Optimal Deadlines for Agreements

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Abstract. We provide a welfare analysis of the deadline effect in a repeated negotiation game in which costly delay can produce information that improves the quality of the decision. We characterize equilibrium strategies and the evolution of beliefs in continuous time, and study how the length of the negotiation horizon affects players' behavior and welfare. The optimal deadline is positive if and only if the ex ante probability that the players disagree on the preferred decision is neither too high nor too low. When it is positive, the optimal deadline is given by the shortest time that would allow efficient information aggregation in equilibrium, which is increasing in the ex ante probability of disagreement and is finitely long.

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1. Introduction

When disagreements are resolved through negotiations, the time horizon of the negotiation process may influence the final outcome. In the classical finite-horizon, alternating-offer bargaining game of Stahl (1972), deadlines affect the way players make and accept bargaining demands through the logic of backward induction, even though the deadlines are never reached in equilibrium. In war of attrition games (e.g., Hendricks, Weiss and Wilson 1988), conflicts are gradually resolved with the passage of time. The presence of a deadline not only affects equilibrium behavior along the path, but can also determine the equilibrium outcome by imposing a default decision upon the arrival of the deadline. In both the bargaining and war of attrition models, the negotiating parties disagree because they have opposing preferences over the outcome. In such situation of pure conflict, negotiation may determine the distribution of payoffs between the parties but not their sum, thus protracted negotiation is invariably wasteful, as it introduces costly delay without any benefits. However, when the disagreement is driven by different private information, and could be overcome after information-sharing, protracted negotiation can have positive welfare consequences by facilitating information aggregation. This paper studies the welfare effects of negotiation deadlines in an environment where the negotiating parties disagree both because of diverging preferences and because of different information, and characterizes the deadline that optimally balances the cost of strategic delay and the benefit of strategic information aggregation.

Consider an emerging industry with two dominant firms trying to establish a common standard. A firm whose current standard has a high value to the industry will want to insist on its standard, while a firm with a low-value standard is willing to adopt the other firm's standard only if it is sufficiently convinced that the latter is high-valued. Since firms with low-value standards may want to claim the opposite, sharing private information about the value of their own standards can be impossible if the decision needs to be made without delay. This leads to failure in agreeing to a common standard and welfare loss when the two firms would have agreed to the high-value standard had they shared their private information. If instead the two firms commit to engaging each other repeatedly in reaching an agreement, the cost in delaying the decision can discourage them from exaggerating the value of their own standards, and generate endogenous information that in equilibrium helps improve the quality of the standards adoption decision. The familiar logic of backward induction suggests that the presence of a negotiation deadline affects incentives for a firm with a low-value standard to continue to insist on its standard or agree to switch to the rival's standard. Indeed, from the ex ante perspective the length of the deadline determines in equilibrium the trade-off between the payoff loss due to delay and the improvement in the quality of the decision. This trade-off is the basis of our welfare analysis of the negotiation deadline.[1]

We model negotiation under a deadlines as a symmetric, continuous time repeated proposal game.[2] At any instant two players simultaneously choose one of two choices to propose, paying a flow cost of delay, until either they agree, at which point the agreement is implemented, or the deadline expires and a random decision is made. The two players favor different choices: each is willing to go along with the other player's favorite choice only if he is sufficiently convinced that the state is an agreement state supporting that choice. At any point of the game, each player is either privately "informed," meaning that he knows the state is the agreement state corresponding to his favorite; or "uninformed," meaning that he is unsure whether the state is the agreement state corresponding to his opponent's favorite, or the state is the disagreement state with each player preferring his own favorite choice. Thus, in each agreement state we have one informed player playing against an uninformed player, where the latter based on his own private information may disagree with the former over the decision but would in fact agree if he had all the information. This is the only essential feature for deadlines to have interesting welfare effects, and is captured in the simplest manner by the above assumptions on the information and preference structures. Our way of modeling the deadline is equally stylized: it stops the game with a coin toss

[1] Although in many situations of industry standard adoption negotiations are open-ended, we argue in this paper that there may be potential welfare improvement to impose a binding deadline. Another example fitting the description of our model is bargaining over child custody in divorce settlements. Sometimes parents might be able to agree which of them is the more suitable custodian for the child if they could share their private information. However, because of the private benefits from being the custodian, even an unsuitable parent may want to insist on claiming custody if he or she is not sure that the other parent is more suitable. In this example, negotiation deadlines are commonly imposed by law.

