Problem Set

Solutions to the problems appear at the end of this document.

Unless otherwise stated, any coupon payments, cash dividends, or other cash payouts delivered by a security in the following problems should be assume to be distributed at the end of the year.

1. This question applies to a world of perfect certainty.

(a) A zero coupon bond with a face value of $10,000 and 4 years to maturity is currently selling for $8,041.63. What is the yield to maturity of this bond? (Your answer here will be used in following Parts (b) – (g)).

(b) A coupon bond with a face value of $1,000 and an 8% annual coupon will reach maturity in 4 years. The yield to maturity of this bond is 5.2%. What is its current selling price? (Your answer here will be used in following Parts (b) – (g)).

(c) An investment dealer currently owns the coupon bond described in Part (b). She plans on "stripping" the coupons, which means she will sell the right to receive the right to receive all future coupon payments to some investor for price $A_0$ and she will sell the right to receive the payment of the face value to some other investor for price $B_0$. Determine the values of $A_0$ and $B_0$.

(d) If the current 1-year interest rate is $r_{10} = 0.048$, what will be the price of the zero coupon bond of Part (a) one year from now (at date 1)? What will be the price of the coupon bond of Part (b) one year from now?

(e) If the current 3-year interest rate is $r_{30} = 0.06$, what will be the value of the 1-year interest rate $r_{13}$ that will prevail 3 years form now (at date 3)? What will be the prices of the zero coupon bond of Part (a) and the coupon bond of Part (b) at that time?

(f) The current 1-year rate of interest is $r_{10} = 0.048$, the 1-year interest rate that will prevail one year from now (at date 1) is $r_{11} = 0.062$. What is the 2-year holding period return on the coupon bond of Part (a) between dates 0 and 2?

(g) You are offered the opportunity to buy a 2-year T-Bill with a face value of $100 for current price $90.36. Given your answer to Part (f), is there a profitable arbitrage opportunity here? Explain.

2. The question applies to a world of perfect certainty.
(a) Corporation XYZ pays all of its net earnings out as cash dividends. It will pay $0.75 per share this year and the same amount in every year over the indefinite future. The 1-year rate of interest is currently 5% and will remain constant at that value indefinitely. What is the current price of a share of equity in Corporation XYZ?

(b) The Sullivan Corporation pays all of its net earnings out as cash dividends. It will pay a dividend of $0.60 per share this year, next year, and the year following next year. After that the dividend will grow indefinitely at an annual rate of 2%. The 1-year rate of interest is currently 4%. It will increase to 5% next year and remain constant at that value over the indefinite future. What is the current price of a share of equity in the Sullivan Corporation? What will be the share price 1 year from now? Two years from now? Three years from now?

(c) The assumption that corporations pay all earnings out as cash dividends is made only to simplify the determinants of share prices. It would make no difference to shareholders whether net earnings distributed as cash dividends or are, instead, re-invested by the corporation in other assets. To demonstrate this, consider an infinitely-lived corporation that has only 1 share of equity outstanding and no debt. In that case the share price $e_0$ at represents the entire market value of the firm at date 0. The market value of any corporation must always equal the market value of the firm's asset holdings (plant, equipment, any financial assets it holds, etc.). Let $A_0$ denote the market value of the firm's asset holdings at date 0. We have just stated that $e_0$ must equal $A_0$. The firm's net earnings during the first time period represent the market return on the firm's asset holdings. Under perfect certainty, that return must be the same as the market rate of interest; therefore net earnings of the corporation during the first time period are $NE_0 = r A_0$. (We will assume that the 1-year rate of interest is constant over time at the value $r$).

If this corporation always pays all of its net earnings out as cash dividends, the values of $A$, $NE$, and $e$ will all be constant, and we can readily deduce that $e = \frac{\text{Dividend}}{r}$.

