Advanced Topic 2:
Intertemporal Foundations II

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The material presented here is an extension of the analysis in the previous Advanced Topic entitled Intertemporal Foundations I. Here **we extend the model to deal with deviations of output and employment from their normal or full-employment levels**.

We begin with equations (13), (18) and (20) of the previous Topic which are renumbered below. **The first two are equations the conditions of flow equilibrium and the third is the condition of stock equilibrium. The economy in question is either closed or so world-dominant that it is unaffected by what happens abroad**.

$$r = \frac{1+\varpi}{1-\tau}(1+g) - 1 \tag{1}$$

$$r = \frac{\hat{m}(1-\gamma)-\delta}{1+2\alpha\, g_{kt}}\left[1 - \frac{1}{3\lambda}\left(\varphi - \lambda\frac{L}{K}\right)^3\right] \tag{2}$$

$$r + o = [\hat{m}(1-\gamma)-\delta]\left(\varphi - \lambda\frac{L}{K}\right)^2 \tag{3}$$

These can be treated as the conditions of short-run equilibrium when K is replaced with the quantity of capital actually utilized. The capital stock will be over-utilized or under-utilized to the extent that income is above or below its full-employment level. Letting ΔY represent the excess of the current level of income over its full-employment level, **the quantity of capital utilized** can be expressed as

$$K_U = \frac{Y_f + \Delta Y}{Y_f}K. \tag{4}$$

Then K_U can be substituted for K in (2) and (3) and, assuming that all temporary changes in income are channeled into investment, the full-employment growth rate of capital g can be extended in the short-run to equal $g + \Delta Y/K$ so that

$$r = \frac{\hat{m}(1-\gamma)-\delta}{1+2\alpha\,(g+\Delta Y/K)}\left[1-\frac{1}{3\lambda}\left(\varphi-\lambda\frac{L}{Y_f+\Delta Y}\frac{Y_f}{K}\right)^3\right] \tag{5}$$

and

$$r+o = [\hat{m}(1-\gamma)-\delta]\left[\varphi-\lambda\,\frac{L}{Y_f+\Delta Y}\,\frac{Y_f}{K}\right]^2 \tag{6}$$

where Y_f is the full-employment level of income and $(g + \Delta Y/K)$ is the level of investment as a fraction of the capital stock. This overall growth rate is assumed to be positive. Since the temporary deviation of output from its full-employment level represents a change in the level of investment, consumption remains continually on its long-run equilibrium growth path.

Now let us **choose the units of measurement for capital so that** $K = 1$ so that (4) becomes

$$(K_U - 1)\,Y_f = \Delta Y$$

which, upon substitution into the two equations above, produces

$$r = \frac{\hat{m}(1-\gamma)-\delta}{1+2\alpha\,[g+(K_U-1)\,Y_f]}\left[1-\frac{1}{3\lambda}\left(\varphi-\lambda\frac{L}{K_U}\right)^3\right] \tag{7}$$

and

$$r+o = [\hat{m}(1-\gamma)-\delta]\left(\varphi-\lambda\,\frac{L}{K_U}\right)^2 \tag{8}$$

which together with

$$r = \frac{1+\varpi}{1-\tau}(1+g)-1 \tag{1}$$

and the equation determining Y_f

$$Y_f = [\hat{m}(1-\gamma)-\delta]\left[1-\frac{1}{3\lambda}\left(\varphi-\lambda\frac{L}{K_U}\right)^3\right] \tag{9}$$

can be solved for the four variables g, r, Y_f and L under conditions where $K_U = 1$. This is done numerically in the statistical program R using the input file `GGAAssg.R`. In that file, sets of terms are consolidated into the groups

$$
\begin{aligned}
\texttt{mterm} &= \hat{m}(1-\gamma)-\delta \\
\texttt{acterm} &= 1+2\alpha\left[g+(K_U-1)\,Y_f\right] \\
\texttt{tterm} &= 1-\frac{1}{3\lambda}\left(\varphi-\lambda\frac{L}{K_U}\right)^3 \\
\texttt{dmterm} &= \left(\varphi-\lambda\,\frac{L}{K_U}\right)^2 \\
\texttt{ssgterm} &= \frac{1+\varpi}{1-\tau}(1+g)-1
\end{aligned}
$$

and the values of the underlying parameters chosen in establishing initial full-employment equilibrium are