[2] See Farrell (1996) for a related model of standard adoption, with a richer type space. He does not consider deadlines.

at some fixed future date if there is no agreement yet. We thus abstract from modeling the details of implementing a deadline default to focus on the most salient feature of the ex ante commitment to short circuiting the information aggregation process.

We show that generically there is a unique equilibrium in which the informed types always "persist" by proposing their favorite alternative. The equilibrium is symmetric, with an uninformed type's behavior depending on the time left before the expiration of the deadline and on his belief that the state is the disagreement state. For each belief of the uninformed, there exists a critical time horizon such that if the time to deadline is shorter than that horizon, the uninformed type also persists, until the deadline is reached when he may "concede" to the opponent's favorite alternative with a positive probability. This intuitive deadline play consists of a "persistence phase" until the deadline arrives, followed by the same outcome as when the players must decide without delay at the start of the deadline play. If the time to deadline exceeds the critical horizon, the uninformed concedes at some probability flow rate. This continuous-time version of randomization between conceding and persisting results because the deadline is too long for the uninformed to persist all the way, but at the same time conceding with a strictly positive probability would give the opposing uninformed type incentives to persist just a little longer. Since the informed types always persist, in this "concession phase" the Pareto-efficient decision is made with a positive probability in an agreement state. As the negotiation game continues during the concession phase, the uninformed's belief about the disagreement state continuously falls because, given the equilibrium strategies, he infers his opponent's failure to concede as evidence that the opponent is an informed type. When the time remaining reaches the critical horizon, the concession phase ends and the deadline play takes over.

Extending the deadline only hurts both the informed and the uninformed if the starting point is shorter than the critical time horizon corresponding to the initial belief: it increases the delay without changing the equilibrium play when the deadline arrives. On the other hand, starting from any deadline beyond the critical time horizon, an extension does not change the welfare of the uninformed (whose equilibrium payoff is pinned down by the payoff from concession that does not vary with the length of the deadline), but generally affects the welfare of the informed in two ways. First, by prolonging the concession phase of the negotiation, it increases the chances that the informed gets his favored decision at the cost of longer delay. Second, longer disagreement during the concession phase convinces the uninformed that state is more likely to be the agreement state, so that he may potentially change his the equilibrium play at the deadline by conceding to his opponent.[3] We show that when the uninformed initially has a low belief that the state is the disagreement state, he will concede with probability one upon the arrival of the deadline. Since the uninformed player will eventually concede, a longer concession phase is bad for the informed player by raising the delay cost. The opposite is true for high initial beliefs. In this case, the uninformed will concede with probability zero upon the deadline. Therefore a longer concession phase in the negotiation is good for the informed player by increasing the chance that the agreed decision is his favorite choice.

We provide a complete characterization of the "optimal deadline" that maximizes the ex ante probability-weighted sum of expected payoffs of the players. Naturally, the optimal deadline is zero when the initial belief of the uninformed about the disagreement state is sufficiently low, as the two players can reach the Pareto-efficient decision without delay. For intermediate initial beliefs of the uninformed, the optimal deadline is such that after the shortest concession phase the uninformed persists until the deadline and then concedes with probability one. Thus, the optimal deadline is the shortest time length that achieves efficient information aggregation in equilibrium. This deadline effectively balances the tradeoff between avoiding wasteful delay when disagreements are of fundamental nature, and allowing the parties sufficient time to successfully reconcile disagreements driven by different information. When positive, the optimal deadline is necessarily finite, because given that the uninformed concedes with probability one at the deadline, extending it further would only hurt the informed by unnecessarily prolonging the concession phase. Further, it is bounded away from zero, because the deadline has to be long enough for the uninformed to have the incentive to concede with probability one after the concession and persistence phases. Finally, when positive, the optimal deadline is increasing in the

[3] When the deadline is sufficiently long, with probability one the uninformed types concede before the deadline arrives. In this case, the deadline is not binding, and an extension of the deadline has no welfare effect.

initial belief of the uninformed, because it takes longer to drive the uninformed player's belief down to a level at which he would be willing to concede upon the deadline. When the uninformed has a sufficiently high belief about the disagreement state, the optimal deadline is again zero. The positive welfare effects from information aggregation, obtained by extending the deadline beyond the critical horizon, are not sufficient to compensate the large payoff loss associated with the long deadline play.

