Advanced Topic 4:
Exchange Rate Determination I

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Our first concern is with the time-series properties of real and nominal exchange rates and with a basic outline of the factors determining them in the real world. The **nominal exchange rate is** here defined as **the foreign currency price of domestic currency** and denoted by Π. The real exchange rate is denoted by Q as follows

$$Q = \frac{\Pi P}{\tilde{P}} \tag{1}$$

where P is the domestic price level and $\tilde{P}$ the foreign price level.

In analyzing real exchange rate movements we must distinguish between traded and non-traded components of each country's output and thereby **express the domestic and foreign price levels as geometric indexes of the country's underlying prices of traded and non-traded output components**,

$$P = P_N^{\theta} P_T^{1-\theta} \tag{2}$$

and

$$\tilde{P} = \tilde{P}_N^{\tilde{\theta}} \tilde{P}_T^{1-\tilde{\theta}}\,, \tag{3}$$

where $\theta > 0$ and $\tilde{\theta} > 0$ are the fractions of domestic and foreign output represented by non-traded components. Although the traded components of domestic and foreign output typically will not involve the same goods because the countries may trade with different third countries as well as with each other, we can express the domestic traded component in the foreign country's currency by replacing P_T with $\tilde{P}_{T_D}/\Pi$ where $\tilde{P}_{T_D}$ is the foreign currency price of the domestic traded component of output. Thus we have

$$\begin{aligned} Q &= \frac{\Pi\, P_N^{\theta} P_T^{1-\theta}}{\tilde{P}_N^{\tilde{\theta}} \tilde{P}_T^{1-\tilde{\theta}}} = \frac{\Pi\, P_N^{\theta} (\tilde{P}_{T_D}/\Pi)^{\,1-\theta}}{\tilde{P}_N^{\tilde{\theta}} \tilde{P}_T^{1-\tilde{\theta}}} \\ &= \frac{(\Pi/\Pi^{\,1-\theta})\, P_N^{\theta} \tilde{P}_{T_D}^{1-\theta}}{\tilde{P}_N^{\tilde{\theta}} \tilde{P}_T^{1-\tilde{\theta}}} = \left[\frac{(\Pi P_N)^{\theta}}{\tilde{P}_N^{\tilde{\theta}}}\right] \left[\frac{\tilde{P}_{T_D}^{\;1-\theta}}{\tilde{P}_T^{\;1-\tilde{\theta}}}\right]. \end{aligned} \tag{4}$$

As can be seen from the above equation, **the long-run equilibrium effects of world technological change and capital accumulation on a country's real exchange rate with**

respect to some other country will depend on the effects of these forces on the price of domestic relative to foreign traded output-components and the price of domestic relative to foreign non-traded output-components, where all prices are measured in a single currency.

Since the non-traded components of output are primarily labour services and are less amenable to increases in labour productivity than the traded-components, the relative price of the non-traded components should tend to rise as real income expands and real wages rise, leading us to expect that the real exchange rates of the more rapidly growing countries will tend to rise through time—this is the well-known Balassa-Samuelson hypothesis.

A second force leading to real exchange rate movements is changes in the allocation of world investment among countries. As technology advances, the resources of different countries become favored for development and world investment shifts to those locations. As noted in the *Advanced Topic: Real and Monetary Shocks and Balance of Payments Equilibrium*, **a shift of world investment into a country will cause its real exchange rate to rise.**

Changes in government expenditure as a fraction of output will probably also cause the real exchange rate to rise because political pressure will direct that spending towards non-traded rather than traded goods.

Also, a rise in world prices of traded goods a country specializes in—for example, oil or commodities—will cause its real exchange rate to rise. And, accordingly, we would expect that an improvement of the terms of trade of a country with respect to the rest of the world will be positively associated with an increase in the domestic real exchange rate.

Domestic and rest-of-world nominal interest rates, i and $\tilde{i}$, are related according to the interest parity condition, a rough definition of which is

$$i - \tilde{i} \quad = \quad -\Phi + \rho_c \tag{5}$$

where Φ is the forward premium on domestic currency on the foreign exchange market and ρ_c is the country-specific risk premium on domestic assets. **a precise definition is**

$$F \quad = \quad S\left[\frac{1+id}{1+if}\right] \tag{6}$$

which yields a forward premium on the domestic currency of

$$\phi \quad = \quad \frac{F}{S} - 1 \tag{7}$$

where F and S are the forward and spot prices of foreign currency in units of domestic currency and id and if are the domestic and foreign nominal interest rates. **If the domestic currency is selling forward at a premium, foreign individuals contemplating a reallocation of part of their portfolios in the direction of increased domestic asset holdings will receive a larger amount of foreign currency when they forward-sell the domestic**

currency to cover themselves at the time when their newly acquired domestic investment matures. Accordingly, they will be satisfied with a lower interest rate on domestic assets relative to that on foreign assets by the amount of the forward premium. Also, we would expect a country-specific risk premium on holding domestic as compared to foreign assets to be quite independent of whether the exchange rate is fixed or flexible. **Given the risk premium, equation (5) above will be an arbitrage condition. If the risk premium is zero, covered interest rate parity is said to hold. The interest parity condition (5) always holds, of course, provided that asset holders are rational in the sense that they do not ignore or throw away information.**

Rational behaviour of market participants also implies that the forward premium on the domestic currency equal the expected future change in the exchange rate over the relevant forward contract period plus an adjustment to cover foreign exchange risk

$$\Phi \quad = \quad E_{\Pi} - \rho_x \tag{8}$$

where E_{Π} is here the expected relative change in the value of the domestic currency in terms of the foreign currency and ρ_x is the foreign-exchange risk premium required in the market by individuals taking uncovered positions. **This condition—called the efficient markets condition—holds because whenever market participants expect that the domestic currency is going to increase in price over a future period by more than the forward premium plus an appropriate allowance for risk, they will buy it forward, taking an uncovered position, in the expectation that when the forward contract matures they will be able to sell the currency at a higher price than they will be paying for it under the forward contract**. Note that **this involves speculation rather than arbitrage** because the expected profit depends on the change in the exchange rate over the life of the forward contract being sufficiently in the direction predicted by the individual taking the uncovered position—otherwise, a loss will be suffered. **To compensate for the probability of a loss, a risk premium equal to ρ_x will be required by the marginal participant. Rational behaviour implies that this risk premium will be sufficient, or more than sufficient, for all individuals taking uncovered positions**.