$\hat{m}$	maximum marginal product of capital	0.14
δ	depreciation rate	0.05
ϖ	rate of time preference	0.015
α	adjustment cost parameter	20
γ	misallocation of capital parameter	0.20
τ	implicit tax on saving	0.01
φ/λ	optimum ratio of liquidity to capital	.03
Υ	fraction of output remaining under zero liquidity	0.2
o	actual and expected long-run relative price-level growth	.02

where φ/λ is referred to as `oplkr`, $\Upsilon = 1-\varphi^3/3\,\lambda$ is referred to as `Ups`, $\hat{m}$ is referred to as `hatm`, o is `o`, and the remaining greek symbols are written as the english words describing them.[1]

After some experimentation starting with the calculation of the levels of Y_f, g, r and L and the actual and expected inflation rate in the case of optimal liquidity according to the Friedman rule, a set of results are calculated using various assumptions. In addition to the optimal-liquidity case, results are calculated producing actual and expected inflation rates of, alternatively, zero, 2 percent, 50 percent and 100 percent. And in the case of 2 percent inflation additional equilibria are successively calculated imposing an improvement of resource allocation taking the form of a reduction in γ to .015 and, alternatively, an increase in the tax on savings to $\tau = .015$. All these results are presented in tabular form in the spreadsheet file `GGAAssg.xls`. Other results can, of course, be calculated in `R` using appropriate modifications of the script `GGAAssg.R` which is constructed in such a way that the current code should be obvious to anyone who can understand an ongoing explanation of what is being done. No initial study of the `R` program should be required.

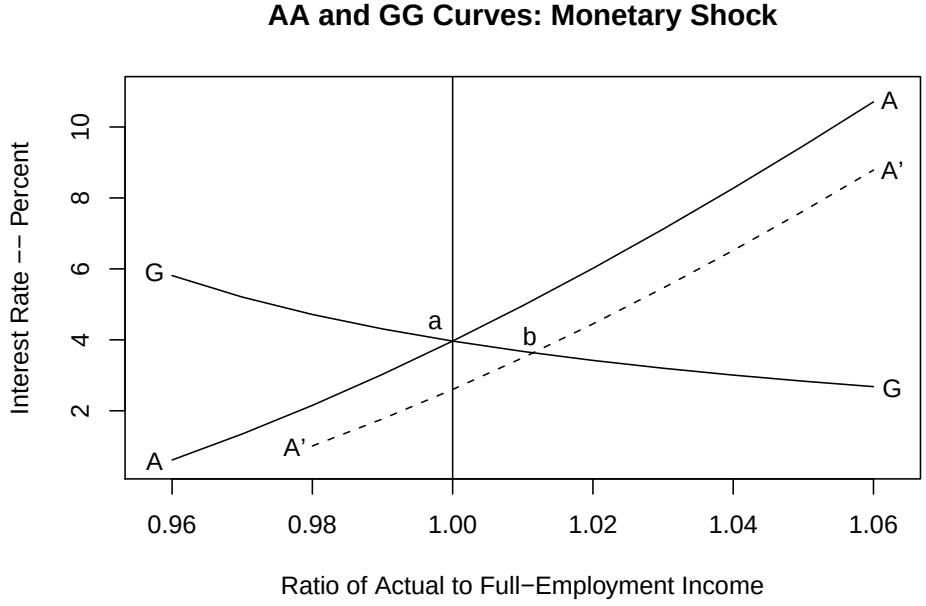
The above full-employment equilibrium results are purely illustrative, indicating only the true direction of the changes in the full-employment equilibrium values in response to the shocks imposed—they tell us nothing about the magnitudes of real-world effects of such shocks. **Not surprisingly, an improvement in resource allocation increases full-employment output, the equilibrium growth rate, the real interest rate and, through a one-shot fall in the price level occurring in the background, an increase in the real stock of**

[1] The implied values of φ and λ are 8.944 and 298.142 respectively.

liquidity. A tax on savings reduces full-employment output and the growth rate, raises the rate of interest because capital growth and associated adjustment costs decline, and reduces the equilibrium real stock of liquidity. High levels of inflation reduce full-employment output and the growth rate as well as the equilibrium stock of liquidity, with the real interest rate remaining unaffected because the effect of the reduced adjustment costs associated with lower investment is offset by the decline in the equilibrium income flow from capital.

