Advanced Topic 3:
Real and Monetary Shocks and Balance of Payments Equilibrium

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The material presented in this Topic adds to the basic analysis in the previous two Advanced Intertemporal Foundations Topics, focusing on the role of the balance of payments in general equilibrium and then on balance of payments equilibrium itself. We begin with the aggregation equations representing income and output. Current output produced in the domestic economy is equal to

\[ X = C' + I'_g + E' \] (1)

where \( X \) denotes aggregate output and \( C' \) and \( I'_g \) represent private plus government production of consumption and investment goods and services that are consumed domestically and \( E' \) is domestically produced consumption and investment goods and services that are exported. Imports of consumption and investment goods and services plus goods that are then re-exported can be expressed as

\[ \tilde{E}' = \tilde{E}'_C + \tilde{E}'_I + \tilde{E}'_E \] (2)

where the subscripts refer respectively to consumption goods and services, investment goods and services and goods that are re-exported. We can then add and subtract \( \tilde{E}' \) from the right side of (1) and manipulate the result to obtain

\[ X = C' + I'_g + E' + \tilde{E}' - \tilde{E}' \\
= C' + \tilde{E}'_C + I'_g + \tilde{E}'_I + E' + \tilde{E}'_E - \tilde{E}' \\
= C + I_g + B_T \] (3)

where \( C = C' + \tilde{E}'_C \) and \( I_g = I'_g + \tilde{E}'_I \) represent private plus public expenditures on consumption and investment goods and and services and \( B_T = E' + \tilde{E}'_E - \tilde{E}' \) is the balance of trade in goods and services, excluding the services of capital. The level of current domestic output (GDP) is thus represented by \( X \) and current gross income of domestic residents (GNP) can be expressed as

\[ Y_g = C + I_g + B_T + DSB \] (4)

where \( Y_g \) equals aggregate gross income of domestic residents and \( DSB \) is the debt service balance, which equals net income of domestic residents from capital employed abroad minus
net income of foreign residents from capital employed in the domestic economy. Subtracting the portion of gross investment representing depreciation from both sides of the above equation yields

\[ Y = C + I + BT + DSB \]  

(5)

where \( Y \) is aggregate net income (Net National Income) and \( I \) is net investment.

Subtracting \( C + I \) from both sides of the above equation, and noting that the current account balance equals the balance of trade in goods and services plus the debt service balance, we obtain

\[ Y - C - I = S - I = BT + DSB = CAB \]  

(6)

where \( CAB \) is the current account balance and \( S \) is the level of domestic savings. Since domestic savings minus domestic investment is the net purchase of assets abroad or the net outflow of capital, the above expression can be rewritten

\[ CAB = BT + DSB = NCO \]  

(7)

where \( NCO \) is the net capital outflow. If domestic net investment exceeds domestic savings, \( NCO \) will be negative and there will be a net capital inflow matched by a current account deficit. When the components of the above equations are viewed as actual levels the equations are true by definition. When the components are viewed as desired magnitudes the equations become equilibrium conditions, with equations (1), (4), (5) and (7) being alternative representation of the condition of domestic aggregate goods market equilibrium.

The excess of desired domestic savings over desired world investment in the domestic economy will depend on domestic full-employment income, the deviation of domestic income from that full-employment level, and the real rate of interest. And that desired level of investment will change through time as the growth of technological capital takes place and has differential and changing effects over time on the profitability of investing in the different countries. The balance of trade in goods and services (excluding the services of capital represented by \( DSB \)) will depend on the same domestic income variables as well as the level of income abroad, which we will assume to be exogenously determined, and on the domestic real exchange rate—that is, the relative price of domestic output in terms of foreign output—which we will designate as \( Q \). Thus,

\[ SI(Y_f, \Delta Y, r, \Phi_{SI}) = BT(Y_f, \Delta Y, Q, \tilde{Y}, \Phi_{BT}) + DSB \]  

(8)

where \( SI() \) and \( BT() \) are the functions determining the desired levels of domestic savings minus investment and the domestic balance of trade in goods and services, \( \tilde{Y} \) is the assumed constant level of net income abroad, and \( \Phi_{SI} \) and \( \Phi_{BT} \) are shift variables that positively affect, respectively, the desired excess of saving over investment and the balance of trade. The former represents increases in desired domestic saving or reductions in desired world investment in the domestic economy while the latter represents
an exogenous expansion of desired exports relative to desired imports. A decline in $Q$ shifts world demand from foreign to domestic output, increasing the current account balance. The debt service balance is a constant determined by what has happened in the past.

