A Dynamic Theory of Parliamentary Democracy[1]

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Abstract

This paper presents a dynamic model of election, government formation, and legislation in a parliamentary democracy with proportional representation in which the policy chosen in one period becomes the status quo for the next period. The electorate votes strategically by taking into account the likely governments that parties would form and the policies they would choose as a function of the status quo. The status quo thus affects both the election outcomes and the bargaining power of the parties during government formation. A formateur party thus has incentives to strategically position the current policy to gain an advantage in both the next election and the subsequent bargaining over government formation and policy choice. These incentives can give rise to centrifugal forces that result in policies that are outside the Pareto set of the parties.

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1 Introduction

How political institutions affect policy choices is of central importance in the field of political economy. Considerable progress has been made in developing comparative models of political institutions to predict the induced policy choices.[1] One persistent obstacle in this research program has been the difficulty of modeling multiparty parliamentary systems, the most prevalent political system in Europe and some parts of Asia. These systems typically arise out of the confluence of two constitutional features. The first is parliamentary governance; i.e., the executive is not directly elected by the popular vote but by the legislature to whom it remains accountable. Second, elections are usually held under proportional representation (PR); i.e., seats in the legislature are assigned to parties proportional to their vote shares.

Parliamentary systems are rich in complex strategic incentives for both voters and politicians. Austen-Smith and Banks (1988), Baron and Diermeier (2001), and Schofield and Sened (2006) study the incentives of strategic voting in single-period, full-equilibrium models that integrate government formation, legislation, and elections.[2] With proportional representation it is rare for one party to capture a majority of seats and parties must form coalitions to govern. Because governments and their policies are the consequence of multilateral bargaining, voters should base their vote not on a party's announced platform or policy preferences but on the policies expected to be chosen by the governing coalitions that may form once a new parliament has been elected. A moderate supporter of a conservative party, for example, may prefer a coalition government of the conservative party with a centrist party over a single-party conservative government. So, in cases in which the conservative party

[1] See Persson and Tabellini (2000; 2003) for extensive surveys of the literature.

[2] For empirical evidence on the use of sophisticated voting strategies in proportional representation systems see Cox (1997) and especially Bawn (1999).

is close to gaining an absolute majority of seats, the voter may prefer voting for the second preferred centrist party.

A parallel line of research has investigated how different aspects of parliamentary democracies provide incentives for policymakers and shape economic policy. Persson, Roland and Tabellini (2000) compare different political regimes and show that legislative cohesion, typical of parliamentary systems (Diermeier and Feddersen 1998), induces politicians to engage in more public goods provision, less pork barrel spending, and more corruption compared to separation of proposal power, typical of presidential systems. Persson and Tabellini (1999) and Lizzeri and Persico (2001) show how proportional representation elections induce political parties to commit to more public goods provision and less redistribution compared to single-member district systems. Milesi-Ferretti, Perotti, and Rostagno (2002) examine incentives for politicians to allocate government spending for local public goods and targeted transfers in parliamentary democracies with different electoral rules. Persson, Roland, and Tabellini (2007) focus on the accountability problem of proportional representation elections and extend their earlier analytical framework to address the incentives for politicians to form political parties and for the parties to form governing coalitions.

Most studies of parliamentary systems have not considered a potentially important incentive: Incumbent governments may strategically position the current government policy to influence the outcome of the next election and the subsequent government formation and policy choice. The absence of research is in part due to the difficulty in formulating tractable and general models of parliamentary institutions in a dynamic setting and is in marked contrast to the rich literature on policy dynamics in a two-party political system.[3]

This paper presents a dynamic theory of representation, government formation

[3]For surveys of the literature see Persson and Tabellini (2000) and Drazen (2000).

and policy choice in a parliamentary democracy with proportional representation, and investigates the implications for the efficiency of policy choices. The theory integrates the principal institutions of multiparty democracies into a full equilibrium, two-period model. Each period begins with a proportional representation election that determines representation and seat shares of the parties in parliament. After the election one party is selected as the "formateur," the party with the right to propose a coalition. Unless a party controls a majority of seats, the probability a party is selected as formateur is proportional to the party's seat share in parliament. The formateur then forms a governing coalition with majority support in parliament and bargains with the other coalition members over policy and the redistribution of office-holding benefits. The model thus integrates three major institutions of a multi-party parliamentary democracy, elections, government formation, and legislation.

We focus on a political system in which a policy is in effect until it is replaced by a new policy. In particular, any policy chosen by the governing coalition must defeat the policy chosen by the previous government, and it then becomes the status quo for the next period. We show that the equilibrium policy outcome in a period is completely determined by the status quo policy at the beginning of that period. This allows us to study policy dynamics in multiparty democracies with strategic voters and politicians. Since the current policy choice has consequences for the next period, the formateur chooses a policy in the current period that trades off current payoffs against potential advantages in the next election and government formation cycle. This paper thus extends the literature of dynamic legislative bargaining by incorporating elections into the model.[4]

[4] The literature on dynamic legislative bargaining with an endogenous status quo was initiated by Baron (1996). For recent development, for example, see Baron and Herron (2003), Battaglini and Coate (2007; 2008), Bowen and Zahran (2007), Fong (2006), Duggan and Kalandrakis (2007), and Kalandrakis (2004; 2007). These papers, however, do not model elections. An exception is Cho (2008) who studies a dynamic bargaining model with elections and a unidimensional policy space.

A key component of our theory is lack of commitment. We assume that both voters and political parties are farsighted and forward looking and cannot commit to future actions. In particular, political parties cannot commit to which government to form or which policy to choose, and voters cannot commit to a voting strategy. Our modeling strategy is in line with Austen-Smith and Banks (1988) and Baron and Diermeier (2001), and different from two other common approaches that have been taken to study proportional representation elections. The first approach assumes commitment by political parties. Persson and Tabellini (1999), Lizzeri and Persico (2001), and Pagano and Volpin (2006) take the view that elections aggregate preferences of voters and assume that office-motivated political parties announce and commit to their policy platforms before an election takes place. The second approach assumes commitment by voters. Persson, Roland and Tabellini (2000; 2007) take the view that elections discipline politicians who have conflicts of interests with voters and may waste public resources for their own benefit. In their models voters commit to their voting strategies before policy making takes place and vote retrospectively to punish or reward parties in the incumbent government. Our approach thus differs from most existing models of proportional representation elections and is appropriate given the subsequent post-election coalition bargaining.

The presence of farsighted politicians in a model with proportional representation elections and government formation creates a new source of inefficiency. This inefficiency arises from the governing parties' ability to strategically position the current status quo to improve their future bargaining strength and electoral prospects.[5] First,

[5] Strategic manipulation of policy and its welfare implication have been studied in a rich literature on the lack of commitment. For example, see Aghion and Bolton (1990), Persson and Svensson (1989), Tabellini and Alesina (1990) for an early development of models with two-party politics, Besley and Coate (1998) in the citizen-candidate framework, and also Fong (2006) and Battaglini and Coate (2007; 2008) for models of legislative bargaining. Our paper adds to this literature by exploring a new mechanism that results from the interaction of bargaining and elections.

parties in the governing coalition have incentives to position themselves favorably for the bargaining over policy and office-holding benefits in the next period. They can do so by disadvantaging the current out party in the bargaining so as to lower its reservation value. This creates a centrifugal incentive to choose an extreme policy, and that policy can be outside the single-period Pareto set of the parties' preferences. This bargaining effect is also studied by Fong (2006) in a model with fixed seat shares.

Second, parties have electoral incentives to obtain greater representation in parliament, since the likelihood of being selected as the formateur is weakly increasing in representation. The party that is most disadvantaged by the status quo would on average receive the least votes in the next election. This results because as a "cheap" coalition partner the current out party would always be included in the next government, and some of its natural constituents have an incentive to vote for another party to increase that party's probability of being selected as formateur and forming a government with the out party. Disadvantaging the out party in the next bargaining over government formation and legislation yields an electoral advantage for the incumbent parties, especially the formateur. With such an electoral advantage, the current formateur expects to head the next government with a greater probability. Hence, it has a stronger incentive to disadvantage the out party in future bargaining so that it could extract more office-holding benefits from the out party if selected as formateur in the next period. Conditional on the type of government formed, the electoral effect thus amplifies the centrifugal force due to the bargaining effect, and leads to more extreme and inefficient policy outcomes.

One may wonder whether voters could elect a government in the first period that would make more efficient policy choices for a given initial status quo. We show that such moderation cannot occur if the initial status quo is sufficiently close to the center of voter preferences. The centrifugal force arising from the period-two election cannot be counterbalanced by a period-one election when the status quo is centrally located.

Policy making in our theory is coalition efficient in the sense that the policy maximizes the aggregate multi-period utility of the governing coalition, but since the policy can be extreme, it can be strongly inefficient for voters. Moreover, social welfare measured as the average multi-period utility of voters decreases as the parties care more about their future utilities. While centrifugal forces arise from coalition bargaining and proportional representation elections, electoral incentives also limit the extent of those forces and of the resulting inefficiency, since a policy too extreme can lead to an incumbent formateur losing its electoral advantage.

The paper is organized as follows. Section 2 presents the model. Section 3 solves the model by backward induction, explains its key mechanisms, and decomposes the total effect of institutions on policy choice into a bargaining effect and an electoral effect. Section 4 discusses normative and positive implications of the model, and the last section concludes. All proofs are relegated to the appendix.

2 The Model

We consider a two-period spatial model of elections, government formation and policy choice.[6] The political system consists of a large finite number N of voters, and three political parties labeled a, b, and c. The political system selects a two-dimensional policy $x \in \Re^2$ in each of two periods, where a period corresponds to an interelection period.[7] The policy choice in a period is made by a government formed among those parties that have representation in parliament as determined by a proportional representation election. A government consists of a coalition of parties with a majority

[6] A two-period model is sufficient to identify the incentives for strategic manipulation of policy on the part of the political parties.

[7] Budge et al. (2001) provide evidence that political competition in post-war Europe is well represented by a two-dimensional policy space.

of seats in parliament.

A political party may be thought of as consisting of a leader supported by a group of party activists with similar preferences. In period t party $i \in \{a, b, c\}$ derives utility from both policy x_t and the redistribution of office-holding benefits, $y = \left(y_t^a, y_t^b, y_t^c\right) \in \Re^3$, where $y_t^a + y_t^b + y_t^c = 0$, and $y_t^i \in \Re$ denotes the net benefits transferred to i from the other parties.[8] Office-holding benefits correspond to things that matter to parties but not to voters. Such benefits include patronage positions, public financing of party activities, and perquisites of office that will be consumed by whichever parties form the government. Moreover, these benefits can be viewed as accruing to party activists who would vote for their party regardless of the distribution of benefits, so electoral outcomes are not affected by the distribution. In forming a government a party can use both policy concessions and the redistribution of office-holding benefits to secure a bargain. For example, in his study of government formation, Strom (1990) refers to a variety of "office and policy inducements" used in forming a government. The redistribution of these benefits can be interpreted as adjustments from the long-tern norm regarding how parties in government divide the aggregate office-holding benefits. An important assumption in this model is that the reallocation of office-holding benefits can only be made among the parties in government. That is, $y_t^i = 0$ if some party i is not in government. This implies that the parties in government can neither extort benefits from nor credibly promise to compensate the out party.[9]

[8] If $y_2^i < 0$, office-holding benefits are transferred from i to the other party(ies) in the government.

[9] It is implicitly assumed that each party is originally endowed with sufficient office-holding benefits to satisfy any proposal made by the formateur. This assumption simplifies the analysis and yields efficient bargaining within coalitions. This assumption also implies that how the parties weigh utilities derived from the policy and transfers of benefits has no effect on the equilibrium. See Baron and Diermeier (2001, p. 935) for more details and examples of office-holding benefits.

The single-period utility function of a party $i \in \{a, b, c\}$ is given by

$$y_t^i + u^i(x_t),$$

where

$$u^i(x_t) = -\left\|x_t - z^i\right\|^2$$

represents party i's single-period policy preferences and $z^i \in \Re^2$ denotes its ideal point.[10] Parties can have preferences over ministries, but those preferences are associated with implementing policies and hence are to be thought of as incorporated in a party's policy preferences. Parties prefer policies closer to their ideal points, but they are more averse to policy changes the farther those changes are from their ideal points. All parties are farsighted. They discount future utilities by $\beta \in [0, 1]$. The discount factor may be interpreted as the political patience of party leaders.[11]

So that no party has an inherent bargaining advantage, the ideal points of the parties are assumed to be symmetrically located in the policy space.[12] This specification allows the dynamics induced by the parliamentary institutions to be isolated from preference alignment effects. The policy space is normalized so that $\|z^i - z^j\| = 1$ for all distinct $i, j \in \{a, b, c\}$. In particular, let $z^a = \left(0, \frac{1}{2}\right)$, $z^b = \left(0, -\frac{1}{2}\right)$, and $z^c = \left(\frac{\sqrt{3}}{2}, 0\right)$.