Substitution of (8) into (5) yields

$$i \quad = \quad \tilde{i} - E_{\Pi} + \rho_x + \rho_c \quad = \quad \tilde{i} - E_{\Pi} + \rho \tag{9}$$

were $\rho = \rho_c + \rho_x$ is the combined risk premium. **When this risk premium is zero, uncovered interest rate parity is said to hold. A higher expected rate of increase in the domestic nominal exchange rate leads to an increase in the forward premium, which is compensated for in the covered interest rate parity condition by a fall in the domestic nominal interest rate.**

A corresponding relationship between domestic and foreign real interest rates can be obtained by substituting for each nominal interest rate the Fisher-condition which specifies that the nominal interest rate must equal the respective domestic real interest rate plus the expected home inflation rate. Equation (9) then becomes

$$\begin{aligned} r \quad &= \quad \tilde{r} - E_P - E_{\Pi} + E_{\tilde{P}} + \rho \\ &= \quad \tilde{r} - E_Q + \rho \end{aligned} \tag{10}$$

where E_P and $E_{\tilde{P}}$ are the expected rates of inflation in the domestic and foreign economies and $E_Q = E_P - E_\Pi + E_{\tilde{P}}$ is the expected rate of change in the real exchange rate. **An actual (expected) increase in the real exchange rate increases the actual (expected) increase in the international value of domestically employed capital. The expectation of a capital gain on domestically-employed real capital will result in a fall in the domestic net-of-capital-gain real interest rate**.

Now consider the conditions determining a country's nominal exchange rate. An appropriate rearrangement of (1) yields

$$\Pi \quad = \quad \frac{Q\tilde{P}}{P} \tag{11}$$

from which it is clear that under full-employment conditions **the nominal exchange rate, when allowed to float, will move in proportion with the full-employment level of the real exchange rate and will rise and fall relative to the real exchange rate in proportion to the ratio of the foreign to domestic price levels which will change through time in accordance with ongoing domestic and foreign inflation**.

It is obvious from the above equation that **in the short-run when the price levels cannot change the real exchange rate will move in step with the nominal exchange rate and the level of employment will therefore also change—movements of the nominal exchange rate brought about by attempts of the public to maintain portfolio equilibrium will drag the real exchange rate along with it.** Confronted with excess money holdings, people will attempt to re-balance their portfolios by exchanging this excess money for non-monetary assets. In a small open economy, income and employment will adjust in response to the resulting real exchange rate movements that re-establish asset equilibrium, as can be seen from the demand function for liquidity

$$L \quad = \quad P\,L(\tilde{r} + \rho - E_Q + E_P, Y, \Phi_m)\,. \tag{12}$$

As will be later shown in more detail, nominal exchange rates will move in the short-run far beyond their new long-run equilibrium levels because, while portfolio re-allocations will be immediate, it will take the adjustments in income and employment necessary to re-establish portfolio equilibrium longer to occur. Such overshooting movements in response to monetary shocks are likely to be substantial in the very short run.

Much of the traditional foreign exchange market analysis has viewed the **exchange rate as an asset price**, with deviations from some constant purchasing power parity level seen as consequences of the evolution of asset prices in the face of policy shocks and other news affecting asset returns. **Asset theory suggests that the risk premiums on foreign exchange holdings should depend on how the returns to those assets co-vary with the return to a market portfolio** which in this type of analysis is very difficult to determine.

Although foreign currency holdings are obviously assets whose value is represented by the exchange rate, exchange rates have a much less direct role in pricing bonds, equities and other assets whose earnings are denominated in a currency foreign to their owner. Observed exchange rate movements may simply reflect other

factors determining the values of asset holdings such as domestic/foreign inflation differentials.[1] The changes in the values of domestic and foreign residents' out-of-country asset holdings as a result of real exchange rate movements will be the same in the long-run whether nominal exchange rates are fixed or flexible. Such **real exchange rate movements, which may or may not be matched by corresponding nominal exchange rate movements in the long run, are negatively related to the real value to domestic residents of foreign asset holdings of all types. If those real exchange rate movements are poorly correlated with changes in the return to the market portfolio, the portfolio risk of holding foreign assets can only be diversified away by limiting those holding to a tiny fraction of domestic wealth. This could explain the significant "home bias" that is in fact widely observed in the market portfolios of investors throughout the world**.

While it is unquestionable that real exchange rate movements will be related in various ways to observed changes in countries' asset returns, **it makes little sense to view exchange rates primarily as asset prices. Instead, we have to think of countries' real exchange rates as the relative price of domestic in terms of foreign output, while keeping in mind that exchange rate movements will have effects of various magnitudes on, as well as sometimes being affected by, the returns to holding particular assets both at home and abroad.**

Does it make any sense to assume purchasing-power-parity—that is, that Q is constant through time? Consider the real and nominal exchange rates and the associated movements in the domestic relative to foreign price levels for the country-combinations in the Figures below.