(i) But now suppose that, instead of paying cash dividends, this corporation always re-invests 100% of its net earnings in additional assets that earn the market rate of return. Show that in this case the values of $A$, $NE$, and $e$ will all grow over time at the rate $r$. Determine the relationship that exists between $e_t$ and $NE_t$ at any date $t \geq 0$.

(ii) Finally, suppose that the corporation uses part of its net earnings for re-investment in new assets and pays the remainder out as cash dividends. Let the re-investment at any date $t$ be equal to $gA_t$, where $g$, where $g$ is constant over time and $g < r$. Then the dividend payout for the time period will be $D_t = NE_t - gA_t$. Show that in this case the values of $A$, $NE$, $e$, and $D$ will all grow over time at the rate $g$. Determine the relationship that exists between $e_t$ and $D_t$ at any date $t \geq 0$.

Explain why the shareholder will be indifferent among situations (i), (ii), and (iii).

3. The question applies to a world of perfect certainty.
Star Limousines Inc. is a small corporation specializing in providing limousine services for weddings and other special events. Business has been good and management is considering adding 10 new limousines to its fleet at a total purchase cost of $1 million. If the limousines are purchased, net revenues will increase by $200,000 for each of the next 5 years, starting with the current year. At the end of the 5th year (date 5) the used limousines will be sold for a total amount $225,000. The current 1-year rate of interest is 4 % and will remain constant at this value over the indefinite future..

(a) Should Star Limousines purchase these new vehicles? Why or why not?

(b) There are currently 10,000 shares of common stock outstanding in the corporation. If does not purchase the new vehicles, the current dividend will be $D_0 = $4 per share and the share price will be $e_0 = $100. Both the annual dividend and the price will remain constant through the indefinite future. If management does purchase the new vehicles and pays for the purchase by borrowing $1 million now and repaying the principal plus all accrued interest at the end of the fifth year, what will be the values of the share price today, 1 year from now, and 2 years from now, i.e., at dates 0, 1, and 2?

(c) Assuming management does purchase the new vehicles and using your solutions to Part (b), compute the 1-year holding period yield from buying 1 share of equity in Star Limousines Inc. at date 1 and re-selling this share at date 2. Determine not only the value of the total holding period yield but also the values of the dividend yield and the proportionate capital gain/loss.

4. The question applies to the real world.

Here is a problem that will require some serious thought and will also demonstrate that a forward rate of interest is not just an academic curiosity.

It is currently Oct. 01, 2006 and Martha has won a lottery that will pay her a prize of $1,000 on Oct. 01, 2007. To celebrate, Martha goes to Leon's (appliance and furniture store) and buys a new refrigerator which she does not have to pay for until 2 years from now. On Oct. 01, 2008 Martha must deliver $1,050 to Leon's. It is Martha's intention to invest the $1,000 she will receive in 2007 for one year and earn enough interest to cover the $1,050 she will have to deliver in 2008, but she has no way of knowing whether the 1-year interest rate that will prevail on Oct. 01, 2007 will be high enough (5 % or more) to generate $50 in interest over the following year. Fortunately, Martha took ECO358H. She observes that the current 1-year interest on T-Bills is 4 % and the current 2-year interest rate on T-bills is 4.5%. From these values Martha computes that the 1-year forward rate of interest is $f_1 = 5 \%$. She decides to take actions that will guarantee that she will be able to earn exactly this rate of interest over the 1-year holding period from Oct. 01, 2007 to Oct. 01, 2008. What actions does she undertake?
5. Pete has a logarithmic utility function \( U(W) = \ln(W) \), and the current level of his (certain) wealth is $5,000.

(a) Suppose Pete is exposed to a situation that results in a 50/50 chance of either winning or losing $1,000. If he can buy insurance that completely removes the risk for a price of $125, should Pete buy the insurance or take the gamble?

(b) Suppose Pete did take the gamble of Part (a) and lost, so that his current wealth is reduced to $4,000. If Pete has an option to repeat the gamble or buy insurance for $125, what will he do this time?

The following information applies to both Questions 6. and 7.

