5. a. Call the aggressive stock A and the defensive stock D. Beta is the sensitivity of the stock’s return to the market return, i.e., the change in the stock return per unit change in the market return. Therefore, we compute each stock’s beta by calculating the difference in its return across the two scenarios divided by the difference in the market return:

$$\beta_A = \frac{-2 - 38}{5 - 25} = 2.00$$
$$\beta_D = \frac{6 - 12}{5 - 25} = 0.30$$

b. With the two scenarios equally likely, the expected return is an average of the two possible outcomes:

$$E(r_A) = 0.5 \times (-2 + 38) = 18\%$$
$$E(r_D) = 0.5 \times (6 + 12) = 9\%$$

c. The SML is determined by the market expected return of $0.5(25 + 5)$ = 15%, with a beta of 1, and the T-bill return of 6% with a beta of zero. See the following graph.
The equation for the security market line is:

\[ E(r) = 6 + \beta (15 - 6) \]

d. Based on its risk, the aggressive stock has a required expected return of:
\[ E(r_A) = 6 + 2.0 (15 - 6) = 24\% \]
The analyst’s forecast of expected return is only 18%. Thus the stock’s alpha is:

\[ \alpha_A = \text{actually expected return} - \text{required return (given risk)} \]
\[ = 18\% - 24\% = -6\% \]

Similarly, the required return for the defensive stock is:
\[ E(r_D) = 6 + 0.3 (15 - 6) = 8.7\% \]
The analyst’s forecast of expected return for D is 9%, and hence, the stock has a positive alpha:

\[ \alpha_D = \text{actually expected return} - \text{required return (given risk)} \]
\[ = 9 - 8.7 = +0.3\% \]

The points for each stock plot on the graph as indicated above.

e. The hurdle rate is determined by the project beta (0.3), not the firm’s beta. The correct discount rate is 8.7%, the fair rate of return for stock D.

6. Not possible. Portfolio A has a higher beta than B, but its expected return is lower. Thus, these two portfolios cannot exist in equilibrium.

7. Possible. If the CAPM is valid, the expected rate of return compensates only for systematic (market) risk represented by beta rather than for the standard deviation which includes nonsystematic risk. Thus, A’s lower rate of return can be paired with a higher standard deviation, as long as A’s beta is lower than B’s.

8. Not possible. The reward-to-variability ratio for portfolio A is better than that of the market, which is impossible according to the CAPM, since the CAPM predicts that the market is the most efficient portfolio. Using the numbers supplied,

\[ S_A = \frac{16 - 10}{12} = .5 \]
\[ S_M = \frac{18 - 10}{24} = .33 \]

The numbers would imply that portfolio A provides a better risk-reward tradeoff than the market portfolio.

9. Not possible. Portfolio A clearly dominates the market portfolio. It has a lower standard
deviation with a higher expected return.

10. Not possible. The SML for this situation is: \( E(r) = 10 + \beta (18 - 10) \)
    Portfolios with beta of 1.5 have an expected return of \( E(r) = 10 + 1.5 \times (18 - 10) = 22\% \).
    A's expected return is 16\%; that is, A plots below the SML (\( \alpha_A = -6\% \)), and hence, is an
    overpriced portfolio. This is inconsistent with the CAPM.

11. Not possible. The SML is the same as in problem 10. Here, portfolio A's required return
    is: \( 10 + .9 \times 8 = 17.2\% \), which is still higher than 16\%. A is overpriced with a negative
    alpha: \( \alpha_A = -1.2\% \).

12. Possible. The CML is the same as in problem 8. Portfolio A plots below the CML, as
    any asset is expected to. This situation is not inconsistent with the CAPM.

13. Since the stock's beta is equal to 1.2, its expected rate of return is \( 6 + 1.2(16 - 6) = 18\% \)
    \[
    E(r) = \frac{D_1 + P_1 - P_0}{P_0}
    \]
    \[
    .18 = \frac{6 + P_1 - 50}{50}
    \]
    \[
    P_1 = $53
    \]

14. Assume that the $1,000 is a perpetuity. If beta is .5, the cash flow should be discounted
    at the rate
    \[
    6 + .5 \times (16 - 6) = 11\%
    \]
    \[
    PV = 1000/.11 = $9,090.91
    \]
    If, however, beta is equal to 1, the investment should yield 16\%, and the price paid for
    the firm should be:
    \[
    PV = 1000/.16 = $6,250
    \]
    The difference, $2,840.91, is the amount you will overpay if you erroneously assumed
    that beta is .5 rather than 1.
    If the cash flow lasts only one year:
    \[
    PV(beta=0) = 1000/1.11 = 900.90
    \]
PV\( (\beta=1) \) = \frac{1000}{1.16} = 862.07

with a difference of $38.83.