As can be seen in the middle and bottom panels of Figure 1 below, Canada's real exchange rates with respect to the United States and the United Kingdom were higher or lower than their 1950 base levels on frequent occasions by amounts ranging up to more than 30 percent over the past 135 years or so. The variability of real exchange rate of the United Kingdom with respect to the United States over the past 200 years, shown in the top panel, was even greater. **These real exchange rate variations are clearly contrary to the implications of the purchasing-power-parity theory. Yet the three real exchange rate series do not show any overall trend. This is consistent with the analysis above which suggests that considerable variation in real exchange rates would be expected from time to time. The absence of overall trends suggests, when we apply the Balassa-Samuelson hypothesis, that the real incomes of these three countries grew by roughly similar relative amounts.**

The price level ratios in the three panels indicate that there was greater inflation in the United Kingdom than in the United States and Canada since the first world war and that these inflation differences grew substantially after 1950. Canada and the United States have had similar inflation rates on average although the Canadian price level rose about 10 percent relative to the U.S. price level in the 1970's and 1980s and then fell by the same amount thereafter.

[1] See pages 108 through 112 of *Interest Rates, Exchange Rates and World Monetary Policy* for a more complete discussion.

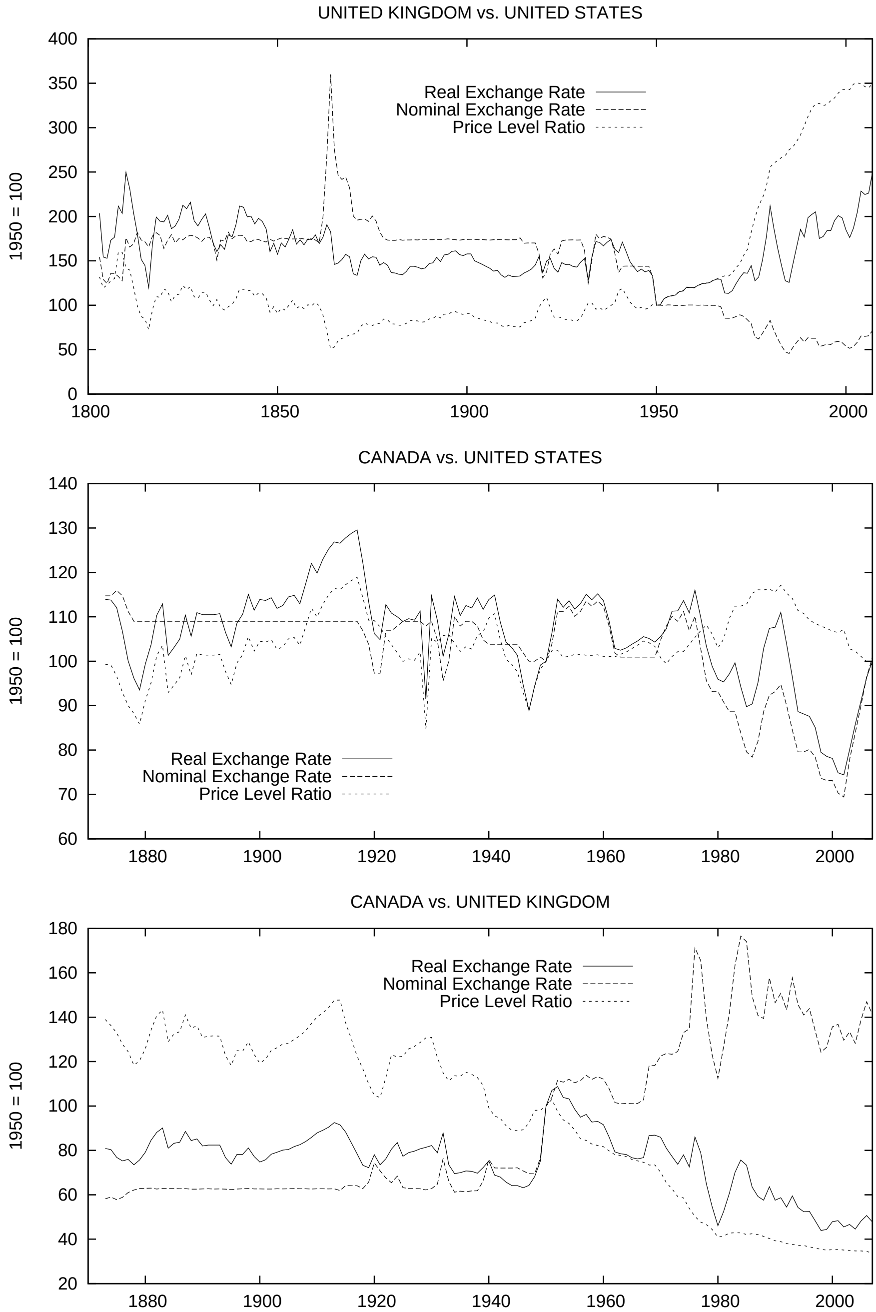


Figure 1: Real exchange rates, nominal exchange rates and price level ratios: Annual data, 1950 = 100

Notice also that when the nominal exchange rates were fixed during the gold standard period before the first world war, and during the Bretton-Woods period after 1950 for the U.K. vs. the U.S., the price level ratios and the real exchange rates move in proportion to each other, as we would predict. And, not surprisingly, during the remaining years when exchange rates were flexible the real and nominal exchange rates moved together, differing only by the relatively smooth changes in relative price levels.

Figures 2 and 3 below show the real and nominal exchange rates and price level ratios for Canada, the United Kingdom, Japan, France and Germany with respect to the United States using monthly data for the period from 1957 onward. In all cases the movements in the real and nominal exchange rates are extremely highly correlated after the fall of the Bretton-Woods system in the early 1970s. While the U.K. price level rose substantially relative to that of the U.S. once the exchange rate was allowed to float, and then flattened out after 1990, these **price-level differences were smooth through time**. Similar price level increases occurred in France relative to the United States from the fall of Bretton-Woods to the mid-1980s, with a decline relative to the U.S. occurring thereafter as France prepared for and ultimately adopted the Euro as its currency. Germany had a lower inflation rate on average than the United States for the entire post-Bretton-Woods period while Japan's price level increased relative to that in the U.S. to the late 1970s and then declined quite substantially thereafter.