To analyze the effects of changes in employment, here viewed as utilization of capital, we set all the parameters at the levels associated with a 2 percent equilibrium actual and expected inflation rate and then impose variations in K_U between 0.96 and 1.06 and calculate the levels of r that result in equations (7) and (8). These respective levels of r trace out the GG and AA curves in the Figures that follow. The first of these gives the combinations of the real interest rate and output that maintain goods market or flow equilibrium and the second give the combinations that maintain stock or asset equilibrium. AA corresponds with the traditional LM curve and GG incorporates those features of the traditional IS curve that survive the imposition of appropriate intertemporal foundations. To incorporate shocks, we calculate the new levels of r associated with the new full-employment levels of output and adjust the relevant curve upward or downward in proportion to the change in r at that level of output. The calculations are performed in `R` using the script `GGAA.R`.

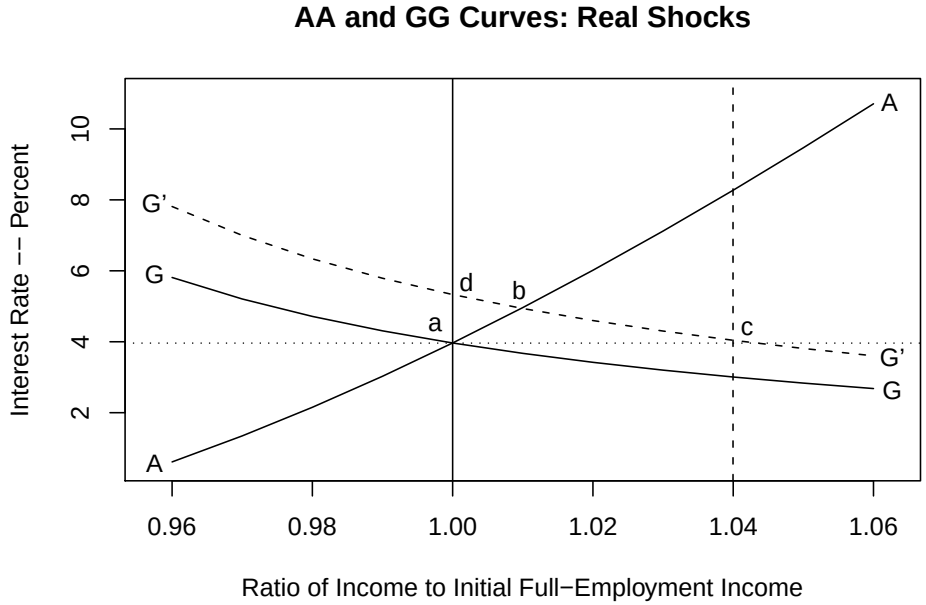
Not surprisingly, the GG curve is negatively sloped and the AA positively sloped. This can be verified without any calculations by simply looking at equations (7) and (8). An increase in employment (capital stock utilization) increases the denominator of the left-most term on the right side of equation (7), and also reduces the term in the square brackets by reducing the real stock of liquidity. For both reasons, r must fall. The same reduction in real liquidity increases the return to holding it in equation (8) causing the nominal interest rate to increase as employment expands. The positive slope of AA follows directly from the fact that the demand for liquidity is positively related to the level of real income and negatively related to the level of liquidity as a fraction of that income. And the negative slope of GG arises from the fact that an expansion of the level of investment increases the adjustment costs of adapting it to the existing capital stock, lowering the return to investment. It turns out that the costs of adjustment to temporary increases in investment are likely to be higher than those to the normal full-employment rate of capital stock expansion with the result that α is likely to be higher in the short-run. Our underlying assumption here is that all the expansion of output takes the form of investment, given the fact that the level of consumption is strictly determined by inter-temporal optimization. This assumption does not allow for the fact that, if firms and workers do not have sufficient information to set real wages appropriately, consumers are also unlikely to have sufficient information about their long-run permanent income levels. Accordingly, a part of the expansion of output may be due to increased consumption in response to the rise in current income, with the result that investment expansion may be



only some fraction of $(K_U - 1)/Y_f$, making the GG flatter.[2]