Voters care only about policy outcomes and not the distribution of office-holding benefits.[13] A voter v is characterized by his ideal point $z^v \in \Re^2$, and his single-

[10] Quadratic preferences are assumed for the sake of tractability. They allow a simple characterization of the equilibrium policy outcomes.

[11] Political patience could differ across countries and also within a country depending on the tenure and age of party leaders as well as other constitutional factors.

[12] This precludes the use of a unidimensional policy space.

[13] The aggregate office-holding benefits can be viewed as fixed, and whatever government is formed will exhaust those benefits. Moreover, the benefits only affect party activists, whose votes will go to their party. Hence, from a voter's perspective the distribution of those benefits is irrelevant to the choice of a voting strategy.

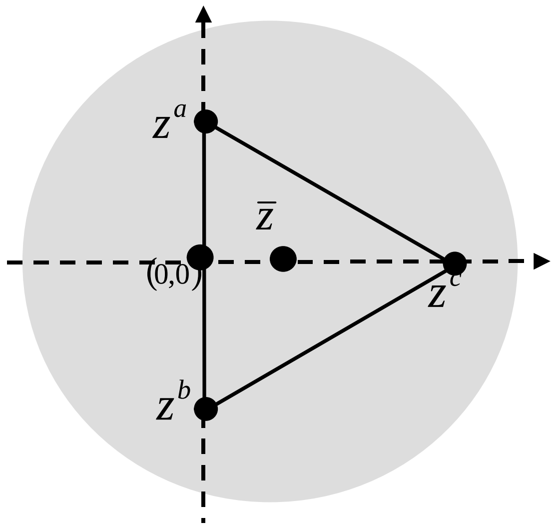

Figure 1: Ideal points of parties and voters.

period preferences are represented by a utility function $u^v(x_t)$ of the same form as those of the parties. That is, the parties are formed among the electorate. Voters are also farsighted and anticipate which party as formateur would form a particular coalition government. In some political systems this ability to anticipate is formalized as pre-election coalitions. Laver and Schofield (1990, pp. 25, 28) give examples of pre-election coalitions in Ireland, the Netherlands, Portugal, and the United Kingdom.

Voters discount future utilities by $\delta \in [0,1]$, a discount factor that may differ from that of the political parties and their leaders. All voters are assumed to vote, and their alternatives are to vote for one of the three parties. To ensure that voter preferences do not favor a particular party or coalition, the ideal points of voters are assumed to be uniformly distributed on a disk $\mathcal{Z} \equiv \{z^v \in \Re^2 : \|z^v - \overline{z}\| \leq L\}$, where $L > \frac{1}{\sqrt{3}}$.[14] Ideal points of the parties and voters are illustrated in Figure 1. The policy $\overline{z} \equiv \frac{1}{3}\sum_{i=1}^{3} z^i$ is the center of party and voter preferences.

Policies are continuing, so when a government is formed in period t, the status quo

[14]The assumption $L > \frac{1}{\sqrt{3}}$ guarantees that the ideal points of the parties are not more extreme than those of the most extreme voters.

$q_{t-1} \in \Re^2$ is the policy in place under the previous government. Similarly, the policy chosen by the new government becomes the status quo for the following period. If a new policy is not enacted, the status quo remains in place. In the first period there is an initial status quo q_0.

An interelection period consists of three stages. The first stage involves a parliamentary election that determines the seat shares of the parties in the parliament. The second stage involves government formation, and the third stage is legislative and involves the choice of a policy by the parliament. The game has complete information, and no player can commit to any action in future stages.

The electoral system is proportional representation with a minimum vote share m required for representation, where $m \in \left(0, \frac{1}{4}\right)$.[15] The electoral rule is viewed as a mapping from vote shares of the parties to seat shares in parliament. If the vote shares $\left(\rho_t^a, \rho_t^b, \rho_t^c\right)$ of all parties are at least m, their seat shares are equal to the vote shares. If only party i's vote share is less than m, it is not represented in parliament and any other party j has a seat share of $\frac{\rho_t^j}{1-\rho_t^i}$. If two parties have vote shares less than m, the third party has a seat share of 1. A parliament in which no party has a majority is referred to as a *minority parliament*, whereas in a *majority parliament* one party has a majority of the seats.

After an election one party is selected as the formateur. Selection is proportional to the party's seat share in parliament, unless one party has a majority of seats in which case it is selected as the formateur.[16] The formateur in period t has the opportunity to form a government, which must be a majority coalition.[17] Therefore, a government

[15] The upper bound on m allows all three parties to be represented in parliament even if there is a majority party.

[16] Diermeier and Merlo (2004) present empirical evidence supporting a proportionality rule with some support for an incumbency advantage.

[17] The model effectively precludes the formation of minority governments. For a theory that accommodates and accounts for the occurrence of minority governments, see Diermeier and Merlo (2000).

coalition C_t is a non-empty subset of the parties represented in parliament such that $\sum_{i \in C_t} s_t^i > \frac{1}{2}$. A *consensus government* includes all three parties, a *majoritarian government* is composed of two parties, and a *single-party government* is formed by a single majority party.

In forming a government the formateur is assumed to make a take-it-or-leave-it offer to the other members of the coalition, which allows us to identify the maximal effect of institutions on policies. The offer specifies a policy proposal x_t and a redistribution $\left(y_t^a, y_t^b, y_t^c\right)$ of office-holding benefits. If all coalition members accept the offer, the government is formed, the proposed policy is implemented, and the office-holding benefits are allocated as proposed. A new period $t+1$ then begins with the status quo $q_t = x_t$. If any party in the coalition rejects the offer, the status quo q_{t-1} is the policy outcome in period t, and no redistribution of the office-holding benefits is made.[18] The status quo for period $t+1$ then is $q_t = q_{t-1}$.

We characterize a subgame perfect equilibrium. In the election stage we follow Baron and Diermeier (2001) and seek a strong Nash equilibrium. That is, an allocation of vote shares $\left(\rho_t^a, \rho_t^b, \rho_t^c\right)$ is such that no *group of voters* has an incentive to deviate from their equilibrium voting strategies. A few technical assumptions are made for a formateur to break indifference, and these assumptions will be introduced sequentially as the analysis proceeds.

[18] The government may be understood as being of cabinet form in which all government parties must agree on the policy choice. An offer to form a consensus government is thus conditional on all coalition partners accepting the offer.

3 Results

3.1 The Second Period

The equilibrium of the two-period multi-stage game is analyzed by backward induction. The game in the second period is a one-shot game, which was fully characterized by Baron and Diermeier (2001). This subsection summarizes intuitions and results relevant for our dynamic analysis.

3.1.1 Policy Choice in the Second Period

Redistribution of office-holding benefits allow parties in government to bargain efficiently. Suppose that party $k \in \{a, b, c\}$ as formateur has decided to form a government C_2 in the second period when the status quo is q_1. The formateur selects a policy x_2^* that maximizes the joint utility of all government members, i.e.,

$$x_2^* = \arg\max_{x \in \Re^2} \sum_{i \in C_2} u^i(x),$$

and then redistributes office-holding benefits so that each of its coalition partner(s) is just willing to vote for the government, i.e., for all $j \in C_2 \backslash \{k\}$,

$$y_2^j + u^j(x_2^*) = u^j(q_1).$$

The symmetric location of ideal points and the quadratic policy utility function jointly imply the first lemma.

Lemma 1 In the second period: (A) A consensus government chooses the center of party preferences $\overline{z}$. (B) A majoritarian government formed by parties j and k chooses the midpoint of the parties' contract curve, $z^{jk} \equiv \frac{1}{2}\left(z^j + z^k\right)$.[19] (C) A single-party government formed by party k chooses its ideal point z^k. (D) Any non-formateur party j in government receives a redistribution $y_2^j = u^j(q_1) - u^j(x_2^*)$ of office-holding benefits. The period-two utility of j is exactly its reservation value $u^j(q_1)$. (E) The period-two utility of formateur k is

$$\sum_{i \in C_2} u^i(x_2^*) - \sum_{i \in C_2 \setminus \{k\}} u^i(q_1).$$

The equilibrium policy choice depends only on which parties are in the government, not on the proposer or the location of the status quo. The redistribution of benefits and the formateur's utility, however, depend on the status quo. Note that the status quo determines a party's reservation value. Therefore, the formateur can extract more benefits the more the status quo disadvantages its possible coalition partners. The status quo thus plays an important role in government formation.

3.1.2 Government Formation in the Second Period

In the government formation stage the formateur chooses a majority coalition to maximize its utility.[20] Suppose that party a, for example, has been selected as formateur in a parliament with all three parties represented.[21]

If party a decides to form a majoritarian government, it chooses the party that is more disadvantaged by the status quo, since that party has a stronger incentive

[19] For example, government ab chooses policy $z^{ab} = (0, 0)$.

[20] For technical convenience, we assume that if a formateur is indifferent between two majoritarian governments, it flips a coin. Moreover, if it is indifferent between a majoritarian government and a consensus government, it chooses the latter. As will be evident in the analysis, the equilibrium policy choice conditional on a consensus government leads to a greater aggregate utility of all parties as well as all voters than the policy choice conditional on any majoritarian government. The selection of a more efficient equilibrium thus strengthens our main results on policy inefficiency.

[21] Baron and Diermeier (2001) show that in any equilibrium all three parties are represented in parliament, whether it is a minority or majority parliament.

to induce the formateur to choose a policy more favorable to it. For expositional purposes, suppose this is party c and the period-two status quo q_1 is such that $u^b(q_1) > u^c(q_1)$.

Formateur party a prefers to form a consensus coalition rather than a majoritarian coalition if and only if

$$\sum_{i=a,b,c} u^i(\overline{z}) - \sum_{i=b,c} u^i(q_1) \geq \sum_{i=a,c} u^i(z^{ac}) - u^c(q_1),$$

where the left-hand-side is a's utility if it forms a consensus government with policy $\overline{z}$, and the right-hand-side is its utility if it forms majoritarian government ac with policy z^{ac}. This condition is equivalent to

$$u^b(q) \leq -\tfrac{1}{2} = \sum_{i=a,b,c} u^i(\overline{z}) - \sum_{i=a,c} u^i(z^{ac}).$$

Intuitively, if the reservation value of the non-formateur party with status-quo advantage (i.e., party b in the example) is sufficiently low, the formateur chooses a consensus coalition. This occurs if both the non-formateur parties strongly dislike the status quo and hence are willing to make sizable concessions of office-holding benefits in exchange for a centrist policy.

To facilitate the statement of results in Lemma 2, define $D^i \equiv \left\{x \in \Re^2 : u^i(x) > -\frac{1}{2}\right\}$, for any $i \in \{a, b, c\}$, as the set of policy alternatives that yield party i a period utility greater than $-\frac{1}{2}$. The preceding analysis implies that if the status quo q_1 is outside D^i and $u^i(q_1) \geq u^j(q_1)$, party k as formateur prefers to form a consensus government rather than a majoritarian government with party j.

Lemma 2 In the government formation stage of the second period with status quo q_1, party k as formateur (A) forms a consensus government if $q_1 \in \Re^2 \backslash (D^i \cup D^j)$, (2)

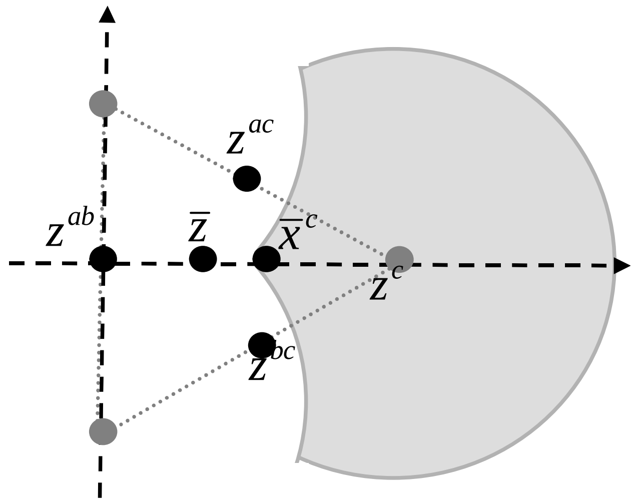

Figure 2: Illustration for Examples 1 and 2.

forms a majoritarian government with party i if $q_1 \in D^i \cup D^j$ and $u^i(q_1) < u^j(q_1)$, and (3) forms a majoritarian government with party i or j with probability $\frac{1}{2}$ if $q_1 \in D^i \cup D^j$ and $u^i(q_1) = u^j(q_1)$.

3.1.3 Parliamentary Election in the Second Period

We next consider the strong Nash electoral equilibria in the second period. Since voters know the status quo, q_1, they can anticipate which coalitions and policies will result for any distribution of seats in parliament. As shown by Baron and Diermeier (2001), in equilibrium some voters may not vote for the parties whose ideal points are closest to theirs. The intuition of strategic voting is explained through the following two examples. These examples will be useful when we analyze the equilibrium policy choice in the first period.