As can be seen from the top panel in Figure 2, Canada's real exchange rate with respect to the United States fell by about 30 percent between the mid-1970s and mid-1980s, rose by around 20 percent between the mid-1980s and the early 1990s, fell again by about 35 percent by around 2002, and then increased by around 40 percent by 2010. In comparing the Canadian case with that of the other countries, it is very important to keep in mind that the scales on the vertical axes differ very substantially from the scale of the vertical axis in the Canada/U.S. panel. The increase in Japan's real exchange rate with respect to the United States was 350 percent between the early 1970s and the mid-1990s and the subsequent fall by 2009 was greater than 50 percent of the mid-1990s levels.

The real exchange rate variations of France and Germany with respect to the United States were more than 40 percent downward in the early 1980s and again in the late 1990s, with upward movements occurring in the intervening and subsequent years.

There is clearly no basis for the purchasing-power-parity theory. And if we view the levels of the observed real exchange rates as not much out of line with their full-employment levels, the adoption of fixed exchange rates with respect to the U.S. dollar would make no sense whatsoever.

The enormous variability of the above real exchange rates raises the important question of whether countries' real exchange rates are stationary over time. Suppose that the real exchange rate changes through time in a manner that could be described by the equation

$$q_t \;=\; \beta\, q_{t-1} + \epsilon_t \tag{13}$$

where ϵ_t **is a white noise process with mean equal to zero. If** $\beta = 0$**, this becomes**

$$q_t \;=\; \epsilon_t . \tag{14}$$

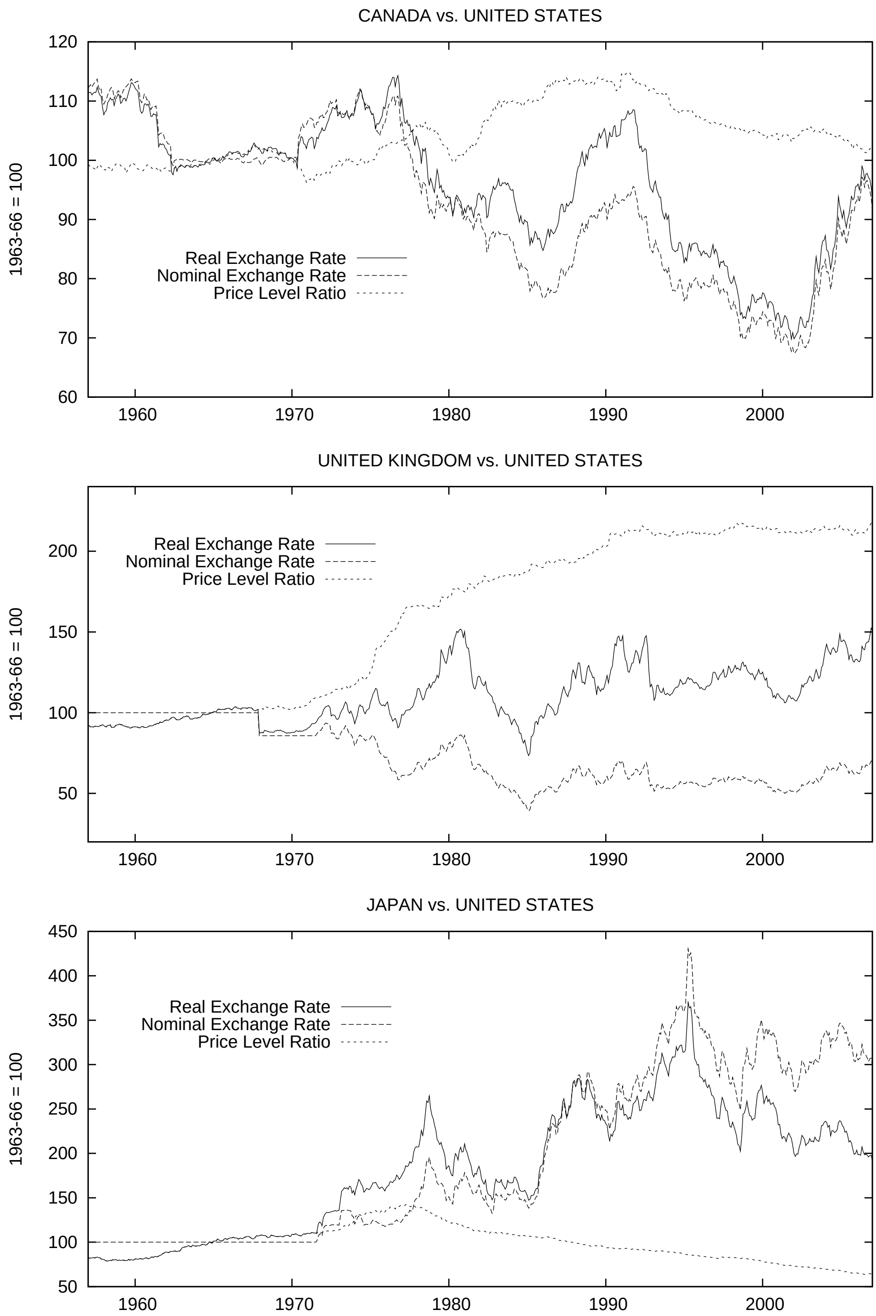


Figure 2: Real exchange rates, nominal exchange rates and price level ratios: Monthly data, 1963-66 = 100