There is a sizable theoretical literature in war of attrition and bargaining games on the "deadline effect," the idea that players make no attempt at reaching an agreement just before the deadline, but when the deadline arrives there are sudden attempts to resolve their differences.[4] Hendricks, Weiss and Wilson (1988) characterize mixed-strategy Nash equilibria of a continuous time, complete information war of attrition game, in which there is a mass point of concession at the deadline and no concession in a time interval preceding it. Spier (1992) shows that in pretrial negotiations with incomplete information, the settlement probability is U-shaped. Ma and Manove (1993) find strategic delay in bargaining games with complete information by assuming that there may be exogenous, random delay in offer transmission. As early offers are rejected and the deadline approaches, there is an increasing risk of missing the deadline and negotiation activities pick up. Also in a bargaining game with complete information, Fershtman and Seidmann (1993) introduce the assumption that, by rejecting an offer, players commit to not accepting poorer offers in the future. They show that when players are sufficiently patient, there is a unique subgame perfect equilibrium in which players wait until the deadline to reach an agreement. Ponsati (1995) studies a war of attrition game in which each player has private information about his payoff loss incurred by conceding to the opponent and must choose the timing of concession. She shows that there is a unique pure strategy equilibrium in which both players never concede before the deadline is reached if their payoff losses are sufficiently large. Sandholm and Vulkan (1999) consider a bargaining game in which two players make offers continuously and an agreement is reached as soon as the offers are compatible with

[4] See also Roth, Murnighan and Shoumaker (1988) for an experimental investigation of eleventh hour agreements in bargaining. In the auction literature, "sniping" refers to bidding just before the auction closes. This has been analyzed by Roth and Ockenfels (2002).

each other. The only private information a player has is the deadline he faces. They show that the only equilibrium is each player persisting by demanding the whole pie until the deadline and then switching to concede everything to his opponent. Finally, Yildiz (2004) shows that when players in a bargaining game are overly optimistic about their bargaining power at the deadline, it is an equilibrium to persist until close to the deadline to reach an agreement. However, when there is uncertainty about when the deadline arrives, the deadline effect disappears.

Broadly consistent with the above papers, we offer a theory of the deadline effect, but because our theory is based on asymmetric information about common values in negotiations under a deadline, we are able to provide a welfare analysis of the deadline. Section 2 presents the benchmark continuous-time repeated proposal game. Section 3 contains analysis of the game with no deadline. This analysis corresponds to the equilibrium play in the concession phase in a game with a finite deadline, and are building blocks for the main results. Section 4 is the main section of the paper. We first construct a symmetric equilibrium that displays the deadline effect, and then prove that it is unique. A complete characterization of the optimal deadline for the benchmark game is then given as the main result of the paper. Two extensions of the benchmark model are briefly analyzed in Section 5. The first one reconsiders the game with no deadlines abut allows for exogenous probabilistic negotiation breakdowns; the second assumes that the players have to pay a penalty if the decision is reached at the deadline. The extensions provide further insights to the deadline effect, and demonstrate that our welfare analysis of deadlines is robust. Section 6 concludes with remarks about future directions for the present line of research.