Under full-employment conditions, all the variables in $SI()$ are exogenous, as are all the variables except $Q$ in $BT()$. The level of the real exchange rate must therefore adjust, either through changes in the domestic price level or in the nominal exchange rate, to bring the current account balance into line with the exogenously determined net capital outflow. Shifts in $\Phi_{SI}$ will cause the real exchange rate to adjust to produce a change in the current account balance equal to the change in the net capital outflow. And any effects on the desired current account balance of shifts in $\Phi_{BT}$ will be offset by adjustments in the real exchange rate. When there is full employment, the current account balance is determined entirely by the desired capital flow. One must therefore take into account desired capital flows when analyzing or predicting long-run movements in the balance of trade and current account balance.

The full-employment situation can be portrayed graphically in the Figure above. The vertical $SI$ line represents the exogenous level of investment minus savings while the negatively sloped $CB$ line represents the negative relationship between the current account balance and the real exchange rate. An increase in savings will shift $SI$ to the right and an increase in domestic investment arising from a shift of world investment toward the domestic economy will shift it to the left. An increase in desired exports will shift $CB$ to the right while an increase in desired imports will shift it to the left.
Under conditions where the domestic price level is fixed and full employment does not necessarily hold, variations in $\Delta Y$ will affect both the desired net capital outflow and current account balance. In the simplest case where the nominal exchange rate is fixed and the domestic price level cannot immediately adjust, the level of domestic output and employment will be the equilibrating variable. This is shown in the Figure below where the positively sloped $NL$ line incorporates the fact that income and employment will rise when the current account balance increases at a given level of the real exchange rate and the negatively sloped $CA$ line takes into account the fact that a rise in income and employment will increase the demand for imports and reduce the supply of exports at a given real exchange rate. A positive shift of desired exports minus imports will shift $CA$ to the right, causing both the current account balance and the level of income and employment to increase. And a reduction of desired savings or a shift of world investment into the domestic economy will shift $NL$ to the left, increasing domestic income and employment and reducing the current account balance.

![Income and Employment and Balance of Trade](image)

To analyse balance of payments equilibrium we have to distinguish between two types of transactions—autonomous and induced. Induced transactions are those that are the result of government attempts to influence the market equilibrium exchange rate while autonomous transactions are those which are undertaken by the private sector and by government for purposes unrelated to officially desired nominal exchange rate levels. Balance of payments equilibrium occurs when induced transactions are zero, as under conditions of untempered exchange rate flexibility. The most common form of induced transactions are the purchase and sale of foreign exchange reserves to maintain a fixed exchange rate or influence the level of a flexible exchange rate. Governments may also indirectly attempt to control the exchange rate by limiting the amount of domestic expenditures abroad—for example, by shipping domestic beer to domestic military personnel abroad so that they will not buy foreign beer. In what follows, it is assumed that the exchange rate is fixed and foreign exchange reserve changes are the only induced transactions.
The maintenance of a fixed exchange rate requires the domestic authorities to supply any changes in domestic currency demanded on the foreign exchange market either by adjusting the stock of foreign exchange reserves or otherwise changing the domestic money supply. The stock of base or high-powered money $H$, which is what the authorities directly control, is equal to the quantity of cash in the hands of the public $CP$ plus the reserves held by the domestic banking system $BR$. And the total amount of domestic money in circulation $M$ is equal to cash in the hands of the public plus deposits in the commercial banks $BD$. The money multiplier—that is, the ratio of the money supply to high-powered money—is

$$mm = \frac{M}{H} = \frac{CP + BD}{CP + BR} = \frac{c + 1}{c + f} \tag{9}$$

where $c = CP/BD$ is the ratio of the public’s currency to deposit holdings and $f = BR/BD$ is the reserve to deposit ratio of the banking system. The demand for nominal money balances can be represented by the equation

$$M_D = P L(r, Y, \Phi_m) \tag{10}$$

where $L()$ is the demand function for real money balances, $P$ is the price level and $\Phi_m$ is an exogenous shift variable, and the actual and expected inflation rate is zero. If the expected rate of domestic inflation is positive, the interest rate in (10) will be the nominal rate, which will equal the real interest rate plus the expected rate of inflation. Any excess supply of money, which can be represented as a rise in the quantity of money relative to the demand for it, will lead as asset equilibrium is re-established to a purchase of assets abroad which, when the authorities maintain a fixed exchange rate, will require them to sell foreign exchange reserves, thereby reducing their official holdings. And correspondingly, an excess demand for money will be accommodated by an increase in official reserve holdings.