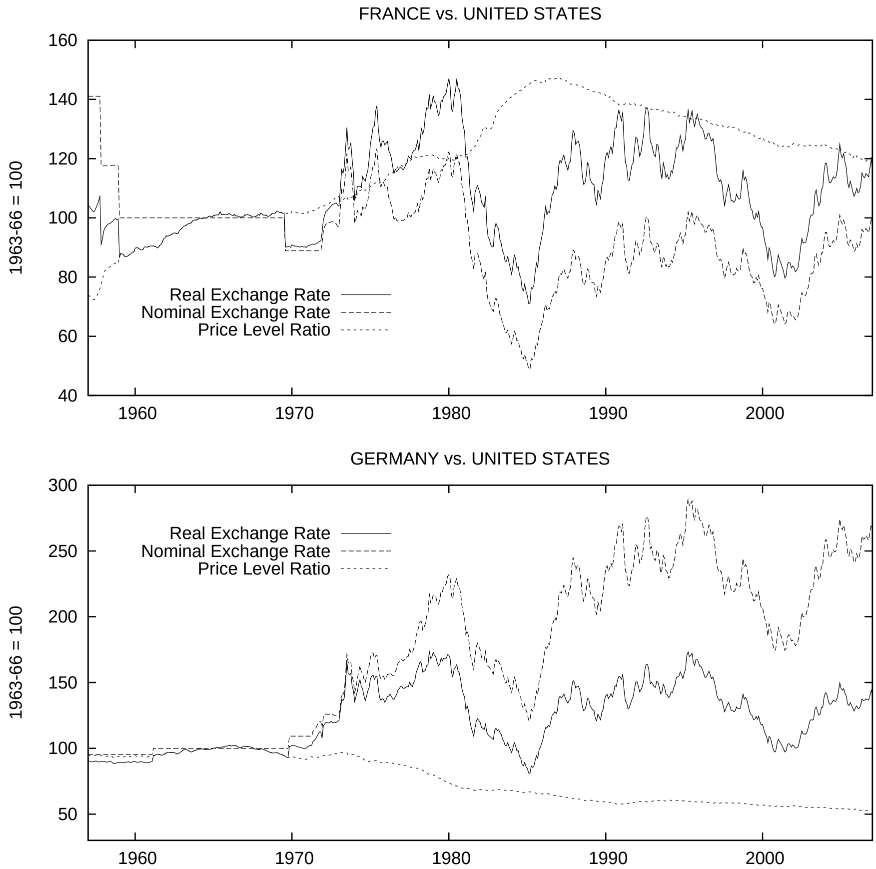


Figure 3: Real exchange rates, nominal exchange rates and price level ratios: Monthly data, 1963-66 = 100. After 1998 the Euro/Dollar exchange rate was spliced to the Franc/Dollar and Deutschmark/Dollar exchange rates.

and q_t is itself a white noise process which must necessarily be stationary. Alternatively, if $\beta = 1$, we have

$$q_t - q_{t-1} \quad = \quad \epsilon_t \tag{15}$$

and the series becomes a random walk, wandering without limit as q_t moves either up or down in each period compared to its previous value by the amount ϵ_t. The series is non-stationary and can be described mathematically as having a unit root. Since, in this case, q_t can also be expressed as

$$q_t \quad = \quad \epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \epsilon_{t-3} + \cdots\cdots + \epsilon_0\,, \tag{16}$$

its variance will equal $(\sigma^2 + \sigma^2 + \sigma^2 + \cdots\cdots)$ which will grow in proportion to the number of periods over which it is being calculated. **Its expected level at any future point in time is its current level and its variance in the limit is infinity. Its future path**

need never pass through the level at which it started or any other level previously achieved, although there is no reason why it could not eventually return to those levels. When $0 < \beta < 1$ the series is stationary with a degree of mean reversion depending on the size of β, the effects of each ϵ_t shock dissipating with time. This can be seen from the fact that

$$q_t \;=\; \epsilon_t + \beta\,\epsilon_{t-1} + \beta^2\,\epsilon_{t-2} + \beta^3\,\epsilon_{t-3} + \beta^4\,\epsilon_{t-4} + ... + \beta^{\infty}\,\epsilon_{t-\infty} \tag{17}$$

so that the effects of a shock in a particular period on the level of q in subsequent periods gets increasingly smaller since β^{t+i} gets smaller as i increases when $\beta < 1$.

The real exchange rate specified in (13) above is a first-order autoregressive process. More generally, the time-series properties of real exchange rate movements could involve additional lags of the real exchange rate as well as one or more lags of the error term. An example of an ARMA(2,2) series (having two autoregressive and two moving-average lags) is

$$q_t \;=\; \beta_1\,q_{t-1} + \beta_2\,q_{t-2} + \,\xi_0\,\epsilon_t + \xi_1\,\epsilon_{t-1} + \xi_2\,\epsilon_{t-2}\,. \tag{18}$$

It turns out that **any equation like the above that includes both autoregressive and moving average terms can be expressed in the form of a pure autoregressive process containing an infinite number of autoregressive lags. Such a process, with a constant and time trend added, can be expressed in the form**[2]

$$\begin{aligned}\Delta q_t \;=\;& \alpha + \gamma\,t - (1-\rho)\,q_{t-1} + \delta_1\,\Delta q_{t-1} + \delta_2\,\Delta q_{t-2}\\ &+ \delta_3\,\Delta q_{t-3} + \cdots\cdots\cdots + \epsilon_t\end{aligned} \tag{19}$$

which will be stationary if $\alpha = \gamma = 0$ and $\rho < 1$. If $\rho < 1$ and $\alpha \neq 0$, the series is stationary around a drift through time of α per period. If $\rho < 1$ and $\gamma \neq 0$, the series is stationary around a drift that is increasing or decreasing through time depending on the sign of γ. In the case where there is no drift or trend, the degree of mean reversion will be greater the closer ρ is to zero.