The Figure above gives the effects of an increase in the quantity of or decline in the demand for liquidity. Not surprisingly, the level of employment increases and the real interest rate falls as equilibrium moves from point `a` to point `b`.

[2]This makes the result consistent with the presence of an underlying Keynesian multiplier effect.



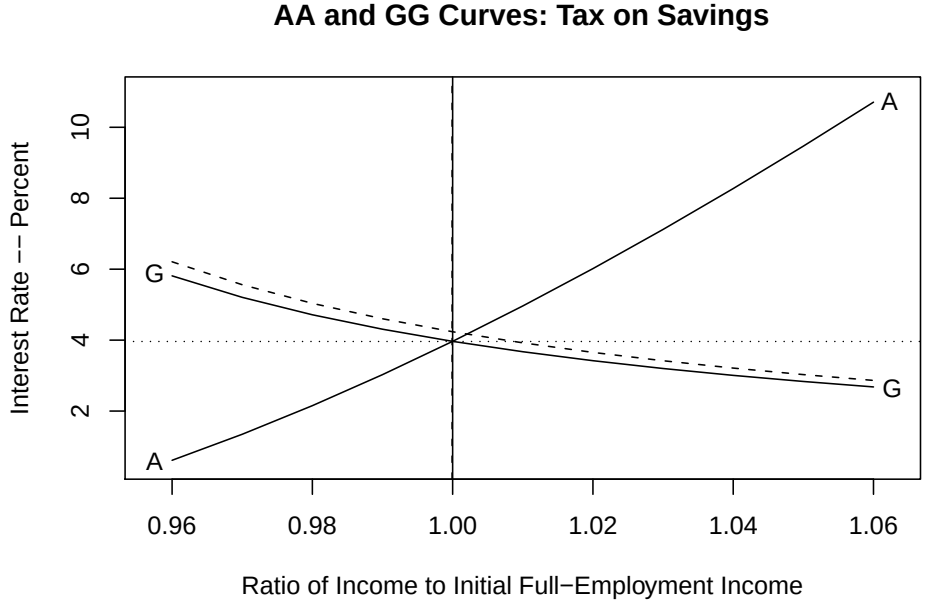
The Figure above gives the effects of an improvement in the allocation of resources that raises the full-employment levels of output and the real interest rate, although the increase in the latter is too small to see on the graph. The new full-employment level of income is given by the dashed vertical line and the new level of the GG curve is G′G′. The equilibrium level of output increases by less than the full-employment level in the short-run with equilibrium being achieved at point `b`. Substantial expansion of real liquidity as a result of either an expansion of the stock of money or a decline in the price level will be required to re-establish full-employment output at point `c`. Output increases by four percent of its initial level and the real interest rate increases by about 2 percent of its initial level. The level of investment as a fraction of the capital stock increases with the equilibrium growth rate, raising the adjustment costs of adding to the capital stock and thereby moderating the increase in the real interest rate. Although the magnitudes of the changes depend on the magnitudes of the parameters chosen, it turns out that the new full-employment interest rate will always be below the intersection of the original **AA** curve with the new vertical full-employment line. A simple representation of the demand for real liquidity function

$$L = L(r, Y)$$

can be differentiated totally, holding the stock of liquidity constant, to yield an expression which can be manipulated to represent the elasticity of the **AA** curve with respect to the horizontal axis as equal to minus the ratio of the income elasticity of demand for liquidity over the interest elasticity of demand for liquidity,

$$\frac{dr}{dy}\frac{r}{Y} = -\frac{\partial L}{\partial Y}\frac{Y}{L} \div \frac{\partial L}{\partial r}\frac{r}{L}.$$