2. A Repeated Proposal Game

Two players, called L and R, have to make a joint choice between two alternatives, l and r. We refer to l as the favorite alternative of L; and r the favorite of R. There are three possible states of the world: L, M, and R. Both state L and state R are "agreement states," in which the mutually preferred alternative is l in state L and r in state R. State M is the "disagreement state," in which player L prefers l and player R prefers r. For player L, the payoffs in state L are $\overline{\pi}_F$ if l is chosen and $\underline{\pi}_F$ if r is chosen, with $\overline{\pi}_F > \underline{\pi}_F$;

the payoffs in state R are $\overline{\pi}_D$ if r is chosen and $\underline{\pi}_D$ if l is chosen, with $\overline{\pi}_D > \underline{\pi}_D$; and the payoffs in state M are $\overline{\pi}_M$ if l is chosen and $\underline{\pi}_M$ if r is chosen, with $\overline{\pi}_M > \underline{\pi}_M$. The payoffs to player R are symmetrically defined.

Each player is either "informed" or "uninformed." If informed, player L knows that the state is L. If uninformed, he knows that the state is either M or R. The information structure for player R is symmetric. Let $\gamma^0 < 1$ be the common belief of the uninformed types that the state is M; we assume that it is common knowledge. The prior probability that the state is the disagreement state M is then given by $\gamma^0/(2-\gamma^0)$. The prior probability of state L and the prior probability of state R are the same, and are both equal to $(1-\gamma^0)/(2-\gamma^0)$.

The repeated proposal game is modeled in continuous time, running from $t = 0$ to the deadline T. We allow T to be infinite. The two players simultaneously propose l or r at each instant t, until the game ends. The game may end before the deadline if the two proposals by the two players agree, in which case the agreed alternative is implemented immediately. If instead the deadline T is reached, the game ends with the decision made by a fair coin flip. Until the game ends, each player incurs an additive payoff loss due to delay at a flow rate of δ.

As mentioned in the introduction, the above preference and information structures are the simplest to capture the essential idea that players in a negotiation disagree over the joint decision based on their private information but would agree if their information were public. In particular, based on his own initial private information the uninformed player L strictly prefers his favorite choice l if

$$\gamma^0 > \gamma_* \equiv \frac{\overline{\pi}_D - \underline{\pi}_D}{\overline{\pi}_D - \underline{\pi}_D + \overline{\pi}_M - \underline{\pi}_M},$$

although the state may be the agreement state R. Note that $\gamma_* \in (0,1)$ by assumption. An initial belief γ^0 about the disagreement state M higher than γ_* means that there is a great degree of conflict between the two players. Symmetrically, an uninformed player R strictly prefers r to l if $\gamma^0 > \gamma_*$, but would agree with player L if the latter is known to be informed (so the state is L). Further, we assume throughout the paper that the benefit of implementing his favorite alternative for each player is at least as large in the corresponding agreement state as in the disagreement state.

Assumption 1. $\overline{\pi}_F - \underline{\pi}_F \geq \overline{\pi}_M - \underline{\pi}_M$.

The above assumption ensures that the informed types have greater incentives to insist on their favorite alternative than the uninformed types. This allows us to focus on a particularly simple, and natural, equilibrium. It will be clear from our analysis that this assumption is sufficient but not necessary.

Our modeling of the deadline amounts to specifying state-contingent default payoffs if the last attempt at an agreement fails. To see this, note that when $T = 0$ our model reduces to a static game in which each player can propose either l or r, and the outcome is that an agreement is implemented immediately and a disagreement results in a decision made by a coin flip. When the belief γ of the uninformed that the state is M is strictly higher than γ_*, this game has a unique equilibrium with each player proposing his favorite alternative. The equilibrium outcome is a coin flip, as the degree of conflict is too large to allow any information sharing.[5] For any belief of the uninformed $\gamma < \gamma_*$, there is a unique equilibrium in which the informed types propose their own favorite and the uninformed propose the favorite alternative of their opponent. At $\gamma = \gamma_*$, there is a continuum of equilibria, in which the informed always propose their favorite while the uninformed propose their favorite with a probability between 0 and 1. Denoting as $U^0(\gamma)$ and $V^0(\gamma)$ the equilibrium payoffs of the uninformed and informed types respectively, we have