One method of attempting to determine whether a real exchange rate series is stationary or stationary around drift and trend is to run an OLS regression based directly on the equation above and then testing whether the coefficients α, γ and $(1-\rho)$, are significantly different from zero, negatively so in the case of $-(1-\rho)$. This test, known as a Dickey-Fuller Test, may or may not be "augmented" by including lagged values of the change in q_t. The issue that immediately arises is how may such lags to include. This can be decided by **choosing a number of lags that includes all statistically significant ones, or by choosing the number of lags that results in the smallest value of either the Akaike Information Criterion (AIC) or Schwartz Bayesian Information Criterion (SBC). These give calculated optimal balances between the gain associated with the reduction in the residual sum of squares when a lag is added and the loss associated with having one less degree of freedom**. The relevant formulae to be calculated for each regression as lags are added are

[2]See pages 119 through 121 in *Interest Rates, Exchange Rates and World Monetary Policy* for details on how to do this.

$$AIC(n) = ln\left(\frac{SSR(n)}{T}\right) + (n+1)\frac{2}{T} \quad (20)$$

$$SBC(n) = ln\left(\frac{SSR(n)}{T}\right) + (n+1)\frac{ln(T)}{T} \quad (21)$$

where $SSR(n)$ is the sum of squared residuals, n is the number lags and T is the number of observations and where $ln()$ is represents the natural logarithm of the expression in the brackets. **Three regressions are typically run, one of which drops the trend term, another which drops both the trend and drift terms, along with a the third that includes both of these terms**.

It turns out that **under the null-hypothesis that $\rho = 1$ the estimated coefficient of $(1-\rho)$ is not distributed according to the t-distribution. A table of critical values constructed by Dickey and Fuller must be used instead of the standard t-tables.** These and other tables to be subsequently referred to are presented in the file `statabs.pdf` which is accessible off the main course page.

The Dickey-Fuller tests assume that the errors ϵ_t are statistically independent of each other and have a constant variance. An alternative procedure, called the Phillips-Perron test, can be used under the assumption that there is some interdependence of the errors and they are heterogeneously distributed. The following equations are estimated by ordinary-least-squares:

$$q_t = \alpha_1 + \rho_1\, q_{t-1} + \gamma\,(t - T/2) + \upsilon_t \quad (22)$$

$$q_t = \alpha_2 + \rho_2\, q_{t-1} + \nu_t \quad (23)$$

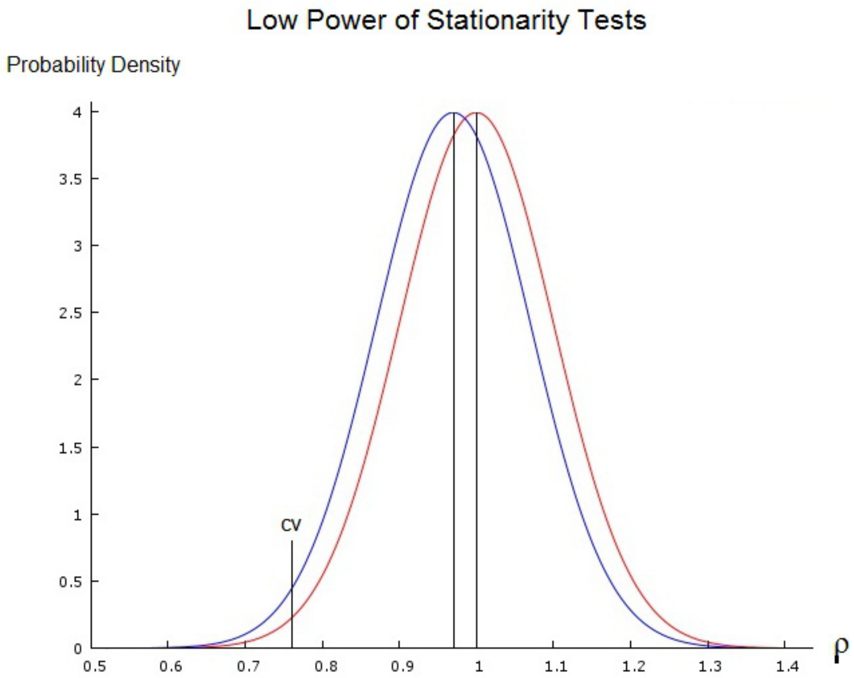
$$q_t = \rho_3\, q_{t-1} + \omega_t \quad (24)$$

where T is the number of observations and υ_t, ν_t and ω_t are error terms. Test statistics based on modifications of the conventional t-statistics are then calculated to allow for heterogeneity and interdependence of the error process. The critical values are the same as those for the corresponding statistics estimated using the Dickey-Fuller approach and the objective is to determine the circumstances, if any, under which the coefficient of q_{t-1} is significantly less than unity, in full recognition of the possibility that there may be stationarity with drift and trend.

A major problem is that the two test procedures just outlined have low power in that they have poor ability to detect stationarity when the true value of ρ is close to unity. When we test the null-hypothesis that ρ equals unity, we use the Dickey-Fuller table to obtain the appropriate critical value of the t-statistic for the estimate of $-(1-\rho)$. This critical value will be some negative number below which the estimated t-value has some small probability, say .05, of falling if ρ is really unity. The Phillips-Perron test directly estimates ρ.

As can be seen from the distribution plotted below,[3] if ρ in fact equals unity the probability of rejecting the null-hypothesis of non-stationarity and concluding

[3] The curves in this figure were constructed using the `tools` menu in `Gretl` and saved as a `.png` file which was then loaded into `MS Paint` where the title, vertical lines and labels were added. It was there saved as a `.jpg` file which was then loaded into `Gimp` and saved as the `.eps` file which was loaded above as the figure.



incorrectly that the series is stationary is equal to the small area to the left of the vertical cv line denoting the critical value—this area will equal, say, .05 indicating that the chance of incorrectly rejecting the null hypothesis is 5% with a 95% chance of correctly concluding that the series is non-stationary. Suppose now that the true value of ρ is slightly less than unity, as indicated by the left-most density curve, implying stationarity with a very small degree of mean reversion. Since the critical value will be the same, application of the test will still lead to the conclusion that the real exchange rate series is non-stationary not far from 95% of the time because the t-statistic will still be below the critical value a small proportion of the time. So there a small risk, 5% in the example above, of concluding that the real exchange rate is stationary when it is not, and a very high risk of concluding that it is non-stationary when it is in fact stationary at true values of ρ not far below unity. We must therefore view these tests with caution.