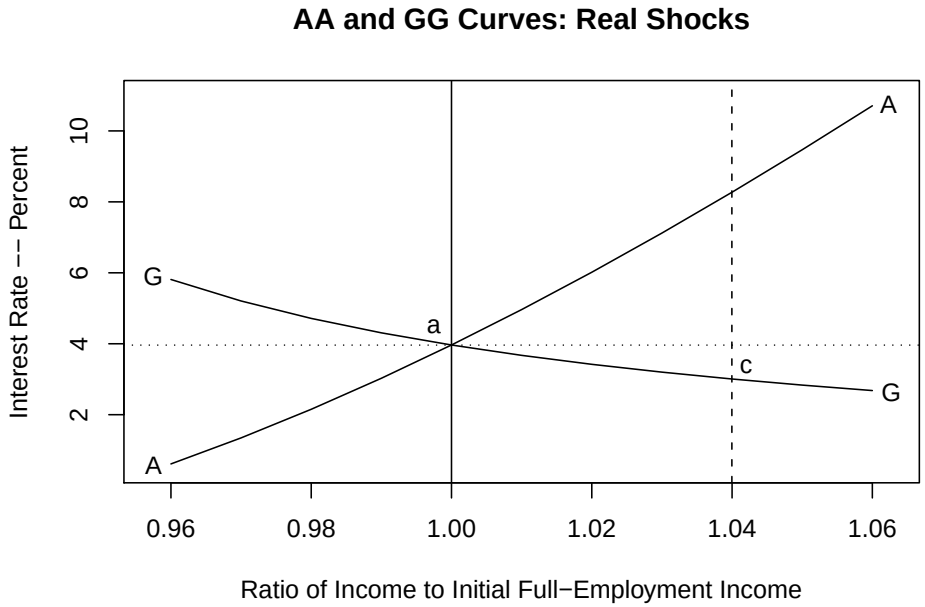
Available empirical evidence clearly indicates that the interest elasticity of demand for money tends to be well below unity in absolute value while the income elasticity of demand for money tends to be above unity.[3] Accordingly, the rise in r along **AA** will always be proportionally greater than the increase in income so that the curve will always cross the new vertical full-employment line above point c.



The Figure above gives the effect of an increase in the tax on future consumption. The real interest rate rises because future consumption and capital accumulation decline and adjustment costs are therefore lower. Also, the level of full-employment income also declines but, given the magnitudes of the parameters selected, the new value is 0.99992 times the original one and the change is therefore too small to show on the graph. **There is no doubt that output and employment will increase relative to their full-employment levels and a decline in real liquidity, either through monetary contraction or inflation, will be necessary to maintain full-employment equilibrium**.

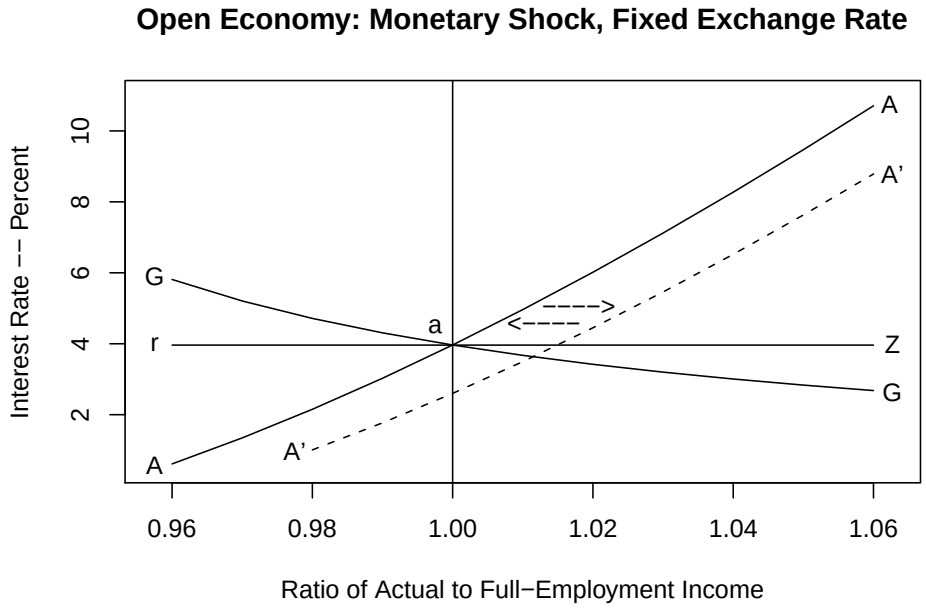
The less-than-full-employment analysis can be pursued further by raising the possibility that investors may form incorrect expectations about the future. An obvious situation would be that they expect the level of full-employment output to increase as indicated in the second Figure above on page 6 when it subsequently turns out that no such change occurs. The short-run equilibrium will be at point b and a decline in the stock of liquidity would be required to prevent that output expansion from occurring. As time passes, of course, output and real interest rates will return to their full-employment equilibrium levels at point a, and any policy-induced decline in liquidity will have to be reversed.