Consider a 2-period portfolio choice problem. In the first time period an individual has saving of amount \( S_0 \). This is to be invested in a portfolio consisting of a risky asset and a risk-free asset. The risk-free asset has a certain rate of return \( r_f = 0.04 \). The risky asset will deliver the random return \( r_i \), which will either be +0.24 or -0.12, each with probability \( \frac{1}{2} \). Let \( w_t \) denote the proportion of \( S_0 \) that is invested in the risky asset. (No restrictions are to be placed on the value of \( w_t \); it can be positive, negative or zero). The portfolio will deliver the payout \( S_0(1 + r_f + w_t(r_i - r_f)) \) in the second time period and all of this will be consumed during that period (after which the world ends). Find the value for \( w_t \) that will maximize expected utility under the following conditions.

6. The individual has a logarithmic utility function with expected utility given by

\[
E \left[ \ln(S_0(1 + r_f + w_t(r_i - r_f))) \right].
\]

7. The individual has an expected utility function of the sort introduced in Ch. 5 of Bodie, where expected utility is defined over the mean \( \bar{r}_p \) and variance \( \sigma_p^2 \) of the rate of return on the portfolio.

\[
E[\text{utility}] = \bar{r}_p - \frac{1}{2} \sigma_p^2.
\]

8. A risky security that is purchased at date 0 can be liquidated at date 1 for an amount with an expected value equal to \( E[X] \). A risk neutral individual would be indifferent between having this risky security and receiving the amount \( E[X] \) with certainty. [Why? Because a risk neutral individual has a utility function of the form \( U(W) = aW \), where \( a \) is a positive constant. So \( E[U(W)] = aE[W] \), which implies that the certainty equivalent of risky \( W \) is always equal to \( E[W] \). And any individual is indifferent between having risky \( W \) or receiving its certainty equivalent with certainty.] Show that this implies that if all
agents are risk neutral, the date 0 price of the risky security must be \( \frac{E[X]}{1+r_f} \), where \( r_f \) is the one-period risk-free rate of interest prevailing at date 0.

9. The following describes a 2 State world in which there are two traded securities – an equity (Security A) and a risk-free, discount bond.

<table>
<thead>
<tr>
<th></th>
<th>Date 0</th>
<th>Date 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>State 2</td>
<td></td>
</tr>
<tr>
<td>(Prob. = ( \pi_1 ))</td>
<td>(Prob. = 1-( \pi_1 ))</td>
<td></td>
</tr>
<tr>
<td>Security A</td>
<td>Price, ( P_A = $3.50 )</td>
<td>Payoff = $5.00</td>
</tr>
<tr>
<td>Risk-free Bond</td>
<td>Price, ( P_B = $0.92 )</td>
<td>Payoff = $1.00</td>
</tr>
</tbody>
</table>

It is clear that the security market is complete here because it is impossible to multiply the payoffs of Security A by a constant and exactly replicate the payoffs of the risk-free bond.

(a) Determine the time period 0 prices of the (A-D) pure securities associated with each of the two states. (Recall that a pure security pays $1 if a particular state occurs and 0 otherwise).

(b) If economic agents in this world had preferences that were risk neutral, what would be the value of \( \pi_1^{RN} \) (the risk neutral probability that State 1 will occur)?

(c) Consider a non-traded Security \( X \), which will pay $1.50 if State 1 occurs and $2.00 if State 2 occurs. Find the hedge portfolio that consists of Security A and the risk-free bond that replicates the payoffs of Security \( X \); then use this portfolio to determine the time period 0 price, \( P_X \), for this security.

The following information is used in Questions 10. – 13.
Consider the following 3-state world in which there exist 3 traded securities with prices and payoffs described below.
10. It is not clear whether the security market is complete here. If it is complete, there does not exist a unique portfolio consisting of 2 of the securities that can replicate the payoffs of the third security (meaning that the 3 sets of payoffs are linearly independent). To show that this security market is indeed complete, show that there exists no unique combination of Security $A$ and T-Bills that will replicate the payoffs of Security $B$.