Example 1. Suppose that the period-two status quo is $q_1 = \overline{x}^c \equiv \left(\frac{1}{2}, 0\right)$. (See Figure 2.) Observe that $\overline{x}^c \in \overline{D}^c \backslash \left(D^a \cup D^b\right)$ (the shaded area in the figure), where $\overline{D}^c$ denotes the closure of D^c. By Lemma 2, given any parliament in which all three parties are represented, party c as formateur would form a consensus government with the centrist policy $\overline{z}$, whereas either party a or b as formateur would form a majoritarian government ab with policy z^{ab}. Note that more than half of the voters (for example, those whose ideal points are to the right of the perpendicular hyperplane passing through $\overline{z}$) strictly prefer $\overline{z}$ to z^{ab}. Therefore, if party c is not elected as the majority party, there must exist a group of voters who want to switch their votes from either a or b to c. If too many voters in the natural constituencies of parties a and b vote strategically for c, however, both parties a and b would lose representation in parliament. Then, as the majority party c would choose its ideal point z^c instead of $\overline{z}$. The argument in this example can be generalized for any status quo $q_1 \in \overline{D}^i \setminus \left(D^j \cup D^k\right)$, for any distinct $i, j, k \in \{a, b, c\}$, and leads to statement (B) in Lemma 3 at the end of this subsection.

Note that in Example 1 there are multiple electoral equilibria. This is because voters do not care about the exact vote shares as long as all three parties are represented and c is elected the majority party.

Example 2. Suppose that the period-two status quo is $q_1 = z^{ab} = (0, 0)$. (Again, see Figure 2.) Given that all three parties are represented in parliament, by Lemma 2, either party a or b as formateur would form a majoritarian government with party c, which is more disadvantaged by the status quo and hence in a weaker bargaining position. Once represented in parliament, party c would be included in any government and the policy choice would be either z^{ac} or z^{bc}. Foreseeing such a consequence, some natural supporters of c who strictly prefer z^{ac} to z^{bc} would strategically vote for a instead of c. In this way, they increase the probability that party a is selected as formateur and therefore the probability that z^{ac} becomes the policy outcome. Similarly, some natural supporters of c who strictly prefer z^{bc} to z^{ac} switch their votes from

c to b. The extent of strategic voting by the supporters of c, however, is bounded by the representation threshold m. In equilibrium voters give party c just enough votes so that it is represented in the parliament and, by the symmetric distribution of preferences, the vote shares of parties a and b are $\left(\frac{1-m}{2}\right)$ each. The argument in this example can be generalized for any status quo $q_1 = (-h, 0)$, where $h \in \left[0, \frac{1}{2}\right)$, and leads to statement (D) in Lemma 3.

Lemma 3 presents a complete characterization of the strong Nash electoral equilibria.

Lemma 3 *Consider the parliamentary election in the second period with a status quo* q_1. *(A) If the status quo is such that any party as formateur would form a consensus government with policy choice* $\overline{z}$, election of any parliament with all three parties represented is a strong electoral equilibrium. (B) If the status quo is such that only party i as formateur would form a consensus government with policy choice $\overline{z}$, every strong electoral equilibrium results in a majority parliament with three parties represented, where party i is the majority party. (C) If the status quo is such that *party* i *as formateur would randomize between coalitions* ij and ik *and the other two parties (*j and k) as formateur *would form governments with party* i, *any strong electoral equilibrium results in a minority parliament with vote shares* $\rho_2^i = m$ and $\rho_2^j = \rho_2^k = \frac{1}{2}(1-m)$. *Majoritarian governments* ij and ik *and policy outcomes* z^{ij} *and* z^{ik} *then result with probability one-half. (D) If the status quo is such that party* i *as formateur would form a majoritarian government with each of the other parties with probability* $\frac{1}{2}$, *and the other two parties* j *and* k *would form majoritarian governments with each other, a minority parliament results with a strong electoral equilibrium with vote shares* $\rho_2^i = \frac{1}{2}$, $\rho_2^j + \rho_2^k = \frac{1}{2}$, *and* $\rho_2^j, \rho_2^k \in \left[m, \frac{1}{2} - m\right]$. *(E) If* $q_1 = \overline{z}$, *the unique strong equilibrium is equal vote shares for all three parties and an* *even lottery over z^{ab}, z^{ac}, and z^{bc}.*[22]

Lemma 3 implies a relationship between the bargaining advantage from the status quo and the advantage in a parliamentary election. In particular, the party that in the bargaining is relatively favored by the status quo obtains a (weakly) higher expected seat share than the other parties and therefore a greater chance of being selected as formateur. Similarly, the party that in the bargaining is relatively disadvantaged by the status quo obtains a (weakly) lower expected seat share than the others. This indicates that, when the period-one formateur strategically positions the policy to gain a period-two bargaining advantage, it may also gain an advantage in period-two election and hence increase its probability of heading the government again. This will be made clear in Sections 3.2.1 and 3.3 when we identify the electoral effect on policy choice.

3.2 Policy Choice in the First Period

For any period-two status quo q_1, let the continuation value $v^i(q_1)$ of party i be its expected utility calculated prior to the period-two election.[23] For any period-one

[22] This lemma is a modified version of Proposition 4 in Baron and Diermeier (2001). Part (B) has been restated and (E) added. Moreover, the equilibrium vote shares in part (D) are different due to a different population structure. Baron and Diermeier (2001) assumed that voters' ideal points were uniformly distributed in the single-period Pareto set of the parties, whereas here voters' ideal points are assumed to be uniformly distributed in the disk $\mathcal{Z} \equiv \left\{ x \in \Re^2 : \|z^v - \overline{z}\| < L \right\}$.

[23] Lemma 3 implies that, in the second period for a large set of status quo alternatives, there exist multiple electoral equilibria that may lead to the same policy outcome but different distributions of period-two utilities for the parties. To simply the analysis, we make two assumptions about the selection of period-two electoral equilibria. First, if a period-two status quo is such that party k as formateur would form a consensus government with the central policy $\overline{z}$ whereas any of the other parties as formateur would be indifferent between forming a majoritarian government and a consensus government, the period-two electoral equilibrium is such that party k is elected as the majority party. This assumption applies only to a period-two status quo on the boundary of D^i for some $i \in \{a, b, c\}$ and outside the set of D^j for all $j \neq i$. Second, for any other period-two status quo, if there are multiple electoral equilibria, all equilibria identified in Lemma 3 occur with equal probability. These selection rules assure that each period-two status quo is associated with unique expected period-two utilities of the parties.

status quo q_0, let $U^i(q_0) = u^i(q_0) + \beta v^i(q_0)$ be the reservation value of party i in the first period, so $U^i(q_0)$ is the sum of party i's period-one utility with the status quo policy q_0 and no transfers, plus its discounted continuation value for the second period with a status quo $q_1 = q_0$. The expected discounted sum of utilities of party i in the first period is therefore $y_1^i + U^i(x_1)$ if a policy x_1 is chosen and it receives a redistribution y_1^i of office-holding benefits.

As in the second period, if party k as formatuer decides to form a government coalition C_1 in the first period, it must choose a policy x_1^* that maximizes the joint two-period utility of all government members, i.e.,

$$x_1^* \in \arg\max_{x \in \Re^2} \sum_{i \in C_1} U^i(x).$$

The formateur then redistributes office-holding benefits so that each of its coalition partner(s) is just willing to vote for the government. If two distinct combinations of policies and office-holding benefits attain the maximal two-period utility for the formateur, it is assumed to choose the one with which it is more likely to be selected as formateur in the second period. This assumption helps eliminate additional equilibria in the first period. Substantively, it amounts to a party's lexicographic preference to head a government.

In what follows, we characterize the equilibrium policy choice conditional on the type of government and the formateur selected.

3.2.1 Policy Choice - Majoritarian Government

Consider a majoritarian government ab formed by party a in the first period. If the parties are solely concerned about their period-one utilities, i.e., $\beta = 0$, the period-one policy choice would be the midpoint $z^{ab} = (0,0)$ of the contact curve of parties a and

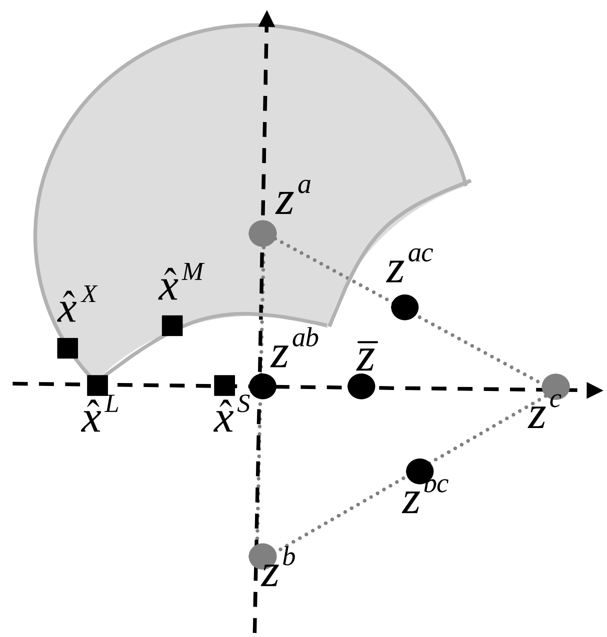

Figure 3: Policy choices by majoritarian government ab formed by party a in the first period.

b. Although strategically irrelevant when $\beta = 0$ this policy advantages parties a and b in the period-two election yielding a vote share of $\frac{1-m}{2}$, as explained in Example 2.

For β positive but sufficiently small, the formateur chooses a policy, as illustrated by $\widehat{x}^S$ in Figure 3, that is equidistant from z^a and z^b but farther away from z^c. This policy sacrifices period-one utility of parties a and b but increases their joint period-two utility, since policy $\widehat{x}^S$ as status quo in the second period further disadvantages the out party c in the period-two bargaining over government formation and policy choice, allowing any of the government members as period-two formateur to obtain more office-holding benefits from c. Also, parties a and b expect a vote share of $\frac{1-m}{2}$ in the period-two election. Since each is more likely than c to be the period-two formateur, a and b have a stronger incentive for strategic policy manipulation to disadvantage c than what they would do if the seat shares were fixed and equally distributed in the second period. As β increases, the parties care more about the their future utilities, so a majoritarian government chooses a more extreme policy.

If the discount factor of the parties is sufficiently high, the future is sufficiently important that the formateur sacrifices efficiency in the first period to position strategically the status quo to gain electoral advantage and induce the efficient policy in the second period. Specifically, the formateur forms a government with b at a policy in $\overline{D}^a \backslash (D^b \cup D^c)$, illustrated as the shaded area in Figure 3. Such a policy, for example $\widehat{x}^M$, substantially lowers the joint period-one utility of the government parties, but it is sufficiently far from z^b and z^c that a receives a majority vote share in the next election because voters understand that if either b or c were to be the formateur they would choose police z^{bc} that is less central than the policy $\overline{z}$ that party a would choose with a majority. As β increases the policy moves along the boundary of D^b until at $\beta = \widetilde{\beta} = \sqrt{3} - 1$ it reaches $\widehat{x}^L \equiv (-\frac{1}{2}, 0)$, which is the intersection of the boundaries of D^a and D^b. As β increases further, the policy moves along the boundary of D^a, but farther from both parties b and c. At $\beta = 1$ the policy is $\widehat{x}^X \equiv (-\frac{\sqrt{3}}{2\sqrt{2}}, \frac{1}{2}(1 - \frac{1}{\sqrt{2}}))$.

The policy is outside the single-period Pareto set of party preferences for all $\beta > 0$, and it is farther away from the center of preferences as β increases. The institutions of the parliamentary system thus give rise to a centrifugal force that results in Pareto inefficient policies. The next proposition formalizes the period-one equilibrium policy and its consequences. The proof is presented in Appendix A.

Proposition 1 Consider a majoritarian government ab formed by party a in the first period. (A) If the discount factor of the parties satisfies $\beta < \beta^* (m)$, where $\beta^* (m)$ is a decreasing function in representation hurdle m, the period-one policy is

$$x_1^* = \left(-\frac{\sqrt{3}(1-m)\beta}{2(2-(1-2m)\beta)}, 0 \right),$$

which is equidistant from the ideal points of parties a and b and farther from the ideal point of the out party c.[24] In the second period both parties a and b obtain a vote share $\frac{1-m}{2}$, and party c obtains a vote share m. Any period-two formateur forms a majoritarian government that includes party c and the policy outcome x_2^* is either z^{ac} or z^{cb} with probability one-half each. (B) If the discount factor of the parties satisfies $\beta \geq \beta^*(m)$, the period-one policy outcome is farther from the ideal point of the out party c, biased toward the formateur a, and given by

$$x_1^* = \left(-\frac{\sqrt{3}\beta}{2(2-\beta)}\kappa, \frac{1}{2}\left|\kappa - 1\right|\right),$$

where

$$\kappa \equiv \left(\frac{1}{2}\right)^{\frac{1}{2}}\left[\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\left(\frac{\beta}{2-\beta}\right)\right)^2\right]^{-\frac{1}{2}}.$$

In the second period there is a majority parliament with all parties represented, where the period-one formateur a receives a majority of the vote and forms a consensus government with policy $\overline{z}$. (C) Regardless of the discount factor the period-one policy outcome is outside the single-period Pareto set of the three parties.