[3]See D.E.W. Laidler, *The Demand for Money*, 2nd. edition, Dun-Donnelly, New York, 1977.

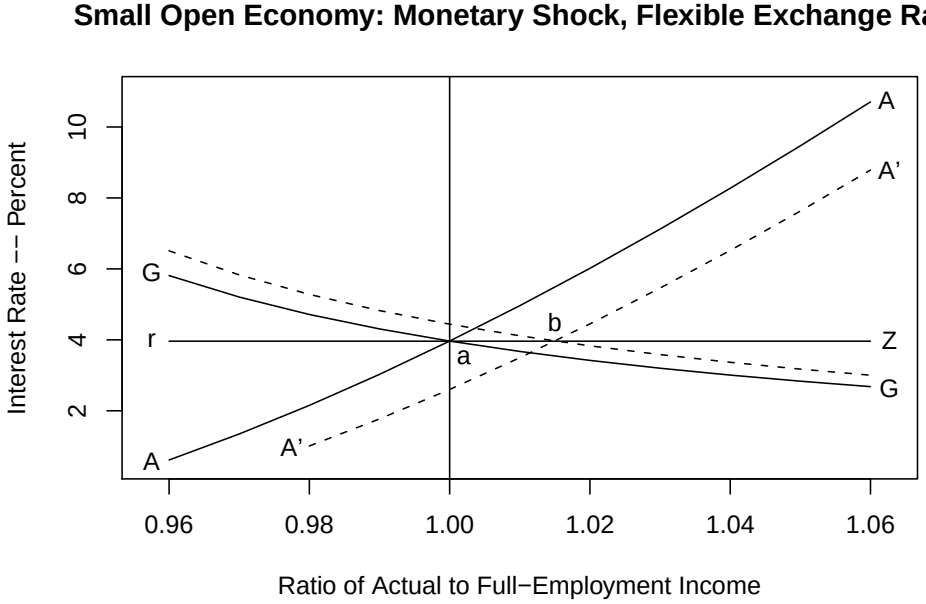


It is also reasonable to expect that random variations in $\hat{m}$ will occur, an obvious example being the effects of weather on agricultural output. Since the expected future levels of $\hat{m}$ will not change, there will be no shift of the GG curve in the Figure above—the shocks will simply involve shifts of the full-employment level of output to the right and left. It would appear from the figure above that actual output will not increase in response to a temporary four-percent expansion of full-employment output unless the authorities expand liquidity to shift the AA to the right to pass through point c. An increase in aggregate output will increase the demand for liquidity and unless this increase in demand is satisfied, the random expansion of agricultural output will be offset by declines in output and employment elsewhere in the economy. Or will it?