11. Determine the risk neutral probabilities associated with each of the 3 states, then use these to determine the prices of the A-D (pure) securities associated with each of the states.

12. Use your answers to Q. 11. to determine the date 0 price of a security that will pay $10 if State 1 occurs and $6 if either States 2 or 3 occurs. Explain why the price of this security does not depend upon the true probabilities of the three states.

13. Find a hedge portfolio consisting of Securities $A$, $B$, and T-Bills that will replicate the payoffs of the security described in Q. 12.

14. (This is a CCAPM problem from the December 2005 Final Examination)

In a 2-period world in which all agents have the same preferences and endowments a representative agent has expected lifetime utility given by the following:
\[
E[U] = -\frac{1}{c_0} - 0.95 E\left(\frac{1}{c_1}\right),
\]
where $c_0$ and $c_1$ denote the agent's consumption in the current and future time periods, respectively.

(a) Suppose the agent has endowments of $Y_0 = 8$ units of consumption in time period 0 and $Y_1 = 10$ units of consumption with perfect certainty in time period 1. Determine the value for the interest rate in time period 0 at which the agent would want to neither borrow nor lend \textit{i.e.} find the interest rate at which optimal $c_0 = Y_0$. 

| Date 0 | \multicolumn{3}{c|}{State 1} | \multicolumn{3}{c|}{State 2} | \multicolumn{3}{c|}{State 3} |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|        | (Prob = 0.6)  | (Prob. = 0.1) | (Prob = 0.3)  |                |                |                |                |                |
| Security A | $P_A = $2.50 | Payoff = $5.00 | Payoff = $3.00 | Payoff = $1.00 |                |                |                |                |
| Security B | $P_B = $4.00 | Payoff = $6.00 | Payoff = $4.00 | Payoff = $3.00 |                |                |                |                |
| Risk-free T-Bill | $P_T = $0.98 | Payoff = $1.00 | Payoff = $1.00 | Payoff = $1.00 |                |                |                |                |
(b) Now suppose that the time period 0 endowment remains at \( Y_0 = 8 \) but there is some uncertainty about the value of the endowment in time period 1 as described below.

<table>
<thead>
<tr>
<th>Possible value for ( Y_1 )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>10</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>12</td>
<td>( \frac{1}{3} )</td>
</tr>
</tbody>
</table>

Determine the value for the (risk-free) interest rate in time period 0 at which the agent would want to neither borrow nor lend with these endowments.

The Following Information is used in Questions 15. & 16.

Consider a CCAPM world where the representative household is infinitely lived and has a lifetime expected utility function given by

\[
E_0[U_0] = u(c_0) + E_0 \sum_{t=1}^{\infty} \beta^t u(c_t),
\]

where the time discount factor is \( \beta = 0.90 \) and the utility function is \( u(c) = \ln(c) \).

15. In each time period \( t \) the representative household has a consumption endowment that is either "high" with \( c_t = 16 \) or "low" with \( c_t = 10 \). Each possibility occurs with probability = \( \frac{1}{2} \), and the outcomes are independent from time period to time period. For the current time period the endowment is \( c_0 = 10 \) ("low").

(a) Determine the value of the current 1-period, risk-free rate of interest.

(b) Determine the value of the current 2-period, risk-free rate of interest.

(c) Determine the current price of security \( A \) that will pay 1 unit of consumption at date 1 if \( c_1 = 10 \) and will pay 2 units of consumption at date 1 if \( c_1 = 16 \).

(d) Determine the current price of security \( B \) that will pay either 1 unit of consumption or 2 units of consumption at date 1, irrespective of the value of \( c_1 \).

(e) Use your solutions to Parts (c) and (d) to determine the expected rates of return (between dates 0 and 1) on securities \( A \) and \( B \). The use your solution from Part (a) to compute the risk premium associated with each of securities \( A \) and \( B \). Can you explain why the two risk premiums differ?
16. Determine the current price of a share of stock that will pay a dividend in each future time period of 2 if the period's consumption is "low" or 3 if the period's consumption is "high". What is the expected value of the price of this stock at date 1?