3.2.2 Policy Choice - Consensus Government

Next, consider a consensus government formed by party c in the first period. The formateur faces an intertemporal tradeoff. If the discount factor of the parties is sufficiently low, it prefers the policy $\overline{z}$ that is efficient in the current period. As a consequence in the second period each party receives one-third of the vote, and a majoritarian government is formed with policy at the midpoint of the contract curve of the government parties.

Similar to the case of majoritarian government, for β sufficiently large, party c as

[24] The value of $\beta^*(m)$ ranges from approximately 0.17 to 0.26. The function $\beta^*(m)$ is characterized in Appendix A.

formateur chooses $\overline{x}^c = (\frac{1}{2}, 0)$, illustrated in Figure 2, which as the status quo induces strategic voting in favor of party c causing it to receive a majority of the vote. With a majority, party c forms a consensus government in the second period with the efficient policy $\overline{z}$. In contrast, if either party a or b was the period-two formateur, they would form a majoritarian government with each other and choose a more extreme policy. Voters thus give c a majority. The policy $\overline{x}^c$ disadvantages parties a and b in the next election and also disadvantages them in the bargaining over government formation and policy choice in the second period. Party c thus must provide the other parties with sufficient benefits in the first period to obtain $\overline{x}^c$ rather than $\overline{z}$.

Consensus governments always choose a first-period policy that is interior to the single-period Pareto set of the parties, but the institutions of the parliamentary system create incentives that give rise to a centrifugal force that moves policy away from $\overline{z}$. Proposition 1 formalizes this intuition, and the proof is presented in Appendix A.

Proposition 2 Consider a consensus government formed by party c in the first period. (A) If the discount factor of the parties is sufficiently small, $\beta < \hat{\beta} \equiv 4 - 2\sqrt{3}$, the period-one policy outcome is the center of preferences $\overline{z}$. As a consequence, in the second period there is a minority parliament with majoritarian governments, each of which is formed with probability one-third. The policy for any government jk is the midpoint z^{jk} of the contract curve. (B) If the discount factor of the parties is sufficiently large, in particular $\beta \geq \hat{\beta}$, the period-one policy outcome is $\overline{x}^c = \left(\frac{1}{2}, 0\right)$. In the second period party c receives a majority vote share and as formateur forms a consensus government with policy $\overline{z}$.

3.2.3 Policy Choice - Single-Party Government

Finally, consider a single-party government formed by a majority party c in the first period. For $\beta = 0$ party c chooses its ideal point, $z^c = \left(\frac{\sqrt{3}}{2}, 0\right)$, and remains the majority party in the second period because it would in the second period choose the centrist policy $\bar{z}$ whereas parties a and b would choose a more extreme policy. As β increases, it chooses a policy equidistant but farther from the ideal points of the other two parties, and hence outside the Pareto set. Such a policy sacrifices party c's period-one utility but allows it to obtain more office-holding benefits in the second period once it is selected as the period-two formateur. At the same time, the policy remains in $\overline{D}^c$, so c remains a majority party in the second period.

The period-one policy cannot be too extreme, however, since then any party as period-two formateur would form a consensus government with $\bar{z}$, and c would lose its electoral advantage. The formateur prefers to retain that electoral advantage that yields it a majority and hence restrains its period-one policy choice within $\overline{D}^c$. Again, electoral considerations bound the extent of the inefficiency.

The next proposition formalizes the preceding analysis. The proof is presented in Appendix A.

Proposition 3 In the first period any single-party government, for example, formed by majority party c, chooses a policy that favors itself but is far and equally distant from the ideal points of the other parties and outside the Pareto set for $\beta > 0$. In particular, the period-one policy is in the set $\overline{D}^c \setminus \left(D^a \cup D^b\right)$ and is given by

$$x_1^* = \begin{cases} \left(\frac{\sqrt{3}}{2(1-2\beta)}, 0\right), & \text{if } \beta < \beta^o \equiv \frac{1}{2}\left(\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}}\right) \\ \left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}, 0\right), & \text{if } \beta \geq \beta^o. \end{cases}$$

In the second period party c remains the majority party and forms a consensus government with policy $\bar{z}$.

3.2.4 Dynamics of Governments and Policies

Propositions 1-3 identify a rich set of dynamics of government coalitions and their policy choices. Consider first the case in which a majoritarian government is formed in the first period. As a function of political patience, begin with a low β, and assume that a is the formateur and b is its government partner. As β increases the policy moves away from the mid-point of the government contract curve but remains equidistant from the ideal points of the government parties, as illustrated by the point $\hat{x}^S$ in Figure 3. In the second period a minority parliament and a majoritarian government result with the policy at the mid-point of the government parties' contract curve.

As β increases above $\beta^*(m)$, the period-one policy jumps to the boundary of D^b and toward the ideal point of a, as illustrated by $\hat{x}^M$ in Figure 3. This disadvantages party b as well as party c. As β increases further, the first-period policy moves along the boundary until it reaches $\hat{x}^L$ in Figure 3. Further increases in β cause the period-one formateur to choose a policy farther from the ideal points of both the other parties, as illustrated by $\hat{x}^X$. For all $\beta > \beta^*(m)$, if selected as formateur in period two, the period-one formateur a would choose $\bar{z}$, whereas the other parties would choose a more extreme policy. Hence, party a receives a majority of the vote in the election, and forms a consensus government in period two with policy $\bar{z}$.

Now consider a consensus government formed by party c in the first period. For $\beta < \hat{\beta}$ the period-one formateur c chooses a policy $\bar{z}$, and neither the government nor the policy is sustainable in the second period. A minority parliament results in the second period, and majoritarian governments are formed with policies at the mid-point of the government parties' contract curve. As β increases above $\hat{\beta}$, the policy jumps towards the formateur's ideal point to the policy $\overline{x}^c = (\frac{1}{2}, 0)$ shown in Figure 2. This policy disadvantages both of the other parties and results in a majority vote for party c. In period two party c forms a consensus government with policy $\bar{z}$. The type of government thus persists but not its policy.

Government transition and policy change thus should be expected in parliamentary systems. Moreover, whenever majoritarian governments are formed and there are future elections, the policies chosen by the governments formed are inefficient.

3.3 Electoral versus Bargaining Effects

To understand the incentives of parties to position strategically the status quo for the second period, it is useful to identify two different effects that drive our results. The *bargaining effect* isolates how the incentive to gain bargaining advantage in the future leads to a more extreme policy in the first period, assuming exogenous and equal seat shares of the parties in the next period. The *electoral effect* captures how the presence of a parliamentary election with strategic voters leads to a more extreme policy in the first period.

To identify these two effects we compare our result to the case where each party exogenously receives one-third of the votes in each election. This case has been analyzed in Fong (2006), where it is interpreted as a model with sincere voting. Fong (2006) shows that for a consensus government there is no strategic positioning of the status quo, no matter how much parties care about the future. The parties choose the center $\overline{z}$ of preferences, which leads to a majoritarian government in period two with policy at the mid-point of the government parties' contract curve. In contrast, a majoritarian government, formed by parties a and b, for example, chooses a policy that is equidistant from the ideal points of the government parties, far from the ideal point of the out party c, and outside the single-period Pareto set. In particular, there exists $\overline{\beta} \in (0,1)$ such that the equilibrium policy choice $\widetilde{x}_1$ is

$$\widetilde{x}_1 = \begin{cases} \left(-\frac{2\sqrt{3}\beta}{2(6-\beta)}, 0\right), & \text{if } \beta \in \left[0, \overline{\beta}\right), \\ \left(-\frac{1}{2}, 0\right), & \text{if } \beta \in \left[\overline{\beta}, 1\right]. \end{cases}$$

Strategically positioning the status quo reduces the joint period-one utility of parties a and b, but in this case each government member, if it is the formateur in period two, will form a government with party c and receive greater office-holding benefits from c.[25] The formateur thus has an intertemporal tradeoff. As the parties care more about the future, the period-one formateur chooses a more extreme policy. As the discount factor increases above the cutoff $\overline{\beta}$, the period-one policy jumps to a sufficiently extreme policy, $\left(-\frac{1}{2}, 0\right)$, that as the status quo would result in a consensus government in the second period. This allows party a, if recognized as the period-two formateur, to maximize the joint period-two utility of all three parties and receive office-holding benefits from both of the other ones.

The electoral effect can be identified by subtracting the bargaining effect characterized above from the total effect of institutions on policy choice. To quantify the bargaining and electoral effects, we define a metric to measure the extremeness of the policy choice and use this metric to compare equilibrium policies in models with and without elections. For any policy $x \in \Re^2$, let $DTC(x) \equiv \|x - \overline{z}\|$ be the distance from the policy x to the center of preferences $\overline{z}$. The bargaining effect then is measured by $DTC(\widetilde{x}_1) - DTC(x_2^*)$, where $\widetilde{x}_1$ denotes the equilibrium period-one policy choice in the model with no election and x_2^* denotes the equilibrium policy choice in a single-

[25] If party c is selected as formateur, it forms a majoritarian government with either party a or b and is able to extract more office-holding benefits from its government partner than if the status quo were $z^{ab} = (0,0)$. Due to strict concavity of the utility functions, however, this is a second-order effect and is always dominated.

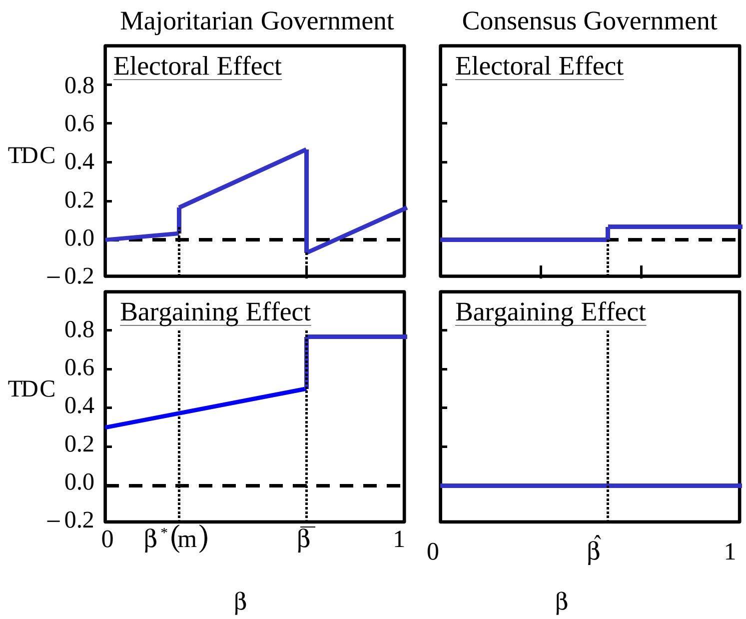

Figure 4: The bargaining effect and the electoral effect conditional on the types of government. Calculation is based on $m = 10\%$.

period model (or in the second period of our model). The electoral effect is measured by $DTC(x_1^*) - DTC(\widetilde{x}_1)$, where x_1^* denotes the equilibrium period-one policy choice in our model with strategic voters. These effects are identified in Figure 4 for the types of government in the first period with the representation hurdle $m = 10\%$.

As indicated in Propositions 2 to 3, the incentives created by the institutions of a parliamentary democracy exert centrifugal forces on the policy in the first period, and those forces are generally stronger, and the period-one policy more extreme, the higher is the discount factor β of the parties.[26] One centrifugal force is due to the incentives created by the institutions of government formation and legislation. This centrifugal force plays a role only under a majoritarian government, and the bargaining effect is weakly increasing in β. The second centrifugal force is due to the institution of elections. For example, consider a period-one majoritarian government with a low

[26] Schofield and Sened (2006, p.63) also identify a centrifugal force.

β. As implied by Lemma 3, the party disadvantaged by the period-one bargaining effect is also disadvantaged in the election because both parties in the incumbent government would form a majoritarian government with the out party from the first period. This attracts some voters located close to the out party and induces the incumbent to choose an even more extreme policy in the first period. The electoral effect thus is in general positive, leading to a more extreme policy outcome, except for the the values of β around $\overline{\beta}$, where there is a jump in the bargaining effect. The electoral effect also plays a role under a consensus government, since for β sufficiently large, the formateur of a consensus government chooses a non-central policy biased toward its ideal point to ensure that it receives a majority in the election. Elections under proportional representation thus impose a centrifugal incentive on the political parties that leads to more extreme policies.