Tables 1 and 2 below present the results of tests performed on the real exchange rates of the U.K. with respect to the U.S., Canada with respect to the U.S., and Canada with respect to the U.K. **using annual data spanning periods longer than 100 years. Tables 3 and 4 present the results of tests using monthly real exchange rates for the years 1957 through 2007** for Canada, France, Germany, Japan and the U.K. with respect to the United States.

The equality conditions above the statistical results in the Phillips-Perron tables indicate the null-hypotheses being tested with the test-statistics presented in the respective columns below.[4]

[4] These test were performed in `XLispStat` and `R` using the input files `urootan.lsp`, `urootmo.lsp`, `urootan.R`

Table 1: Dickey-Fuller test results for real exchange rate series: Annual data

Dependent Variable Δq_t	Drift	q_{t-1}	Δq_{t-1}	Trend
U.K. / U.S. 1805–2007	0.504	-0.090	$0.145^{\circ\circ}$	-0.002
	(0.437)	(-2.715)	(2.063)	(-0.252)
	0.247	-0.087^{**}	$0.142^{\circ\circ}$	
	(0.458)	(-2.803)	(2.054)	
		-0.087^{***}	$0.142^{\circ\circ}$	
		(-2.821)	(2.063)	
Canada / U.S. 1874–2007	1.760	-0.145^{*}		-0.013
	(1.558)	(-3.191)		(-1.655)
	-0.060	-0.104^{*}		
	(-0.229)	(-2.711)		
		$-.105^{***}$		
		(-2.722)		
Canada / U.K. 1875–2007	4.793	-0.108	$0.253^{\circ\circ\circ}$	-0.036
	(2.426)	(-3.008)	(2.962)	(-2.128)
	-0.238	-0.066	$0.239^{\circ\circ\circ}$	
	(-0.431)	(-2.174)	(2.771)	
		-0.067^{**}	$0.241^{\circ\circ\circ}$	
		(-2.191)	(2.804)	

Notes and Sources: All series tested are expressed as percentage deviations from their means. The numbers in parentheses below the coefficients are the conventional t-statistics. The superscripts *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively, using the Dickey-Fuller tables and the superscripts $^{\circ}$, $^{\circ\circ}$ and $^{\circ\circ\circ}$ indicate significance at the 10%, 5% and 1% levels according to a standard t-test.

Table 2: Phillips-Perron test results for real exchange rate series: Annual Data

$$q_t = a_0 + a_1 q_{t-1} + a_2 (t - T/2) + u_t$$

$$q_t = \tilde{a}_0 + \tilde{a}_1 q_{t-1} + v_t \qquad q_t = \hat{a}_1 q_{t-1} + w_t$$

	$a_0 = 0$	$a_1 = 1$	$a_2 = 0$	$a_0 = 0$ & $a_1 = 1$	$\tilde{a}_1 = 1$	$\hat{a}_1 = 1$
U.K. / U.S. 1805–2002						
Lags = 1	0.137	-2.774	0.290	4.463	-2.985**	-3.002***
Lags = 4	0.145	-2.637	0.226	4.136	-2.876*	-2.894***
Canada / U.S. 1874–2002						
Lags = 1	-0.201	-3.090	-1.432	4.631	-2.620*	-2.639***
Lags = 4	-0.184	-3.325*	-1.068	5.010	-2.803*	-2.823***
Canada / U.K. 1874–2002						
Lags = 1	-0.547	-2.502	-1.513	2.923	-1.604	-1.628*
Lags = 4	-0.548	-2.498	-1.520	2.910	-1.582	-1.617*

Notes and Sources: All series tested are expressed as percentage deviations from their means. The superscripts *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively, using the Dickey-Fuller tables which are also appropriate for the Phillips-Perron test. The statistics in all columns but the fourth from the left are t-based.

Table 3: Dickey-Fuller test results for real exchange rate series: Monthly data, 1957–2007

Dependent Variable Δq_t	Drift	q_{t-1}	Trend	Δq_{t-1}	Δq_{t-2}	Δq_{t-3}
	-0.049	-0.005	0.0001	$0.207^{\circ\circ\circ}$		
	(-0.427)	(-0.859)	(0.404)	(5.136)		
Canada	-0.006	-0.006		$0.209^{\circ\circ\circ}$		
/ U.S.	(-0.139)	(-1.622)		(5.262)		
		-0.006^{*}		$0.209^{\circ\circ\circ}$		
		(-1.623)		(5.266)		
	Estimation begins with March 1957					
	-0.556	-0.027^{*}	0.002	$0.338^{\circ\circ\circ}$	$-0.129^{\circ\circ\circ}$	$0.092^{\circ\circ}$
	(-2.239)	(-3.196)	2.700	(8.337)	(-3.038)	(2.263)
U.K.	0.069	-0.010		$0.332^{\circ\circ\circ}$	$-0.136^{\circ\circ\circ}$	$0.083^{\circ\circ}$
/ U.S.	(0.768)	(-1.782)		(8.169)	(-3.214)	(2.028)
		-0.010^{*}		$0.333^{\circ\circ\circ}$	$-0.136^{\circ\circ\circ}$	$0.089^{\circ\circ}$
		(-1.791)		(8.206)	(-3.250)	(2.057)
	Estimation begins with May 1957					
	-0.349	-0.011	0.0014	$0.321^{\circ\circ\circ}$		
	(-0.922)	(-1.953)	(1.169)	(8.307)		
Japan	0.072	-0.005		$0.316^{\circ\circ\circ}$		
/ U.S.	(0.620)	(-1.751)		(8.288)		
		-0.005^{*}		$0.317^{\circ\circ\circ}$		
		(-1.751)		(8.259)		
	Estimation begins with March 1957					

