if \( n < \frac{1}{(1-\delta)^2} \), then for all \( p \in [0, V] \), there exists \( \bar{k}(p) \) such that for all \( k > \bar{k}(p) \),

\[
\frac{p}{k} + \frac{1}{k} \sum_{j=1}^{k-1} \delta^j < \frac{1}{nk} \frac{p}{1-\delta} + \sum_{j=1}^{k-1} \frac{1}{nk} \frac{\delta^j p}{1-\delta},
\]

and if \( n \in [\frac{1}{(1-\delta)^2}, \frac{1}{(1-\delta)^2}) \), then for all \( p \in [n (1 - \delta)^2 V, V] \), there exists \( \hat{k}(p) \) such that for all \( k > \hat{k}(p) \),

\[
\frac{p}{k} < \frac{1}{nk} \frac{p}{1-\delta} + \sum_{j=1}^{k-1} \frac{1}{nk} \frac{\delta^j p}{1-\delta}.
\]

If \( n \geq \frac{1}{(1-\delta)^2} \), then tacit collusion cannot be supported as a SCE.

Let \( \bar{k} = \max_{p \in [0, V]} \bar{k}(p) \) and \( \hat{k} = \max_{p \in [n (1 - \delta)^2 V, V]} \hat{k}(p) \). By continuity, for all \( n \leq \frac{1}{(1-\delta)^2} \) and \( p \in [0, V] \), there exist \( \alpha \in (0, 1) \), \( \phi \in (0, \phi) \) for \( \phi \) sufficiently close to zero such that for all \( k > \max\{\bar{k}, \hat{k}\} \),

\[
(\alpha p + (1 - \alpha) V) \left( \frac{1}{k} + 2\phi \right) + \frac{p}{k} \sum_{j=1}^{k-1} \delta^j < \frac{1}{nk} \left( (\alpha p + (1 - \alpha) V) + \delta \frac{p}{1-\delta} \right) + \sum_{j=1}^{k-1} \frac{1}{nk} \frac{\delta^j p}{1-\delta}.
\]

Similarly, for all \( n \in [\frac{1}{(1-\delta)^2}, \frac{1}{(1-\delta)^2}) \) and \( p \in [n (1 - \delta)^2 V, V] \), there exist \( \alpha \in (0, 1) \), \( \phi \in (0, \hat{\phi}) \) for \( \hat{\phi} \) sufficiently close to zero such that for all \( k > \max\{\bar{k}, \hat{k}\} \),

\[
(\alpha p + (1 - \alpha) V) \left( \frac{1}{k} + 2\phi \right) < \frac{1}{nk} \left( (\alpha p + (1 - \alpha) V) + \delta \frac{p}{1-\delta} \right) + \sum_{j=1}^{k-1} \frac{1}{nk} \frac{\delta^j p}{1-\delta}.
\]
Consumers can deviate either by signing a contract different from what the SCE prescribes that he signs in the first period or by not signing a contract in any other period so that the measure of consumers in one particular cohort accumulates. Our concern about the sustainability of the SCE come from the fact that if consumers anticipate that by deviating in sequence eventually there will be many more consumers in one cohort and that a price war will be triggered when this cohort of consumers become available, then they may deviate in sequence to trigger a price war. If firms anticipate that a price war is eventually inevitable, a price war may begin in the first period. Below we construct an SCE in which if consumers deviate from the equilibrium strategy, they will be punished by higher prices. This provide them with a strict incentive to play the equilibrium strategy.

Suppose as part of the equilibrium, whenever the measure of available consumers accumulates to $\frac{1}{k} + 2\phi$ in one cohort, firms will raise the per-period subscription fee from $p$ to $p' = \alpha p + (1 - \alpha) V$ where $\alpha \in (0, 1)$ for one period. Every firm offers the $k$-period contract $\left(p' + \sum_{j=1}^{k-1} \delta^j p, k\right)$. Those firms which would have been under a long-term contract with the deviating consumers in equilibrium offer, besides the same $k$-period contracts, additional contract(s) of different length(s) $\left(p' + \sum_{j=1}^{k' - 1} \delta^j p, k'\right)$. The lengths of these additional contracts are specified such that if a consumer who previously deviated signs one of these additional contracts, he will become available the same time as those in a cohort he previously belonged to. Consumers respond to these contracts by having a measure $\frac{1}{k}$ of non-deviating consumers signing the $k$-period contracts and a measure $2\phi$ of deviating consumers the other contract(s). After that, consumers who previously deviated will fall back into the cohorts they belong to in equilibrium. Suppose any consumer deviates at this point such that in a future period there are $\frac{1}{k} + 2\phi$ available consumers, firms will further raise the price for one period to $p'' = \alpha p' + (1 - \alpha) V$. Anticipating that, the measure $\frac{1}{k} + 2\phi$ of consumers strictly prefer that they sign contracts according to what the equilibrium of the continuation subgame prescribes. In other words, no deviation by one single consumer can eventually cause the size of any cohort to grow beyond $\frac{1}{k} + 2\phi$. If (33) and (34) hold, then if firms offer contracts of lengths longer than $\max\{\bar{k}, \hat{k}\}$, charging $p$ in each period, and raising the price to a level closer to $V$ if the size of any cohort reaches $\frac{1}{k} + 2\phi$, then no single consumer’s deviation will eventually lead to a price war. Anticipating that, consumers will strictly prefer not to deviate in the first place. By choosing $p$ arbitrarily close to $V$ and $k$ arbitrarily large, any profit arbitrarily close to but less than $\min\{\frac{V}{1-\delta}, \frac{n}{n-1} \frac{\delta}{1-\delta} V\}$ can be achieved. $\Box$
References