**SOLUTIONS**

1. (a) 5.6%  (b) $1,098.83  (c) \( A_0 = $282.37, B_0 = $816.46 \)  (d) $8,427.63  

(e) The 1-year interest rate that will prevail at date 3 is \( r_{13} = 0.0441 \) (or 4.41%). The price of the bond form Part (a) will be $9,577.63. The price of the bond from Part (b) will be $1,034.38.

(f) The 2-year holding period return must equal the current 2-year rate of interest which is \( r_{20} = 0.055 \) (or 5.5%).

(g) The 2-year rate of interest is 5.5% from Part (f); so the 2-year TBill shold be selling for a price of $89.85. That it is actually selling for a higher price means that there is a profitable arbitrage opportunity here. Sell short the 2-year TBill and invest the proceeds in either of the bonds from Parts (a) and (b) and at date 2 you will collect a profit even though you made 0 net investment.

2. (a) $15  (b) \( e_0 = $19.44, e_1 = $19.62, e_2 = $20.00, e_3 = $20.40 \) (Note: For Part (b) you must first determine \( e_2 \) using the "formula" \( e_2 = \frac{D_2}{r - g} = \frac{0.60}{0.05 - 0.02} \). Then, work backwards to determine \( e_1 \) and \( e_0 \) in that order. \( e_3 \) just = \((1+g)\) \( e_2 \).

(c) For (i) \( e_t = \frac{NE_t}{r} \), for all dates \( t \geq 0 \). \( A, NE, \) and \( e \) will each grow over time at the rate \( r \) per year.

For (ii) it is up to the student to work out what is to be demonstrated. The relationship between dividends and share prices is \( e_t = \frac{D_t}{r - g} \), for all dates \( t \geq 0 \).

3. (a) \( NPV = +$75,299 \) so the firm should undertake the investment because it will increase the current share price.

(b) New values for share prices when the investment is undertaken are \( e_0 = $107.53 \) \( e_1 = $87.83 \) \( e_2 = $67.34 \)

(c) Buy 1 share at date 1 for $87.83 and receive a cash dividend of $24 at the end of the year, the re-sell the share for $67.34 at date 2. The 1-year holding period rate of return is 4% (equal to the rate of interest). The dividend yield is +27.33% and the proportionate capital gain/loss is -23.33%.
4. Martha should today sell short a 1-year TBill with a face value of $1,000 and invest the proceeds in 2-year TBills (which we will assume also have face value of $1,000). In doing this, Martha will receive exactly enough from the short sale to purchase 1.05 of the 2-year TBills. Note that Martha has made 0 net investment today. One year from now (Oct. 01, 2007), Martha will receive the $1,000 in lottery winnings, which she should use to cover her short sale of a year earlier. (The T-Bill she shorted will reach maturity on Oct. 01, 2007 and Martha will be obligated to deliver $1,000 to whoever bought it). Thus, 1 year from now Martha will have invested $1,000 of her own wealth and will still have 1.05 of the 2-year TBills, which have now become 1-year Tbills. On Oct. 01, 2008, when these TBills mature, Martha will receive $1,050 (=1.05 × $1,000), which she can use to pay off Leons. Note that Martha does not invest any of her own wealth until 1 year from now and we know with certainty that she will earn a 1-year holding period rate of return of 5% on that investment. In other words, her actions insure that she will earn $f_1$ between dates 1 and 2.

5. (a) Pete should refuse the insurance and take the gamble because with the gamble his expected utility is 8.4968, whereas with insurance his utility is only 8.4918.

5. (b) This time Pete should buy the insurance and forego the gamble because with the gamble his expected utility is 8.2618, whereas his utility with insurance is greater at 8.2623.