Changes in official foreign exchange reserve holdings or purchases or sales of domestic assets by the authorities will lead to a change in the stock of high-powered money which, as noted above, equals cash in the hands of the public plus commercial bank reserves. Using equations (9) and (10), we can express the demand for high-powered money as $H_D = (1/mm) M_D$ so that the condition of monetary equilibrium becomes

$$H = \frac{1}{mm} P L(r, Y, \Phi_m). \tag{11}$$

Since the government will have created the existing stock of base money by purchasing stocks of either domestic bonds $BD$ or foreign exchange reserves $R$ and therefore $H = R + BD$, equation (11) can be modified to produce

$$R = \frac{1}{mm} P L(r, Y, \Phi_m) \tag{12}$$

where $BD$ and $R$ can be referred to as the domestic and foreign source components of domestic base money. It follows from this equation that the stock of government-held foreign exchange reserves will be the difference between the quantity of nominal base money demanded and the quantity created through purchases.
of domestic bonds by the monetary authorities. These domestic bonds will typically be government bonds that were previously issued to the public to finance government expenditure, although the monetary authority can buy bonds from the treasury branch of government to directly finance government expenditures, in which case the government is simply printing money and spending it. The balance of payments surplus, which equals the rate of increase through time in the stock of reserves, is simply the change in the stock of reserves per unit time,

\[
\frac{\Delta R}{\Delta t} = \frac{\Delta}{\Delta t} \left( \frac{1}{mm} P_L(r, Y, \Phi_m) \right) - \frac{\Delta B_D}{\Delta t}. \tag{13}
\]

It is clear from equations (12) and (13) that the authorities can accumulate any desired stock of foreign exchange reserves without affecting the level of output and employment by simply selling domestic government bonds to the public and then buying foreign exchange reserves in the foreign exchange market to bring the stock of domestic high powered money, and hence domestic money supply, back up to the level the public wishes to hold. Moreover, there is no causal relationship between the balance of trade or the current account balance and balance of payments equilibrium—balance of payments surpluses and deficits are entirely monetary phenomena. Accordingly, the net capital outflow can be decomposed into two parts, autonomous represented by ANCO and induced represented by \(\frac{\Delta R}{\Delta t}\) as follows

\[
NCO = \frac{\Delta R}{\Delta t} + ANCO \tag{14}
\]

so that the balance of payments surplus can be alternatively expressed as

\[
\frac{\Delta R}{\Delta t} = NCO - ANCO = CAB - ANCO. \tag{15}
\]

Balance of payments deficits or surpluses arise when the autonomous net capital inflow is greater or less than the underlying real equilibrium net capital inflow because the public is attempting to unload or accumulate money balances and the government is forced to acquire or provide those money balances by decumulating or accumulating foreign exchange reserves in order to maintain the fixed level of the nominal exchange rate.

It is often argued that foreign exchange market crises arise because the country in question is running out of foreign exchange reserves. Since the authorities can accumulate reserves by simply selling government debt, there is technically nothing to prevent them from adjusting the stock of foreign exchange reserves to any level desired. So foreign exchange market crises must arise because the authorities do not want to do what is necessary to maintain the stock of reserves at some desired level. Or, to put it differently, there must be underlying reasons why the authorities will not be able to maintain the fixed exchange rates. In fact, there are two such reasons. One arises when the authorities feel politically obliged to finance government expenditures by printing money—they cannot do this and maintain a
fixed exchange rate. The second arises when the equilibrium real exchange rate is falling. From the definition of the real exchange rate

\[ Q = \frac{\Pi P}{\tilde{P}} \]

where \( \Pi \) is the nominal exchange rate defined as the foreign currency price of domestic currency and \( \tilde{P} \) is the foreign price level, the equilibrium domestic price level can be expressed as

\[ P = \frac{Q\tilde{P}}{\Pi} . \]

When the nominal exchange rate is fixed, a fall in the equilibrium real exchange rate will require a proportional fall in the domestic price level which cannot be accomplished without a short-run reduction in the level of employment. Political pressures will therefore ultimately force the authorities to let the exchange rate fall instead of the price level.

As long as domestic residents are allowed to buy and sell capital assets abroad, there is no basis for the argument that a balance of payments deficit is occurring because exports are falling relative to imports and the current account balance is deteriorating. Rather, the argument must be that the demand for domestic output in world markets is declining so that the equilibrium domestic real exchange must fall—it is the unwillingness of the authorities to let the domestic price level fall that causes the problem. Exports will actually fall relative to imports only when there is a reduction in the net capital outflow, or increase in the net capital inflow, which represents a decline in the world demand for domestic output. Alternatively, the world demand for domestic output can decline via a reduction in the world demand for domestic goods with an unchanged net capital flow, in which case the real exchange rate will devalue without any change in the current account balance.