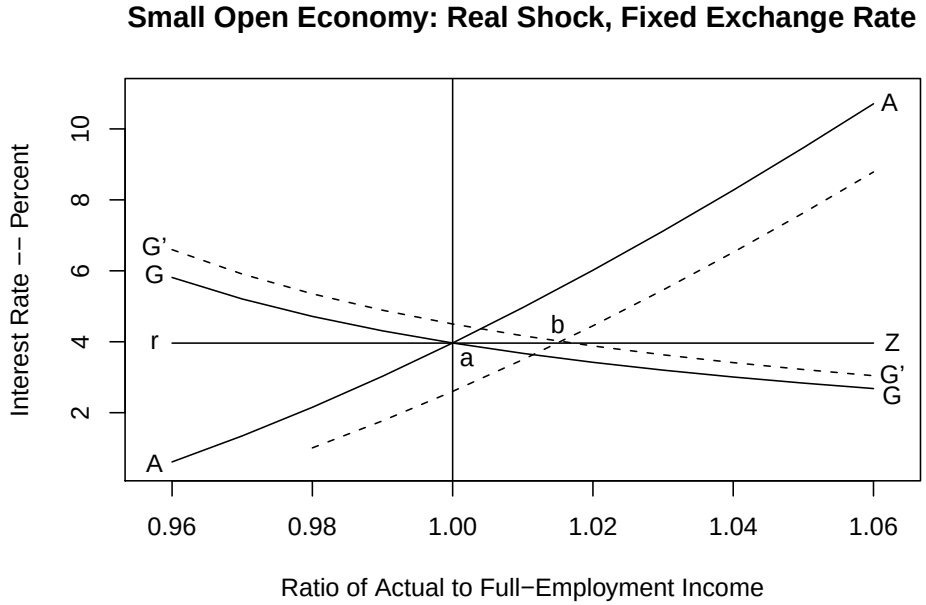
We have to keep in mind that in this situation adjustment to the positive full-employment output shock will take time. Downward pressure on employment in the rest of the economy will only arise when increases in the demand for liquidity resulting from the random expansion of output actually occur. If the randomly expanding sector, be it farm crop production or some other area of the economy, is a small fraction of aggregate output, the upward pressure on the demand for liquidity will not only be relatively small, but will occur only when the individuals involved get around to adjusting their liquidity holdings. And since such shocks occur randomly in both directions and in many areas of the economy, it would probably be an exceptional case for a temporary increase in aggregate full-employment output to persist for any period of time.



We now turn our attention to short-term shocks to output and employment in open economies that are too small to have noticeable effects on the rest-of-world aggregate. The Figure above focuses on monetary shocks in the case where the country maintains a fixed exchange rate with respect to the rest of the world. The world interest rate is indicated by the horizontal line **rZ**. An increase in the supply of or reduction in the demand for liquidity will shift the **AA** curve to the right. As domestic resident attempt to re-balance their portfolios to eliminate their excess liquidity holdings, they will buy assets abroad. To keep the exchange rate from depreciating, the domestic authorities must sell foreign exchange reserves in return for domestic currency, thereby reducing the stock of liquidity and shifting the **AA** curve back to the left to pass through point c. **Regardless of the size of the domestic economy in relation to the rest of the world, the authorities will be required to provide domestic residents with their desired stock of liquidity at the existing world interest rate in order to maintain the exchange rate parity. Monetary policy is ineffective under a fixed exchange rate—the domestic authorities cannot change the world interest rate by creating additional liquidity**.