6. One must find the value for $w_1$ that maximizes the expected utility function:

$$E[U] = \ln S_0 + \frac{1}{2} \ln(1.04 + w_1(0.20)) + \frac{1}{2} \ln(1.04 + w_1(-0.16)).$$

The solution is $w_1 = 0.65$. That is, the individual should invest 65% of wealth in the risky security and the remaining 35% in the risk-free security.

7. The risky security has a return with an expected value $\bar{r}_1 = 0.06$ and a variance $\sigma_1^2 = 0.0324$. Then a portfolio with a proportion $w_1$ invested in the risky security and a proportion $(1-w_1)$ invested in the risk-free security has an expected return

$$\bar{r}_p = r_f + w_1(\bar{r}_1 - r_f)$$

and a variance $w_1^2 \sigma_1^2$.

The optimal value for $w_1$ is the value that maximizes the expected utility function

$$E[U] = 0.04 + w_1(0.02) - \frac{1}{2} (w_1^2 (0.0324)).$$

The solution is $w_1 = 0.6173$. That is, the individual should invest 61.73% of wealth in the risky security and the remainder in the risk-free security.

8. One way of obtaining the amount $E[X]$ with certainty at date 1 is to invest the amount
The price of the risky asset at date 0 must also equal \( \frac{E[X]}{1 + r_f} \); otherwise there exists a profitable arbitrage opportunity. (If the price of the risky asset was > \( \frac{E[X]}{1 + r_f} \), risk neutral agents would sell infinite amounts of the risk-free asset and use the proceeds to acquire the risky asset. This action would force the two prices to equality.)

9. (a) Let \( q_1 \) denote the price of pure security 1 that pays $1 in State 1 and 0 in State 2, and let \( q_2 \) denote the price of pure security 2 that pays $1 in State 2 and 0 in State 1. Then Security A is equivalent to having 5 units of pure security 1 and 3 units of pure security 2. The risk-free bond is equivalent to having 1 unit of each of the pure securities. The "no arbitrage" condition requires that the following 2 equations hold:

\[
5q_1 + 3q_2 = P_A = $3.50 \\
q_1 + q_2 = P_B = $0.92
\]

Solving these equations simultaneously yields \( q_1 = $0.37, \ q_2 = $0.55 \).

(b) We know that if agents are risk neutral the price of Security A must equal the expected value of its payoffs at date 1, discounted by \((1 + r_f)\). Here the expected payoff using risk neutral probabilities is \([5.00\pi_1^{RN} + 3.00(1 - \pi_1^{RN})]\) and \((1 + r_f) = $1/$0.92\. These values imply, \( \pi_1^{RN} = 0.4022 \).
[Alternatively, we know that \( \pi_1^{RN} = q_1(1 + r_f) \), which yields the same value.]

(c) The details are left to the student but the hedge portfolio consists of +2.75 units of the risk-free bond and -0.25 units of Security A. The price of Security X is $1.655.

10. Consider a portfolio consisting of \( a \) units of Security A and \( t \) T-Bills. Can we find unique values of \( a \) and \( t \) that satisfy the following simultaneous equations?

\[
\begin{align*}
\text{State 1} & : 5a + t = 6 \\
\text{State 2} & : 3a + t = 4 \\
\text{State 3} & : a + t = 3
\end{align*}
\]

The equations imply \( 4a = 1 \) and \( 2a = 1 \), which is a contradiction, so there is no unique solution and the market is complete.

11. The risk neutral probabilities solve the following equations
For Security $A$ 
$\$2.50 = \frac{\$5\pi_1^{RN} + \$3\pi_2^{RN} + \$1(1 - \pi_1^{RN} - \pi_2^{RN})}{(1 + r_f)}$

For Security $B$ 
$\$4.00 = \frac{\$6\pi_1^{RN} + \$4\pi_2^{RN} + \$3(1 - \pi_1^{RN} - \pi_2^{RN})}{(1 + r_f)}$

Setting $(1+r_f) = \$1/\$0.98$ and solving yields \( \pi_1^{RN} = 0.3061, \pi_2^{RN} = 0.1633, \) and \( \pi_3^{RN} = 0.5306. \)

12. The price of the security is $7.08 (using the counterpart of the equations in the solution for Q. 11). The price does not depend on the actual state probabilities because they are already reflected in the prices of the traded securities, $A$, $B$ and T-bills. The non-traded security is a derivative of the traded securities and we are simply using prices and payoffs of the latter to determine the price of the former via "no arbitrage" arguments.