For n-year cash flow the difference is \( 1000PA(11\%,n)-1000PA(16\%,n) \).

15. Using the SML: \( 4 = 6 + \beta(16 - 6) \)

\[ \beta = -\frac{2}{10} = -0.2 \]

16. \( r_1 = 19\%; \quad r_2 = 16\%; \quad \beta_1 = 1.5; \quad \beta_2 = 1 \)

a. To tell which investor was a better selector of individual stocks we look at their abnormal return, which is the ex-post alpha, that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (risk-free rate and market rate of return) we cannot tell which investor was more accurate.

b. If \( r_f = 6\% \) and \( r_M = 14\% \), then (using the notation of alpha for the abnormal return)

\[ \alpha_1 = 19 - [6 + 1.5(14 - 6)] = 19 - 18 = 1\% \]

\[ \alpha_2 = 16 - [6 + 1(14 - 6)] = 16 - 14 = 2\% \]

Here, the second investor has the larger abnormal return and thus he appears to be the superior stock selector. By making better predictions the second investor appears to have tilted his portfolio toward underpriced stocks.

c. If \( r_f = 3\% \) and \( r_M = 15\% \), then

\[ \alpha_1 = 19 - [3 + 1.5(15 - 3)] = 19 - 21 = -2\% \]

\[ \alpha_2 = 16 - [3 + 1(15 - 3)] = 16 - 15 = 1\% \]

Here, not only does the second investor appear to be the superior stock selector, but the first investor's predictions appear valueless (or worse).

30. a. \( E(r) = \alpha + 5 + .8(15 - 5) = 14 \), from which we find \( \alpha = 1 \) percent. Therefore, we should invest in the fund.

b. Since the market index portfolio has a beta of 1 and the money market fund a beta of 0, we should invest 0.8 in the market index portfolio and 0.2 in the money market fund. The
expected rate of return on this portfolio is \(0.2 \times 0.5 + 0.8 \times 0.15 = 13\) percent, less than the 14 percent return of the fund by an amount equal to the alpha of the fund.

31. a. 

<table>
<thead>
<tr>
<th></th>
<th>Expected Return</th>
<th>Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>(5% + 0.8(14% - 5%) = 12.2%)</td>
<td>(-12.2% = 1.8%)</td>
</tr>
<tr>
<td>Y</td>
<td>(5% + 1.5(14% - 5%) = 18.5%)</td>
<td>(-18.5% = -1.5%)</td>
</tr>
</tbody>
</table>

b. 
i. Kay should recommend Stock X because of its positive alpha, compared to Stock Y, which has a negative alpha. In graphical terms, the expected return/risk profile for Stock X plots above the security market line (SML), while the profile for Stock Y plots below the SML. Also, depending on the individual risk preferences of Kay’s clients, the lower beta for Stock X may have a beneficial effect on overall portfolio risk.

ii. Kay should recommend Stock Y because it has higher forecasted return and lower standard deviation than Stock X. The respective Sharpe ratios for Stocks X and Y and the market index are:

Stock X: \((17\% - 5\%)/25\% = 0.48\)

Stock Y: \((14\% - 5\%)/36\% = 0.25\)

Market index: \((14\% - 5\%)/15\% = 0.60\)

The market index has an even more attractive Sharpe ratio than either of the individual stocks, but, given the choice between Stock X and Stock Y, Stock Y is the superior alternative.

When a stock is held as a single stock portfolio, standard deviation is the relevant risk measure. For such a portfolio, beta as a risk measure is irrelevant. Although holding a single asset is not a typically recommended investment strategy, some investors may hold what is essentially a single-asset portfolio when they hold the stock of their employer company. For such investors, the relevance of standard deviation versus beta is an important issue.