3.4 Government Formation and Parliamentary Election in the First Period

The characterizations and comparative statics in the previous subsections were conditional on the type of government formed in the first period. This suggests that voters may be able to avoid extreme policies in the first period through the period-one election. Which type of government forms in the first period depends on the initial status quo q_0, the election outcome, and the identity of the formateur. Although the analysis is similar to that for the second period, the mapping from the initial status quo to the period-one election outcome, selection of formateur, and choice of government is both quite complex and discontinuous. So, we focus here on whether voters can avoid extreme policy outcomes through their votes in the parliamentary election. We make the following observations.

First, in the first period a single-party government is never formed. Suppose otherwise that the initial status quo is such that some party i, if elected as the majority party, would form a single-party government. In this case, the electorate would never give a majority of votes to party i. By Proposition 3, any single-party government would choose an extreme policy for its own electoral advantage in the second period. This policy choice is so extreme such that, even if the voters are very patient, a majority of voters would be better off with a minority parliament and with, for example, a randomization of policies chosen by some majoritarian governments. Thus, single-party governments will not be considered further.

Second, in the first period all three parties are represented in parliament. If only two parties, for example a and b, were represented, the policy outcome would be strongly biased against party c. The constituents of party c then would be better off if party c were represented so that it had a chance to be either selected as the formateur or included in the period-one government. Therefore, they will ensure that party c is represented. There are always enough c supporters to meet the representation threshold.

Third, recall that a formateur chooses to form a consensus government with a central policy only if the reservation values of both the other parties are sufficiently low, since then it will receive sizable transfers of office-holding benefits from the other parties in exchange for policy concessions. This requires that the status quo be extreme. Consequently, only i the initial status quo q_0 is sufficiently extreme, some party i as formateur would form a consensus government with a central policy is not possible if q_0 is sufficiently central. This insight is summarized in the next proposition. The proof is presented in Appendix B.

Proposition 4 If the initial status quo is sufficiently close to the center of voter preferences, the electorate can do nothing in the period-one election to limit the extent of the policy extremeness in the first period by any formed government.

In other words, whereas the period-two election exerts a centrifugal force on policy choice in the first period, with a relatively central initial status quo the period-one election cannot counterbalance this force.

4 Discussion

4.1 Welfare Implications and Political Failure

Proposition 1 identifies a political failure when the parties care about the future, where "political failure" is defined as a policy that is outside the single-period Pareto set of party preferences.[27] The political failure is due to both the institution of elections and the institution of government formation and legislation. These failures appear unavoidable, as long as voting is an inalienable right and voters and parties are unable to commit to future actions.

One source of the political failure associated with elections is voters, who are willing to reward centrist policies in the final period even though it induces inefficiency in the previous period. This results because voters cannot commit to how they will vote in future elections. If all voters were loyal to a party, and hence voted sincerely, this source of political failure would be eliminated. A second source of the political failure associated with elections lies with parties, which may have difficulty committing to enact, or not to enact, particular policies. A party's platform or a pre-announced electoral coalition could be credible, but only if voters were to punish a party for deviating. Parties may be able to develop reputations for fulfilling promises, but political

[27] This is consistent with Besley and Coate (1998), who define a political failure as the failure to make a Pareto-improving investment.

temptations to exploit a reputation for short-term gains can be strong, particularly when voters are sophisticated and respond to the anticipated future actions by the parties. Moreover, the centrifugal force is stronger when the future is more important to the parties, which may make reputations more difficult to sustain.

A political failure also results from the incentives created by the institution of government formation and legislation. In the absence of elections the opportunity to disadvantage a party in future bargaining over governments and policy can lead a majoritarian government to position the status quo outside the Pareto set. This political failure resulting from the bargaining effect is generally increasing in political patience.

In addition to identifying political failures, the model can be used to evaluate the welfare of voters and how their welfare responds to the impatience of political parties. Social welfare is defined as the average two-period utility of all voters. The average single-period utility can be shown to be approximately a constant plus that of a hypothetical voter with ideal point $\overline{z}$.[28] Given a policy x, the average single-period utility of all voters is then measured by $-\left\|x-\overline{z}\right\|^2$. Similarly, the average two-period utility of all voters is measured by $-\left\|x_1-\overline{z}\right\|^2-\delta\left\|x_2-\overline{z}\right\|^2$, where $\delta\in[0,1]$ is the discount factor of voters.

As shown above, for most of the domain of β and conditional on any form of the government, the period-one policy is more extreme if the voters vote strategically in the second period than if they vote sincerely. This implies that, conditional on any type of period-one government, on average voters are worse off if they vote strategically than sincerely. That is, voters cannot commit not to anticipate which governments might form or the policies they would choose. The parties anticipate this and choose a policy that is more extreme.

[28] The proof is available from the authors upon request.

By Proposition 1 conditional on a majoritarian government being formed in the first period, the voters on average are worse off for higher $\beta \in [0, \beta^* (m))$ or $\beta \in (\beta^* (m), 1]$. This implies that social welfare is *lower* as the parties care more about the future. This results because the period-one formateur has an incentive to choose a more extreme policy as the future becomes more important. A more extreme policy achieves two purposes for the formateur. First, it reduces the vote shares the other parties are likely to receive in the subsequent election. This increases the probability that the period-one formateur will again be the formateur in the second period. Second, if the period-one formateur is recognized as the period-two formateur, it obtains greater office-holding benefits from its future coalition partners, since they are more disadvantaged in the bargaining.[29]

4.2 Strategic Voting

One implication of the model is the effect of strategic voting. As voters rationally calculate the consequences of their votes and cast their votes accordingly, they become manipulable by political parties through policy choices and they are generally worse off.

Whether voters vote sincerely or strategically in parliamentary elections is an empirical question, but our model sheds light on how such studies could be conducted.[30] The analysis in Section 3.2 with strategic voting implies that on the equilibrium path any party included in an incumbent government receives on average a (weakly) greater

[29] Note that when β moves from slightly below to slightly above $\beta^* (m)$, on average the voters are better off. This is because for $\beta \in (\beta^* (m), 1]$ the period-one policy is so extreme that in the second period a consensus government with a central policy $\overline{z}$ will result with probability one. This leads to a discrete jump of period-two utility for the average voter. The same result obtains if in the first period a consensus government is formed; i.e., replace $\beta^* (m)$ in the above statement by $\widehat{\beta}$.

[30] The few existing empirical studies (Cox 1997, Bawn 1999) have focused on the effects of variations in voting rules on voting behavior. Examples include representation thresholds or mutiple-ballot systems. To our knowledge there is no empirical work that has directly studied strategic voting in the context with government formation.

vote share than the out party in the subsequent election and therefore a higher probability of being recognized as the next formateur.[31] Moreover, an incumbent formateur receives a (weakly) larger vote share than any other party in the subsequent election and therefore it is (weakly) the most likely party to head the government in the next period.

The explanation for these results proceeds in three steps. First, the parties disadvantaged by the status quo have less bargaining power in the parliament and thus are more likely to be included in the new government after the election. Second, foreseeing this, some natural constituents of the disadvantaged parties strategically vote for their second preferred party, which is the party favored by the status quo. Third, an incumbent government has an incentive to propose a policy to advantage itself and disadvantage the out parties to gain not only more bargaining power but also more votes from strategic voters. The incumbent thus attains an electoral advantage with strategic voters that is not present if voters vote sincerely for parties whose ideal points are closest to theirs. Evidence of an incumbency advantage in proportional representation systems is therefore consistent with the implications of strategic voting.

4.3 The Role of the Representation Threshold

Proposition 1 implies that, ceteris paribus, when the discount factor of the parties is sufficiently small, a higher representation threshold leads to a less extreme policy choice by a majoritarian government. Consider the natural constituents of a party

[31] There, however, exist equilibria in which the out party receives a greater vote share than the non-formateur party in a majoritarian government. For example, consider any $m \in \left(0, \frac{1}{4}\right)$, any $\beta \in (\beta^*(m), 1]$, and suppose that in the first period party a as formateur forms a majoritarian government with party b. A majority parliament then results and, for some $\varepsilon > 0$ sufficiently small, $\left(\rho_2^a, \rho_2^b, \rho_2^c\right) = \left(\frac{1}{2} + \varepsilon, m, \frac{1}{2} - m - \varepsilon\right)$ constitute a period-two strong electoral equilibrium in which party c receives more votes than b.

i that is relatively disadvantaged by the period-two status quo. To have their party represented in parliament, at least an m proportion of voters have to vote for party i. A higher threshold thus restricts the extent to which those natural constituents can vote strategically. This lowers the probability that any of the parties in the period-one majoritarian government will be recognized as formateur and mitigates the incentive of a period-one majoritarian government to choose a more extreme policy.

The representation threshold, however, does not affect the policy outcome if the discount factor of the parties is sufficiently large. In that case, the period-one formateur chooses a policy to ensure that it is elected as the majority party in the second period, and period-two outcomes are independent of the representation threshold.

The testable implication then is that among countries with low political patience the policies are less extreme the higher the representation threshold, whereas among countries with higher political patience policy is independent of the threshold.

Social welfare is strictly increasing in the representation threshold if the discount factor is sufficiently small, since a higher threshold benefits an average voter by serving as a commitment device that reduces strategic voting. Although unmodeled here, a different type of incentive for strategic voting may be created by the representation thresholds (Cox 1997). Supporters of parties with an expected vote share too small to cross the threshold may rationally abandon their most preferred party and vote for a party that is assured representation to avoid wasting their vote. Therefore, a potential cost of a higher representation threshold is that parties that represent minority groups may be unable to obtain sufficient votes for representation in parliament.

4.4 An Alternative Interpretation of the Discount Factor

In the model, a government cannot fail before the next regularly scheduled election, so the life span of a government corresponds to the length of an interelection period. Yet, a defining feature of most parliamentary systems is that an incumbent government can be removed by parliament at any time during the interelection period, e.g., by a successful no-confidence motion. In many countries this can also lead to the dissolution of parliament and early elections. In western European multiparty parliamentary systems in the postwar period, the average duration of governments has varied from 13 months in Italy to 45 months in Luxembourg (Laver and Schofield 1990). If politicians all subjectively discount their future utility at similar *annual* rates, politicians in countries with shorter expected life spans of governments should have a higher *per period* discount factor, i.e., a larger β, than politicians in countries with longer life spans.[32] In the model this corresponds to Italians having higher β's than Luxembourgers.

Such differences could also result from some unmodeled constitutional feature, such as the requirements for confidence and censure procedures, that may affect the stability and therefore duration of governments. For example, it may be more difficult to replace a government in a country with a constructive vote of confidence than in a country in which a successful no-confidence motion can end a government.[33] If these factors affect the average duration of government, a higher (lower) discount factor can be interpreted not only as more (less) patience of the parties, but also as a political system that leads to more (less) frequent government turnover. While the former

[32] Suppose that a political party has an *annual* discount factor $\beta_0 \in [0, 1]$. If it expects a government to last for T years, its *per period* discount factor is $\beta = \beta_0^T$, which is decreasing in T.

[33] There is a large empirical literature on the factors that influence cabinet duration including constitutional features. See Diermeier, Eraslan, and Merlo (2003) for a recent example.

may be difficult to measure, the latter can be easily measured.[34]

5 Concluding Remarks

The principal institutions of parliamentary democracies are elections, government formation, and legislatures. Since the government serves with the confidence of the parliament, government formation and legislation are necessarily intertwined and a bargaining perspective is a natural approach to studying coalition formation and policy choice. Both government formation and legislation depend on representation in parliament, and the modal electoral institution is proportional representation. Political incentives arise from all three institutions, and both political parties and voters respond to those incentives.

This paper identifies how the incentives present in a multiparty parliamentary system affect the dynamics of representation, government, and policy. The bargaining over government formation and policy choice creates intertemporal incentives, since the current policy choice affects the bargaining positions of parties in the next period. When parties are politically patient, bargaining incentives can lead a majoritarian government to choose a policy outside the single-period Pareto set of the parties, and the inefficiency is increasing in political patience. Elections determine the representation of parties and also the likelihood that a party will be selected as formateur. This provides incentives for parties to position the current policy to gain an advantage in the next election. The incentives arise because voters anticipate both the governments that could form in the next period and the policies they would choose as a function of representation and the status quo. These electoral incentives can lead to policies farther from the center of voter preferences. Political failures thus

[34] To estimate political patience, another possibility may be to measure the expected tenure of party leaders.

result from both government formation and elections, and those failures provide centrifugal forces on policy choice. These forces are generally stronger the more patient are political parties.

The incentives present in a parliamentary system affect the continuity of governments and policy. These incentives are sufficiently strong that governments generally do not persist from one interelection period to the next and neither do their policies. Government transition and policy change thus should be expected in parliamentary systems, independently of whether there are shocks to the system. How transition and policy changes depend on the time horizon is left to future research.

The incentives leading to policy inefficiency and transitions in government and policy are due in part to commitment problems. If voters could commit to loyalty to a party, the centrifugal force arising from elections would not be present. If parties could commit credibly to the governments they would form and the policies they would choose, the centrifugal force arising from bargaining over government formation and policy would be mitigated, although not eliminated. We assumed that parties cannot commit to future governments or policies, but we implicitly assumed that parties can commit not to dissolve a government before the next scheduled election. We leave endogenous government termination, either voluntarily or through a no-confidence motion, for future work.