14. For CCAPM the risk-free interest rate is the value of $r_f$ that satisfies the following equation.

\[ 1 = \beta (1 + r_f) E_0[\frac{u'(c_f)}{u'(c_0)}]. \]

Here $\beta = 0.95$ and $u'(c) = \frac{1}{c^2}$.

(a) In this part $c_0 = 8$ and $c_1 = 10$ with certainty. The equation yields $r_f = 0.6447$ (64.47%).

(b) In this part $c_0 = 8$ and $E_0[u'(c_f)] = \frac{1}{3} \left( \frac{1}{64} + \frac{1}{100} + \frac{1}{144} \right) = 0.010856$. Then, $r_f = 0.515$.

15. (a) For CCAPM the risk-free interest rate is the value of $r_f$ that satisfies the following equation.

\[ 1 = \beta (1 + r_f) E_0[\frac{u'(c_f)}{u'(c_0)}]. \]

Here $1 = 0.90 (1 + r_f) \frac{1}{2}(1/10 + 1/16) \frac{1}{1/10}$, which implies $r_f = 0.3675$. 

11
(b) A risk-free discount bond that pays 1 unit of consumption at date 2 will sell for a price \( p_2 \) that satisfies

\[
 p_2 = \beta \ E_0[\frac{u'(c_{t+1})}{u'(c_0)}].
\]

Here \( p_2 = 0.9 \left( \frac{1/2(1/10+1/16)}{1/10} \right) \), which implies \( p_2 = 0.658125 \). But \( p_2 \) is also equal to \( \frac{1}{(1+r_{20})^2} \), where \( r_{20} \) is the risk-free, 2-period interest rate. Therefore, \( r_{20} = 0.2327 \).

(c) \( p_A = 0.9 \left( \frac{1/2(1/10+2/16)}{1/10} \right) = 1.0125 \).

(d) \( p_B = 0.9 \left( \frac{1/4(1/10+2/10+1/16+2/16)}{1/10} \right) = 1.096875 \).

(e) \( r_A = 0.4815 \quad r_B = 0.3675 \)

\( r_A - r_f = 0.114 \quad r_B - r_f = 0 \)

Security A is "risky" because its return has a positive correlation with consumption at date 1; whereas security B is not "risky" because its return is independent of (has 0 correlation with) consumption at date 1.

16. For CCAPM the price of the stock is the value of \( S_0 \) that solves the following:

\[
 S_0 = E_0 \sum_{t=1}^{\infty} (0.90)^t \, \frac{Div_t}{u'(c_t)}. \]

For all values of \( t \geq 1 \), the expectation \( E_0[Div_t/\frac{u'(c_t)}{u'(c_0)}] = \frac{1/2(2/10+3/16)}{1/10} = 1.9375 \).

Thus \( S_0 = 1.9375 \sum_{t=1}^{\infty} (0.90)^t = 1.9375 \left( \frac{0.9}{1-0.9} \right) = 17.4375 \).

With probability \( \frac{1}{2} \), \( c \) will again be equal to 10 at date 1, and \( S_1 \) will again equal 17.4375.

But with probability \( \frac{1}{2} \), \( c \) will equal 16 at date 1, in which case \( E_1[Div_t/\frac{u'(c_t)}{u'(c_0)}] \) will be equal to \( \frac{1/2(2/10+3/16)}{1/16} = 3.1 \) and \( S_1 \) will be equal to 27.9.

Therefore, \( \bar{S}_1 = 1/2(17.4375 + 27.9) = 22.66875 \).