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Dependent Variable Δq_t	Drift	q_{t-1}	Trend	Δq_{t-1}	Δq_{t-2}	Δq_{t-3}
	-0.160	-0.018	0.0006	$0.256^{\circ\circ\circ}$	$-.073^{\circ}$	$.097^{\circ\circ}$
	(-0.800)	(-2.705)	(1.085)	(6.325)	(-1.759)	(2.382)
France / U.S.	0.030	-0.016		$0.256^{\circ\circ\circ}$	-0.074°	$0.096^{\circ\circ}$
	(0.317)	(-2.501)		(6.321)	(-1.775)	(2.361)
		-0.016^{**}		$0.256^{\circ\circ\circ}$	-0.073°	$0.096^{\circ\circ}$
		(-2.504)		(6.331)	(-1.774)	(2.367)
	Estimation begins with May 1957					
	-0.193	-0.014	0.0008	$0.284^{\circ\circ\circ}$		
	(-0.866)	(-2.465)	(1.280)	(7.303)		
Germany / U.S.	0.063	-0.104		$0.282^{\circ\circ\circ}$		
	(0.645)	(-2.105)		(7.256)		
		-0.010^{**}		$0.283^{\circ\circ\circ}$		
		(-2.108)		(7.287)		
	Estimation begins with March 1957					

Notes and Sources: All the real exchange rate series are expressed as percentage deviations from their means. The numbers in the brackets below the coefficients are the conventional t-statistics. The subscripts containing the * and ° characters have the same meaning as in Table 1.

Table 4: Phillips-Perron test results for real exchange rate series: Monthly Data, 1957–2002

$$q_t = a_0 + a_1 q_{t-1} + a_2 (t - T/2) + u_t$$

$$q_t = \tilde{a}_0 + \tilde{a}_1 q_{t-1} + v_t \qquad q_t = \hat{a}_1 q_{t-1} + w_t$$

	$a_0 = 0$	$a_1 = 1$	$a_2 = 0$	$a_2 = 0$ & $a_1 = 1$	$\tilde{a}_1 = 1$	$\hat{a}_1 = 1$
Canada/U.S.						
Lags = 1	-0.136	-0.433	1.274	1.651	-1.458	-1.460
Lags = 4	-0.129	-0.614	1.348	1.740	-1.533	-1.534
U.K./U.S.						
Lags = 1	0.872	-2.669	1.423	2.861	-1.361	-1.369
Lags = 4	0.812	-2.897	1.034	3.043	-1.536	-1.547
Japan/U.S.						
Lags = 1	0.745	-1.440	-0.116	1.362	-1.581	-1.582
Lags = 4	0.680	-1.678	-0.405	1.601	-1.648	-1.649*
France/U.S.						
Lags = 1	0.350	-2.269	0.808	2.560	-2.068	-2.062**
Lags = 4	0.323	-2.466	0.644	2.739	-2.256	-2.260***
Germany/U.S.						
Lags = 1	0.753	-2.080	0.626	1.964	-1.769	-1.773*
Lags = 4	0.700	-2.234	0.427	2.194	-1.899	-1.905*

Notes and Sources: All the real exchange rate series are expressed as percentage deviations from their means. Estimation starts at February 1957 in the case of one lag and at May 1957 when two lags are used. The superscripts *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively, using the Dickey-Fuller tables which are also appropriate for the Phillips-Perron test. The statistics in all columns but the fourth from the left are t-based.

It is clear from the annual data that the real exchange rates observed were stationary over the long periods involved. And there is no evidence of drift and trend—the constant and time variable were both statistically insignificant in all the regressions and the lagged real exchange rate variable was statistically significant at the 1% level for the U.K. and Canada with respect to the U.S. in both the Dickey-Fuller and Phillips-Perron tests and at the 5% level for Canada with respect to the United Kingdom under the Dickey-Fuller test, but only at the 10% level in the case of the Phillips-Perron test. Given the low power of these tests to detect stationarity, we need not be concerned about the poor level of statistical significance in the one case.

The Dickey-Fuller tests yield an estimated value of $(1 - \rho)$ of about $.1$ for the Canadian real exchange rate with respect to the U.S. and values somewhat below that level in the other two cases. These values indicate **very slow mean reversion**. The regression coefficient are not reported in the Phillips-Perron tests.

In the case of monthly data over the period since 1957, we can reject non-stationarity at the 10% level in the Dickey-Fuller tests for the real exchange rates of Canada, the United Kingdom and Japan with respect to the United States, and at the 5% level for the real exchange rates of France and Germany with respect to the U.S. Non-stationarity can be rejected at the 5% level or better in the Phillips-Perron tests for France with respect to the United States, but otherwise, only in the cases of Japan and Germany with respect to the U.S. can non-stationarity be rejected even at the 10 percent level. In **no case is there any evidence of drift or trend**. Since the regression coefficients in the case of monthly data give the degree of monthly mean reversion we can multiply them by 12 to get a rough idea of the mean reversion on an annual basis—here it would again seem **reasonable to expect a degree of mean reversion of no more than** 0.1.

Although the case for stationarity gets weaker over shorter as compared to longer periods, it would seem reasonable to conclude, given the low power of the tests and the common-sense economics argument that no country's real exchange rate is likely to ever go zero or infinity, that real exchange rates are stationary. But the degree of mean reversion is surely very low. Over very short time horizons, therefore, it would seem reasonable to expect that the real exchange rate, and given stable domestic and foreign inflation rates, the nominal exchange rate as well, are just about as likely to rise as fall between any current period and the next.

Given the evidence that the value of ρ is around 0.9 on an annual basis, we can conclude that nearly 60% of a shock to the real exchange rate will remain after 5 years and about 35 percent of that shock will still remain after 10 years.

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and `urootmo.R` which produced the four output files `urootan.lou`, `urootmo.lou`, `urootan.Rou` and `urootmo.Rou`. The data files used are `jfdataan.lsp`, `jfdatamo.lsp`, `jfdataan.tab` and `jfdatamo.tab`. The data are also available in Excel `.xls` and Gretl `.gdt` files having the above root names and are described in the two text files `jfdataan.cat` and `jfdatamo.cat`. The XLispStat calculations also require the file `addfuncs.lsp` containing the functions written for the purpose at hand.