Our theory predicts that majority parties can arise. This prediction is contrary to empirical evidence: Majority parties are rare in proportional representation systems. In the context of the model the absence of majority parties is implied by three factors: politically impatient parties, a centrally located status quo, or voters who vote sincerely or are loyal to a party. Extensions of the current model could make it less likely that a majority party would emerge in equilibrium. First, the electorate could be a mixture of voters with different voting behaviors. If the fraction of sincere voters is sufficiently large, a majority party may not be formed even with strategic voting by those sophisticated voters. Second, the current model considers a parliamentary system with only three exogenously-given parties and without entry. With more parties or endogenous entry, a majority would be more difficult to obtain. We also leave this topic for future research.

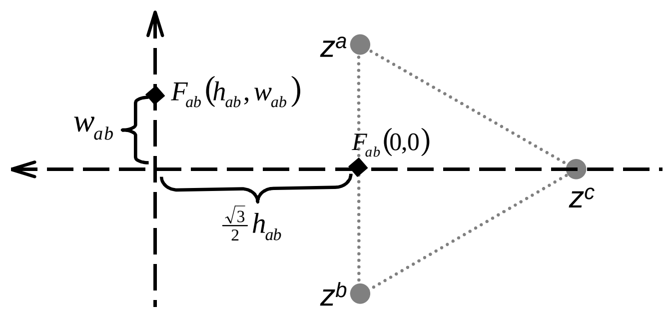

Figure 5: Transformation of coordinates: the F-representation.

Appendix

Throughout the Appendix, we apply a notation system that locates the positions of different policy alternatives. For any $x \in \Re^2$ and for any distinct $i, j = a, b, c$, there exist $h_{ij}, w_{ij} \in \Re$ such that

$$x = F_{ij}\left(h_{ij}, w_{ij}\right) \equiv \frac{1}{2}\left(z^i + z^j\right) + \left(\frac{1}{2}\left(z^i + z^j\right) - z^k\right) h_{ij} + \left(z^i - z^j\right) w_{ij}.$$

(See Figure 5 for an illustration.) With the coordinate system of $F_{ij}(\cdot)$, any policy is described according to its position relative to the ideal points of all three parties. Note that if $h_{ij} > 0$, the policy x is outside the Pareto set. For example, $F_{ab}(\frac{1}{\sqrt{3}}, 0) = (-\frac{1}{2}, 0)$.

Moreover, let $H_i(C)$ denote the optimal policy choice by party i as formateur when it forms a government coalition C in the first period. Since parties care about their status as formateur (or heads of government) in addition to the policy, a formateur may propose a policy that yields a greater chance of getting more votes in the subsequent parliamentary election.

A Proofs of Propositions 1-3

Proof of Proposition 2. Consider a consensus government formed by party a in period one. Note that a formateur will propose a policy that maximizes the joint two-period utility of all members in the government, since it can use redistributions of office-holding benefits as instruments to reallocate the utilities of the parties. Therefore, the proof involves the $H_a(abc)$ that maximizes the joint two-period utility of all three parties and yields party a the highest probability of being recognized as period-two formateur among all policy alteratives that maximize this joint utility.

Partition the policy space into two regions: $R_1^C \equiv \left(D^a \cap D^b\right) \cup \left(D^a \cap D^c\right) \cup \left(D^b \cap D^c\right)$ and $R_2^C \equiv \Re^2 \setminus R_1^C$. Note that neither R_1^C nor R_2^C is convex. In the second period, a status quo in R_1^C or R_2^C leads to a different joint utility for all three parties. The approach is to characterize local maxima in these regions separately and then compare them to identify the globally optimal policy choice for party a. An optimal policy in a region R is denoted by $H_a(abc|R)$.

Region R_1^C Suppose that a consensus government is restricted to choose a policy from region R_1^C. Then by Lemmata 1 to 3 in the second period a majoritarian government will be formed, and the joint period-two utility of all three parties will be $-\frac{5}{4}$. Therefore,

$$\sum_{i=a,b,c} U^i\left(H_a\left(abc|R_1^C\right)\right) = \max_{x' \in R_1^C}\left\{\sum_{i=a,b,c} u^i\left(x'\right) + \beta\left(-\tfrac{5}{4}\right)\right\} = -1 - \tfrac{5}{4}\beta,$$

and $H_a\left(abc|R_1^C\right) = \overline{z}$.

Region R_2^C Suppose that a consensus government is restricted to choose a policy in region R_2^C. Then by Lemmata 1 to 3 in the second period all three parties will be represented in parliament and a consensus government will be formed with policy $\overline{z}$. This implies that the joint period-two utility of all three parties will be -1. Therefore,

$$\sum_{i=a,b,c} U^i \left(H_1 \left(abc|R_2^C \right) \right) = \max_{x' \in R_2^C} \left\{ \sum_{i=a,b,c} u^i \left(x' \right) + \beta \left(-1 \right) \right\} = \tfrac{\sqrt{3}}{2} - 2 - \beta,$$

and the maximum is attained at $F_{ab}\left(-\frac{1}{\sqrt{3}}, 0\right)$, $F_{ac}\left(-\frac{1}{\sqrt{3}}, 0\right)$ and $F_{bc}\left(-\frac{1}{\sqrt{3}}, 0\right)$. Note that only if the last policy alternative is chosen, in the second period party a will receive a majority vote share and be recognized as formateur for certain. Therefore, given our assumption of lexicographic preferences, $H_a\left(abc|R_2^C\right) = F_{bc}\left(-\frac{1}{\sqrt{3}}, 0\right)$.

Comparison Finally, it can be shown that

$$\sum_{i=a,b,c} U^i \left(H_1 \left(abc|R_1^C \right) \right) > \sum_{i=a,b,c} U^i \left(H_1 \left(abc|R_2^C \right) \right)$$

if and only if $\beta \in \left[0, \widehat{\beta}\right)$ where $\widehat{\beta} \equiv 4 - 2\sqrt{3}$. ■

Proof of Proposition 1.

Consider majoritarian government ab formed by party a in period one. The goal is to identify the $H_a(ab)$ that maximizes the joint two-period utility of parties a and b and yields party a the highest probability of being recognized as period-two formateur among all policy alteratives that maximize this joint utility.

Let $Q_{ij} \equiv \{ x \in \Re^2 : u_i(x) > u_j(x) > u_k(x) \}$ for all $i, j, k = a, b, c$, $i \neq j \neq k$, and partition the policy space into 10 regions. In the second period, a status quo in a different region will lead to a different joint expected utility of parties a and b. These regions are:

$R_1^T \equiv \left(\overline{D}^a \backslash \left(D^b \cup D^c\right)\right) \cup \left(\overline{D}^b \backslash \left(D^a \cup D^c\right)\right)$,

$R_2^T \equiv D^a \cap D^b \cap \{\mathbf{x} : u_a(\mathbf{x}) = u_b(\mathbf{x}) > u_c(\mathbf{x})\}$,

$R_3^T \equiv \Re^2 \backslash \left(\overline{D}^a \cup \overline{D}^b \cup \overline{D}^c\right),$

$R_4^T \equiv D^a \cap D^b \cap (Q_{ab} \cup Q_{ba}),$

$R_5^T \equiv D^a \cap D^b \cap \left\{\mathbf{x} : \max_{i=a,b} \{u_i(\mathbf{x})\} > \min_{i=a,b} \{u_i(\mathbf{x})\} = u_c(\mathbf{x})\right\},$

$R_6^T \equiv \left[(D^a \cap Q_{ac}) \cup \left(D^b \cap Q_{bc}\right)\right] \cap D^c,$

$R_7^T \equiv \left(D^a \cup D^b\right) \cap D^c \cap \left\{\mathbf{x} : \max_{i=a,b} \{u_i(\mathbf{x})\} = u_c(\mathbf{x}) > \min_{i=a,b} \{u_i(\mathbf{x})\}\right\},$

$R_8^T \equiv \left(D^a \cup D^b\right) \cap D^c \cap \left\{x : u^a(x) = u^b(x) < u^c(x)\right\},$

$R_9^T \equiv \{\overline{z}\}$, and

$R_{10}^T \equiv \overline{D}^c \backslash \left(D^a \cup D^b\right).$

The approach is again to characterize local maxima or suprema in these regions separately and then compare them to identify the globally optimal policy choice for party a. An optimal policy in a region R is denoted by $H_a(ab|R)$.

Region R_1^T Suppose that government ab is restricted to choose a policy from region R_1^T. In particular, suppose that a policy $x' \in \overline{D}^a \backslash \left(D^b \cup D^c\right)$ is chosen in the first period. Then by Lemma 3 and the equilibrium selection rules assumed, in the second period all three parties will be represented, and party a will receive a majority vote share.[35] By Lemmata 1 and 2 party a as formateur will form a consensus government with policy $\overline{z}$, and the joint period-two utility of parties a and b will be $(-1) - u^c(x)$. Therefore, the joint two-period utility of parties a and b is

$$\sum_{i=a,b} U^i(x') = \sum_{i=a,b} u_i(x') + \beta(-1 - u_c(x')), \tag{1}$$

[35] The equilibrium selection rules apply only if $q_1 = x'$ is on the border of $\overline{D}^a$ and outside D^b and D^c.

and

$$\max_{x'\in\overline{D}^a\setminus\left(D^b\cup D^c\right)}\sum_{i=a,b}U^i\left(x'\right)=\begin{array}{ll}\max\limits_{x'\in\Re^2} & -\left(2-\beta\right)\left\|x'-F_{ab}\left(\frac{\beta}{2-\beta},0\right)\right\|^2-\frac{(1+\beta)(1-\beta)}{2-\beta}\\ s.t. & \left\|x'-z^a\right\|^2\leq\frac{1}{2},\left\|x'-z^b\right\|^2\geq\frac{1}{2},\left\|x'-z^c\right\|^2\geq\frac{1}{2},\end{array}$$

where the objective function on the right-hand side is a simplification of equation (1), and the three constraint inequalities correspond to the constraints that $x'\in\overline{D}^a$, $x'\notin D^b$, and $x'\notin D^c$ respectively. The maximum is attained at

$$H_a\left(ab|\overline{D}^a\setminus\left(D^b\cup D^c\right)\right)=F_{ab}\left(\kappa\left(\frac{\beta}{2-\beta}\right),|\kappa-1|\left(\frac{1}{2}\right)\right),$$

where

$$\kappa\equiv\left(\frac{1}{2}\right)^{\frac{1}{2}}\left[\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\left(\frac{\beta}{2-\beta}\right)\right)^2\right]^{-\frac{1}{2}},$$

and

$$\sum_{i=a,b}U^i\left(H_1\left(ab|\overline{D}^a\setminus\left(D^b\cup D^c\right)\right)\right)=-\left(\frac{2-\beta}{2}\right)\left(\frac{1}{\kappa}-1\right)^2-\frac{(1+\beta)(1-\beta)}{2-\beta}.$$

This is a corner solution, since either the constraint $\left\|x'-z^b\right\|^2\geq\frac{1}{2}$ or that $\left\|x'-z^a\right\|^2\leq\frac{1}{2}$ is binding. Due to symmetry,

$$H_a\left(ab|\overline{D}^b\setminus\left(D^a\cup D^c\right)\right)=F_{ab}\left(\kappa\left(\frac{\beta}{2-\beta}\right),-|\kappa-1|\left(\frac{1}{2}\right)\right)$$

attains the same maximum in R_1^T. However, if this policy is chosen in the first period, by Lemma 3 and the equilibrium selection rules assumed, party a will never be recognized as period-two formateur since party b will be elected as the majority party. Therefore, by the assumption of lexicographic preferences, $H_a\left(ab|R_1^T\right)=F_{ab}\left(\kappa\left(\frac{\beta}{2-\beta}\right),|\kappa-1|\left(\frac{1}{2}\right)\right)$.