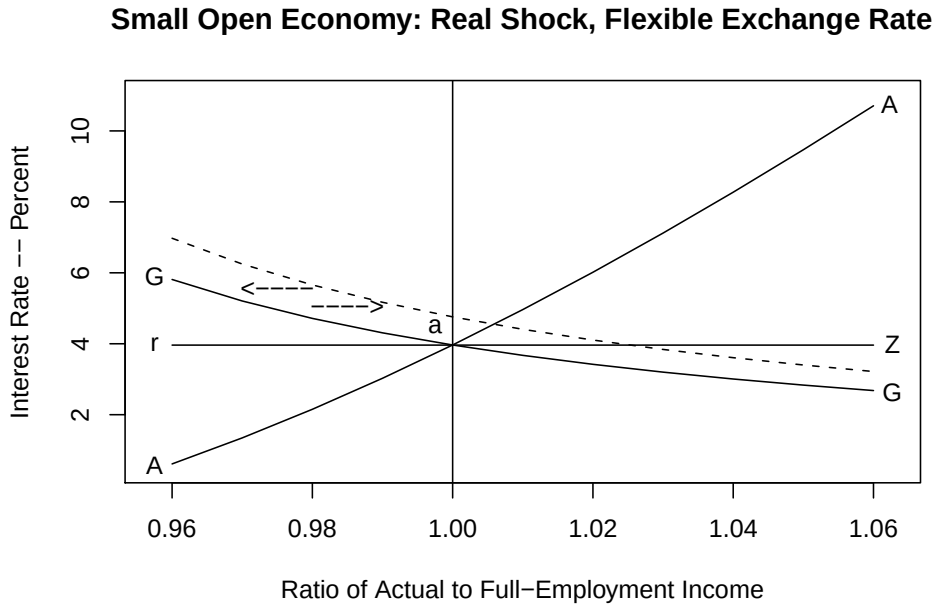


When they allow the domestic exchange rate to float, the authorities will be able to expand the stock of liquidity and shift AA to the right to $A'A'$ in the Figure above and increase output and employment. As domestic residents attempt to rebalance their portfolios by purchasing assets abroad the resulting excess supply of domestic currency on the foreign exchange market causes the domestic currency to devalue. The associated fall in the real exchange rate will shift the GG curve to the right until it passes through point `b` with output having risen sufficiently to induce the public to want to hold the new stock of liquidity.



Finally we turn to the effects of temporary real shocks. The Figure above shows the effect of a positive real shock on a small open economy under a fixed exchange rate—this could take the form of an appropriate temporary expansion of government production of some good, a temporary cut in taxes to lower income credit-constrained members of the community financed by floating government debt,[4] or an increase in market expectations as to the future return to capital in the domestic economy unmatched by an actual increase in full-employment output. In all cases the GG curve shifts to the right to G′G′. As output increases so does the domestic demand for liquidity and the authorities, to keep the domestic currency from appreciating, are forced to accumulate foreign exchange reserves in return for domestic currency, thereby shifting AA to the right to A′A′ to produce a new equilibrium at point b. An appropriate fiscal policy will thus have the desired effect in a small open economy under fixed exchange rates. Though monetary policy will not work under a fixed exchange rate, domestic output can be expanded by appropriate expansionary fiscal policy.

[4]Those who are credit constrained are likely to use transitory increases in income to replace stocks of consumer durables that they let decline during previous periods of transitory income decline. A tax cut, financed by issuing government debt is equivalent to a loan from those who individuals who purchase the bonds to those individuals who do not. The interest rate on these loans is much lower than that in private loan markets because the government guarantees repayment through future taxes.



The effects of equivalent temporary real shocks in a small open economy under a flexible exchange rate are analyzed in the Figure above under the assumption that domestic authorities do nothing to change the level of liquidity in the domestic economy. An increase in real output leads to an increase in domestic residents' demand for liquidity, inducing them to sell assets abroad to maintain portfolio equilibrium. This creates an excess demand for the domestic currency in the international market resulting in an appreciation of the domestic real and nominal exchange rates which shifts world demand off domestic goods, returning the GG curve to its original level and wiping out any increase in domestic income and employment. Fiscal policy cannot work in a small open economy with flexible exchange rates.

Finally, we must address the situation of a country that is big enough for changes in its output, employment, and demand for capital to have significant world-wide effects. Attempts to develop an appropriate analytical program in R must be ruled out on grounds of enormous complexity. Nevertheless, sensible analysis is still possible. **The trick is to recognize that the world GG and AA curves can be viewed as the horizontal sums of the corresponding curves of the individual countries composing it, and that the world real interest rate will be determined by the intersection of those two curves. Monetary and real shocks to a domestic economy of interest will lead to shifts in the world GG and AA curves and corresponding changes in the world interest rate, so analysis of the situation in an individual country must take into account these world interest rate changes. Since such efforts will necessarily be purely graphical extensions of the the analysis above, they can be postponed to later Advanced Topics dealing with applied rather than purely theoretical matters**.