Region R_2^T Suppose that a policy $x'\in R_2^T$ is chosen in the first period. Then by Lemma 3, in the second period party c will receive a vote share of m, and both parties a and b will receive $\frac{1-m}{2}$. As a consequence, with probability m party c will be recognized as formateur and randomize between majoritarian governments ac and bc. If, for example, a majoritarian government ac is formed, the joint period-two utility of parties a and b will be $u^a(x') + \left(-\frac{3}{4}\right)$; party a gets its reservation value since it is included in the new government, and party b gets $\left(-\frac{3}{4}\right)$ since it is excluded from the new government coalition. With probability $1-m$, either party a or b will be recognized, and the formateur will form a majoritarian coalition with party c. The joint period-two utility of parties a and b then will be $\left(-\frac{1}{2} - u^c(x')\right) + \left(-\frac{3}{4}\right)$. Therefore, the joint two-period utility of parties a and b is

$$\sum_{i=a,b} U^i(x') = \sum_{i=a,b} u^i(x') + \beta\left((1-m)\left(\left(-\frac{1}{2} - u^c(x')\right) + \left(-\frac{3}{4}\right)\right) + m\left(u^a(x') + \left(-\frac{3}{4}\right)\right)\right),$$

and

$$\begin{aligned}
&\sup_{x' \in R_2^T} \sum_{i=a,b} U_1^i(x') \\
= &\sup_{h \in \left(-\frac{1}{3}, \frac{1}{\sqrt{3}}\right)} \Phi(h) \equiv -\frac{3(2-(1-2m)\beta)}{4}\left(h - \frac{(1-m)\beta}{2-(1-2m)\beta}\right)^2 - \frac{4+2(1+4m)\beta - \left(5-8m-m^2\right)\beta^2}{4(2-(1-2m)\beta)} \\
= &\begin{cases} -\frac{4+2(1+4m)\beta - \left(5-8m-m^2\right)\beta^2}{4(2-(1-2m)\beta)}, & \text{if } \beta \in \left[0, \widehat{\beta}(m)\right), \\ -1 + \left(\frac{\left(2\sqrt{3}-1\right) - 2\left(2+\sqrt{3}\right)m}{4}\right)\beta, & \text{otherwise,} \end{cases}
\end{aligned}$$

where

$$\widehat{\beta}(m) = \begin{cases} 1, & \text{if } m \in \left[\widehat{m}, \frac{1}{4}\right), \text{ where } \widehat{m} \equiv 3\sqrt{3} - 5, \\ \frac{2}{1+\sqrt{3} - \left(2+\sqrt{3}\right)m}, & \text{otherwise.} \end{cases}$$

For all $\beta \in \left[0, \widehat{\beta}(m)\right)$, the local supremum is attained by an interior policy $F_{ab}(h^*, 0) \in$

R_2^T where $h^* \equiv \frac{(1-m)\beta}{2-(1-2m)\beta}$, and therefore a supremum is a maximum. On the other hand, for all $\beta \in \left[\widehat{\beta}(m), 1\right]$, a maximum does not exist in region R_2^T. To see this, pick any policy $F_{ab}(h,0) \in R_2^T$ and define $h(\varepsilon) \equiv \varepsilon h^* + (1-\varepsilon)h$. Since R_2^T is an open region, $F_{ab}(h(\varepsilon),0) \in R_2^T$ for $\varepsilon > 0$ sufficiently small. Note that $\Phi(h)$ is strictly concave and $h^* = \arg\max_{h' \in \Re} \Phi(h')$. Therefore, $\sum_{i=a,b} U^i(F_{ab}(h(\varepsilon),0)) > \sum_{i=a,b} U^i(F_{ab}(h,0))$. Finally, $\sup_{x' \in R_2^T} \sum_{i=a,b} U^i(x') \leq \max_{x' \in R_1^T} \sum_{i=a,b} U^i(x')$ for all $\beta \in [\beta^*(m), 1]$ and all $m \in \left(0, \frac{1}{4}\right)$, where $\beta^*(m)$ is a decreasing function in m and $\beta^*(m)h \in \left(0, \widehat{\beta}(m)\right)$ for all m. This can be verified by comparing the functional forms of local maxima and/or suprema in regions R_1^T and R_2^T. The function $\beta^*(m)$ will be characterized in the last part of the proof.

Region R_3^T Suppose that a policy $x' \in R_3^T$ is chosen in the first period. Then by Lemmata 1 to 3, in the second period all three parties will be represented, each party will be recognized as formateur with probability one-third (which is the probability they perceive before the period-two election), and a consensus government will be formed. Therefore, with probability one-third, party c will be recognized and the joint period-two utility of parties a and b will be $u^a(x') + u^b(x')$ because both of them will be included in the consensus coalition and receive their period-two reservation values. With probability two-thirds, either party a or b will be recognized, and their joint period-two utility will be $(-1) - u^c(x')$, which is the joint utility of all three parties (that is, -1) net of party c's reservation value. Thus,

$$\sum_{i=a,b} U^i(x') = \sum_{i=a,b} u^i(x') + \tfrac{2}{3}\beta\left((-1) - u^c(x')\right) + \tfrac{1}{3}\beta \sum_{i=a,b} u^i(x'),$$

and

$$\sup_{x' \in R_3^T} \sum_{i=a,b} U^i(x') = \sup_{x' \in \Re^2} \quad -2 \left\| x' - F_{ab}\left(\tfrac{\beta}{3}, 0\right) \right\|^2 - \tfrac{1}{6}(1+\beta)(3-\beta)$$

$$s.t. \quad \|x' - z^a\|^2 > \tfrac{1}{2}, \left\|x' - z^b\right\|^2 > \tfrac{1}{2}, \|x' - z^c\|^2 > \tfrac{1}{2}.$$

The supremum is $-1+\frac{1}{3}\left(\sqrt{3}-1\right)\beta$, which is strictly less than $\sum_{i=a,b} U^i\left(H_1\left(ab|R_1^T\right)\right)$ for all m and all β. Therefore, $H_a(ab) \notin R_3^T$.

Regions R_4^T to R_{10}^T The procedures to characterize local maxima (or suprema) in regions R_4^T to R_{10}^T are similar to those for regions R_1^T, R_2^T and R_3^T. To save space, we only summarize the final utility calculations:

$$\sup_{x' \in R_4^T} \sum_{i=a,b} U^i(x') = -\tfrac{64+64\beta-47\beta^2}{32(4-\beta)},$$

$$\max_{x' \in R_5^T} \sum_{i=a,b} U^i(x') = -\tfrac{40+48\beta-7\beta^2}{8(8-\beta)},$$

$$\sup_{x' \in R_6^T} \sum_{i=a,b} U^i(x') = -\tfrac{5+6\beta}{8},$$

$$\sup_{x' \in R_7^T} \sum_{i=a,b} U^i(x') = -\tfrac{16+(19-2m)\beta}{24},$$

$$\sup_{x' \in R_8^T} \sum_{i=a,b} U^i(x') = -\tfrac{16+19\beta}{24},$$

$$\sum_{i=a,b} U^i(\overline{z}) = -\tfrac{4+5\beta}{6}, \; (R_9^T = \{\overline{z}\}), \text{ and}$$

$$\max_{x' \in R_{10}^T} \sum_{i=a,b} U^i(x') = -(1+\beta).$$

All these local maxima or suprema can be verified to be strictly smaller than $\sup_{x' \in R_2^T} \sum_{i=a,b} U_i^i(x')$ for all m and all $\beta \neq 0$. Therefore, $H_a(ab) \notin \bigcup_{r=4}^{10} R_r^T$.

Comparison This analysis has shown that $H_a(ab) \notin R_r^T$ for $r = 3, 4, ..., 10$.

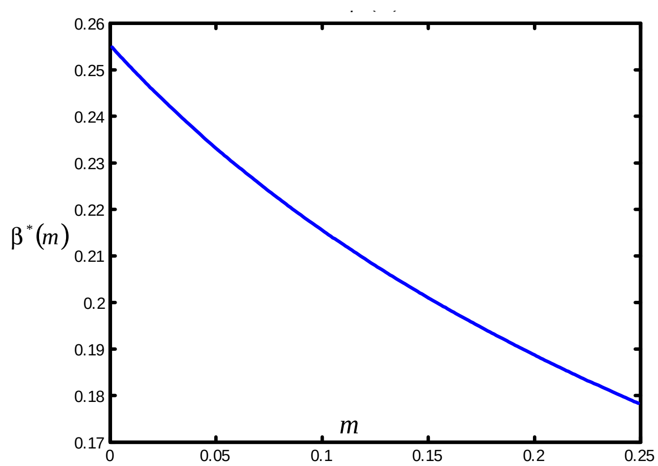

Figure 6: $\beta^*(m)$.

The analysis of local maxima in regions R_1^T and R_2^T also implies that for all $m \in \left(0, \frac{1}{4}\right)$,

$$H_a(ab) = \begin{cases} F_{ab}\left(\frac{(1-m)\beta}{2-(1-2m)\beta}, 0\right), & \text{if } \beta \in [0, \beta^*(m)), \\ F_{ab}\left(\kappa\left(\frac{\beta}{2-\beta}\right), |\kappa - 1|\left(\frac{1}{2}\right)\right), & \text{if } \beta \in [\beta^*(m), 1]. \end{cases}$$

Characterization of $\beta^*(m)$ Consider the claim that $\sup_{x' \in R_2^T} \sum_{i=a,b} U^i(x') \leq \max_{x' \in R_1^T} \sum_{i=a,b} U^i(x')$ for all $\beta \in [\beta^*(m), 1]$ and all $m \in \left(0, \frac{1}{4}\right)$, where $\beta^*(m)$ is a decreasing function in m and $\beta^*(m) \in \left(0, \widehat{\beta}(m)\right)$ for all m. To show this, first of all, observe that for all m and all $\beta \in \left[\widehat{\beta}(m), 1\right]$,

$$\begin{aligned} & \sum_{i=a,b} U^i\left(H_a\left(ab|R_1^T\right)\right) - \lim_{x' \to H_a\left(ab|R_2^T\right)} \sum_{i=a,b} U^i(x') \\ = \; & \left[\left(1 + \frac{\sqrt{3}}{2}\right) m - \frac{\sqrt{3}}{2} + \frac{3}{4}\right] \beta + \sqrt{2\left(\beta - \frac{1}{2}\right)^2 + \frac{3}{2}} - 1 > 0. \end{aligned}$$

Therefore, $\beta^*(m) < \widehat{\beta}(m)$. Second, for all $m \in \left(0, \frac{1}{4}\right)$ and all $\beta \in \left[0, \widehat{\beta}(m)\right)$,

$$\begin{aligned} & \sum_{i=a,b} U^i\left(H_a\left(ab|R_1^T\right)\right) \geq \lim_{x' \to H_a\left(ab|R_2^T\right)} \sum_{i=a,b} U^i(x') \\ \Leftrightarrow \quad & 4(2-(1-2m)\beta)\sqrt{2\left(\beta - \tfrac{1}{2}\right)^2 + \tfrac{3}{2}} \geq \left(7 - 12m - m^2\right)\beta^2 - 2(7-4m)\beta + 12, \end{aligned}$$

which is equivalent to

$$\begin{aligned} \Omega(\beta, m) \quad \equiv \quad & -\left(17 - 40m + 2m^2 + 24m^3 + m^4\right)\beta^4 + 4\left(9 - 16m + 9m^2 + 4m^3\right)\beta^3 \\ & -4\left(19 - 32m - 22m^2\right)\beta^2 + 16(5+4m)\beta - 16 \\ \geq \quad & 0. \end{aligned}$$

Note that for all $m \in \left(0, \frac{1}{4}\right)$, (1) $\Omega(0, m) < 0$, (2) $\lim_{\beta \to \infty} \Omega(\beta, m) < 0$, (3) $\Omega(1, m) = 7 + 168m + 122m^2 - 8m^3 - m^4 > 0$, and (4) $\Omega(\beta, m) = 0$ is a biquadratic equation in β with four roots. There are standard procedures of solving a biquadratic equation, and it can be verified that (5) two of its roots are real and the other two are imaginary. Call the two real roots $\beta_1^*(m)$ and $\beta_2^*(m)$ such that $\beta_1^*(m) \leq \beta_2^*(m)$. By (1), (2), (3) and (5), it follows that $0 < \beta_1^*(m) < 1 < \beta_2^*(m)$, and for all $\beta \in [\beta_1^*(m), 1]$, $\Omega(\beta, m) \geq 0$ and therefore $\sum_{i=a,b} U^i\left(H_a\left(ab|R_1^T\right)\right) \geq \lim_{x' \to H_a\left(ab|R_2^T\right)} \sum_{i=a,b} U^i(x')$. Then, define $\beta^*(m) = \beta_1^*(m)$ and $\beta^*(m) > 0$. Finally, it can be verified that the relationship between $\beta^*(m)$ and m is as shown in Figure 6. ■

Proof of Proposition 3. Consider a single-party government formed by party c in the first period. Party c chooses a policy to maximize its expected discounted sum of utilities. To analyze this maximization problem, partition the policy space into two regions: $R_1^S \equiv \overline{D}^a \backslash \left(D^b \cup D^c\right)$ and $R_2^S \equiv \Re^2 \backslash R_1^S$.

Suppose that party c is restricted to choose a policy x' from the set of R_1^S. Then in the second period the parliamentary election leads to a majority parliament, and the majority party c forms a consensus government with policy $\overline{z}$. This implies that party c's expected discounted sum of utility is

$$\begin{aligned} U^c(x') &= u^c(x') + \beta\left[(-1) - u^b(x') - u^c(x')\right] \\ &= \tfrac{3}{4}(2\beta-1)h^2 - \tfrac{3}{2}h + (2\beta-1)w^2 - \tfrac{3}{4} - \tfrac{1}{2}\beta, \end{aligned}$$

where $h, w \in \Re$ are such that $F(h,w) = x'$. The first-order condition for h is

$$\frac{\partial U^c}{\partial h} = \tfrac{3}{2}\left[(2\beta-1)h^2 - 1\right] \leq 0.$$

For $\beta \geq \frac{1}{2}$, the policy is as extreme as possible while still leading to a consensus government in the second period formed by c. That is, $h^* = \widehat{h} \equiv -1-\sqrt{\frac{2}{3}}$. For $\beta < \frac{1}{2}$, the maximum is attained at an interior solution $h^* = -\frac{1}{1-2\beta}$ if $\beta \leq \beta^o \equiv \frac{1}{2}\left(\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)$, and at a corner solution $h^* = \widehat{h}$ otherwise.

Suppose that party c is restricted to choose a policy x' from the set of R_2^S. Then compared to the case with a policy choice in R_1^S, the probability that party a is recognized as period-two formateur substantially dropped from one to below one-half. Therefore, party a loses some of its expected utility in the second period. At the same time, by choosing a policy outside R_1^S, party a makes the policy farther away from its ideal point and thus lowers its period-one utility. Thus, in equilibrium party a does not choose a policy in region R_2^S. $\blacksquare$

B Proof of Proposition 4

We first show that this proposition is true for $q_0 = \overline{z}$. Suppose that all three parties are represented in parliament and party a has been selected as the formateur. We claim that, forming a consensus government is a strictly dominated strategy for party a, regardless of the values of β and m.

If forming majoritarian government ab, party a as formateur would derive a two-period utility of

$$W^a(ab, \overline{z}) = \sum_{i=a,b} U^a(H_a(ab)) - U^b(\overline{z}).$$

If forming a consensus government, party a as formateur would derive a two-period utility of

$$W^a(abc, \overline{z}) = \sum_{i=a,b,c} U^i(H_a(abc)) - \sum_{i=b,c} U^i(\overline{z}).$$

The reservation value of any party i given the initial status quo $\overline{z}$ is calculated as $U^i(\overline{z}) = -\frac{1}{3}\left(1 + \frac{5}{4}\beta\right)$. Note that $W^a(ab, \overline{z})$ and $W^a(abc, \overline{z})$ are functions of β and m. It can be verified that for all $\beta \in [0, 1]$ and all $m \in \left(0, \frac{1}{4}\right)$, $W^a(ab, \overline{z}) - W^a(abc, \overline{z}) > 0$. This proceeds with three cases: (1) $\beta \in [0, \beta^*(m))$, (2) $\beta \in \left[\beta^*(m), \widehat{\beta}\right)$, and (3) $\beta \in \left[\widehat{\beta}, 1\right]$.

Consider the first case. By Proposition 2, $H_a(abc) = \overline{z}$. Therefore, $W^a(abc, \overline{z}) = 0$. By Proposition 1, $H_a(ab) = \left(-\frac{\sqrt{3}(1-m)\beta}{2(2-(1-2m)\beta)}, 0\right)$. The joint period-one utility of parties a and b is thus

$$\sum_{i=a,b} u^i(H_a(ab)) = 2\left[-\left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}(1-m)\beta}{2(2-(1-2m)\beta)}\right)^2\right].$$

Given $H_a(ab)$ as the period-two status quo, with probability $\frac{1-m}{2}$, formateur a forms a majoritarian government with c with policy z^{ac} and derives a utility of

$$\sum_{i=a,c} u^i(z^{ac}) - u^c(H_a(ab)) = \left(-\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}(1-m)\beta}{2(2-(1-2m)\beta)}\right)^2.$$

In that case, party b as the period-two out party derives a utility of $u^b(H_a(ab)) = -\frac{3}{4}$.

The situation is symmetric if b is the period-two formateur. With probability m, party c as formatuer forms a majoritarian government with either a or b and the joint period-two utility of parties a and b is equal to

$$u^a\left(H_a\left(ab\right)\right)+u^b\left(z^{ac}\right)=\left[-\left(\tfrac{1}{2}\right)^2-\left(\tfrac{\sqrt{3}(1-m)\beta}{2(2-(1-2m)\beta)}\right)^2\right]+\left(-\tfrac{3}{4}\right).$$

The joint continuation value of the parties is calculated as

$$\sum_{i=a,b}v^i\left(H_a\left(ab\right)\right)=(1-m)\left[\left(-\tfrac{1}{2}\right)+\left(\tfrac{\sqrt{3}}{2}+\tfrac{\sqrt{3}(1-m)\beta}{2(2-(1-2m)\beta)}\right)^2\right]-m\left(1+\left(\tfrac{\sqrt{3}(1-m)\beta}{2(2-(1-2m)\beta)}\right)^2\right).$$

If the period-two status quo is $\overline{z}$, by symmetry, the joint continuation value of parties a and b is

$$\sum_{i=a,b}v^i\left(\overline{z}\right)=\tfrac{2}{3}.$$

Given that $W^a\left(abc,\overline{z}\right)=0$, the rest is to verify that

$$\sum_{i=a,b}u^i\left(H_a\left(ab\right)\right)+\beta\sum_{i=a,b}v^i\left(H_a\left(ab\right)\right)-\left(\sum_{i=a,b}u^i\left(\overline{z}\right)+\beta\sum_{i=a,b}v^i\left(\overline{z}\right)\right)>0,$$

which is purely algebra. Calculations for the Cases (2) and (3) can be done in the same way. See Figure 7 for an illustration of $W^a\left(ab,\overline{z}\right)-W^a\left(abc,\overline{z}\right)$ for $m=10\%$.

Therefore, regardless of the parameter values and given that $q_0=\overline{z}$, party a as formateur will not form a consensus government in the first period. By symmetry, no party as period-one formateur will form a consensus government. As a consequence, given that a minority parliament is elected, no matter how seat shares are distributed, some majoritarian government forms in period one with an extreme policy that falls outside the stage game Pareto set of the parties. To show that any period-one electoral equilibrium must lead to a minority parliament, the proof of Proposition 4 in Baron

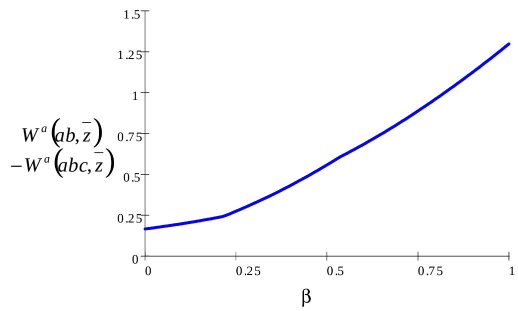

Figure 7: The difference of two-period utilities of party a derived from forming majoritarian government ab and a consensus government, given $m = 10\%$.

and Diermeier (2001) applies, with the parties' utility functions replaced by U^a, U^b, and U^c.

Finally, two observations allow us to generalize the result to any initial status quo q_0 sufficiently close to $\overline{z}$. First, for any $i \in \{a, b, c\}$ and any coalition $C \subseteq \{a, b, c\}$ such that $i \in C$, $W^i(C, q_0)$ is continuous in q_0, holding β and m constant. Second, for any distinct $i, j \in \{a, b, c\}$, $W^i(ij, \overline{z}) - W^i(abc, \overline{z})$ is *strictly* positive, holding β and m constant.

References

[1] Aghion, Philippe and Patrick Bolton. 1990. "Government and the Risk of Default: A Political-economic Model of Strategic Role of Debt." In Rudi Dornbusch and Mario Draghi, eds., *Public Debt Management: Theory and History*, pp.121-145. Cambridge University Press.

[2] Austen-Smith, David and Jeffrey S. Banks. 1988. "Elections, Coalitions, and Legislative Outcomes." *American Political Science Review*, 82(2): 405-422.

[3] Baron, David P. 1996. "A Dynamic Theory of Collective Goods Programs." *American Political Science Review*, 90(2): 316-330.

[4] Baron, David P. and Michael Herron. 2003. "A Dynamic Model of Multidimensional Collective Choice." In Kollman, Ken, John H. Miller and Scott E. Page eds., *Computational Models of Political Economy,* pp.13-47. MIT Press.

[5] Baron, David P. and Daniel Diermeier. 2001. "Elections, Governments, and Parliaments in Proportional Representation Systems." *Quarterly Journal of Economics*, 116(3): 933-967.

[6] Battaglini, Marco and Stephen Coate. 2007. "Inefficiency in Legislative Policy-Making: A Dynamic Analysis." *American Economic Review*, 97(1): 118-149.

[7] Battaglini, Marco and Stephen Coate. 2008. "A Dynamic Theory of Public Spending, Taxation and Debt." *American Economic Review*, 98(1), 201-236.

[8] Bawn, Kathleen. 1999. "Voter Responses to Electoral Complexity: Ticket Splitting, Rational Voters and Representation in the Federal Republic of Germany." *British Journal of Political Science*, 29(3): 487-505.

[9] Besley, Timothy and Stephen Coate. 1998. "Sources of Inefficiency in a Representative Democracy." *American Economic Review*, 88(1): 139-156.

[10] Bowen, Rene and Zaki Zahra. 2007. "On Dynamic Compromise." Manuscript.

[11] Budge, Ian, Hans-Dieter Klingemann, Andream Volkens and Judith Bara. 2001. *Mapping Policy Preferences: Estimates for Parties, Electors, and Governments 1945-1998*. Oxford University Press.

[12] Cho, Seok-Ju. 2008. "Three-Party Competition in Parliamentary Democracy with Proportional Representation." Manuscript.

[13] Cox, Gary W. 1997. *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. Cambridge University Press.

[14] Diermeier, Daniel, Hülya Eraslan and Antonio Merlo. 2003. "A Structural Model of Government Formation." *Econometrica*, 71: 27-70.

[15] Daniel, Diermeier and Timothy J. Feddersen. 1998. "Cohesion in Legislatures and the Vote of Confidence Procedure." *American Political Science Review*, 92(3): 611-621.

[16] Diermeier, Daniel and Antonio Merlo. 2000. "Government Turnover in Parliamentary Democracies." J*ournal of Economic Theory*, 94: 46-79.

[17] Diermeier, Daniel and Antonio Merlo. 2004. "An empirical investigation of coalitional bargaining procedures." *Journal of Public Economics*, 88: 783-797.

[18] Drazen, Allan. 2000. *Political Economy in Macroeconomics.* Princeton University Press.

[19] Duggan, John and Kalandrakis, Tasos. 2007. "A Dynamic Model of Legislative Bargaining." Manuscript.

[20] Fong, Pohan. 2006. "Dynamics of Government and Policy Choice." Manuscript.

[21] Kalandrakis, Anastassios. 2004. "A Three-Player Dynamic Majoritarian Bargaining Game." *Journal of Economic Theory*, 16: 294-322.

[22] Kalandrakis, Anastassios. 2007. "Majority Rule Dynamics with Endogenous Status Quo." Manuscript.

[23] Laver, Michael and Norman Schofield. 1990. *Multiparty Governments.* Oxford University Press.

[24] Lizzeri, Alessandro and Persico, Nicola, 2001. "The Provision of Public Goods under Alternative Electoral Incentives." *American Economic Review*, 91: 225-239.

[25] Milesi-Ferretti, Gian Maria, Roberto Perotti, and Massimo Rostagno. 2002. "Electoral Systems and Public Spending." *Quarterly Journal of Economics*, 117(2): 609-657.

[26] Pagano, Marco and Paolo F. Volpin. 2006. "The political economy of corporate governance." *American Economic Review*, 95(4): 1005-1030.

[27] Persson, Torsten and Lars Svensson. 1989. "Why a Stubborn Conservative Would Run a Deficit: Policy with Time-inconsistency Preferences." *Quarterly Journal of Economics*, 104: 325-345.

[28] Persson, Torsten and Guido Tabellini. 1999. "The size and scope of government: Comparative politics with rational politicians." *European Economic Review*, 43: 699-735.

[29] Persson, Torsten and Guido Tabellini. 2000. *Political Economics: Explaining Economic Policies*. MIT Press.

[30] Persson, Torsten and Guido Tabellini. 2003. *The Economic Effects of Constitutions*. MIT Press.

[31] Persson, Torsten, Gerard Roland, and Guido Tabellini. 2000. "Comparative Politics and Public Finance." *Journal of Political Economy*, 108(6): 1121-1161.

[32] Persson, Torsten, Gerard Roland, and Guido Tabellini. 2007. "Electoral Rules and Government Spending in Parliamentary Democracies." *Quarterly Journal of Political Science*, 2: 155-188.

[33] Schofield, Norman and Itai Sened. 2006. *Multiparty Democracy: Elections and Legislative Politics*. Cambridge University Press.

[34] Strom, Kaare. 1990. *Minority Government and Majority Rule.* Cambridge, UK: Cambridge University Press.

[35] Tabellini, Guido and Alberto Alesina. 1990. "Voting on the Budget Deficit." *American Economic Review*, 80(1): 37-49.