$$U^0(\gamma) \begin{cases} = \frac{1}{2}\gamma(\overline{\pi}_M + \underline{\pi}_M) + (1-\gamma)\overline{\pi}_D & \text{if } \gamma \in [0, \gamma_*), \\ \in \left[\gamma_* \underline{\pi}_M + (1-\gamma_*)\overline{\pi}_D, \frac{1}{2}\gamma_*(\underline{\pi}_M + \overline{\pi}_M) + (1-\gamma_*)\overline{\pi}_D\right] & \text{if } \gamma = \gamma_*, \\ = \frac{1}{2}\gamma(\overline{\pi}_M + \underline{\pi}_M) + \frac{1}{2}(1-\gamma)(\overline{\pi}_D + \underline{\pi}_D) & \text{if } \gamma \in (\gamma_*, 1]; \end{cases} \tag{1}$$

and

$$V^0(\gamma) \begin{cases} = \overline{\pi}_F & \text{if } \gamma \in [0, \gamma_*), \\ \in \left[\frac{1}{2}(\underline{\pi}_F + \overline{\pi}_F), \overline{\pi}_F\right] & \text{if } \gamma = \gamma_*, \\ = \frac{1}{2}(\overline{\pi}_F + \underline{\pi}_F) & \text{if } \gamma \in (\gamma_*, 1]. \end{cases}$$

Due to the symmetry of the model, any outcome in the disagreement state is Pareto-efficient. Thus, if $\gamma \in [0, \gamma_*)$, both the informed and the uninformed receive their first

[5] There is no mechanism that Pareto-improves on this outcome. More precisely, for any $\gamma > \gamma_*$, in any incentive compatible outcome of a direct mechanism without transfers the probability of implementing a fixed alternative is constant across the three states. See Damiano, Li and Suen (2009) for a formal argument.

best expected payoffs. However, when $\gamma \in (\gamma_*, 1]$, the equilibrium outcome is inefficient, as the expected payoffs for both types would increase if the uninformed types agree to his opponent's favorite alternative instead of a coin flip.[6]

In our model of negotiation under a deadline, the deadline simply means deciding by a coin flip at a fixed future date T if no agreement has been reached. In practice, reaching the negotiation deadline without an agreement may instead trigger a binding arbitration process by an independent outside party that may involve activities such as presentations by each player or fact-finding by the arbitrator. We have taken a reduced-form approach by abstracting from such details of deadline implementation. The essential feature of the deadline we are trying to capture in this model is the ex ante two-part commitment: the negotiating parties commit to both not terminating the negotiation process before the fixed date T, and to not extending it beyond T. Although in reality both parts of this commitment are vulnerable to ex post renegotiation, we assume away the credibility issues in order to take the first step towards understanding welfare implications of deadlines.

3. Preliminary Analysis

We refer to as "persisting" the act of a player proposing his own favorite alternative (player L proposing l or player R proposing r), and "conceding" the act of proposing his opponent's favorite alternative. We will restrict our analysis to perfect Bayesian equilibria in which the informed types persist with probability one throughout the game. We impose no restriction on the strategies of the uninformed types and consider both strategies in which the uninformed is mixing between persisting and conceding at a given instant of time, as well strategies where the uninformed mixes continuously over an interval of time. For histories of "regular disagreement" where both players have persisted since the beginning of the game, such strategies can be described through two functions $y : [0, T] \to [0, 1]$ and $x : [0, T] \to [0, \infty)$, with the convention that $x(t) = 0$ whenever $y(t) > 0$. At any instant

[6] The specification of the default decision as a coin flip when the deadline expires implies stark payoff discontinuities in the no-delay game when the belief of the uninformed types about the disagreement state M is exactly γ_*. Our characterization of the optimal deadline turns out to be robust with respect to the payoff discontinuities. Section 5.2 presents an extension of the model with an alternative specification of the deadline default payoffs that eliminates the discontinuities. All our results are qualitatively unchanged.

$t \in [0, T]$ reached by the game, $y(t)$ is the probability that the uninformed concedes upon reaching time t. When y is zero on a small time interval, $x(t)$ denotes the flow rate of concession at any t in the interval $[t, t + \mathrm{d}t)$. That is, upon reaching time t, the probability of an uninformed type proposing his rival's alternative in the interval is $x(t)\mathrm{d}t$. How the game is played after a "reverse disagreement," where both players simultaneously concede, does not matter to our equilibrium construction and the welfare analysis.[7] For convenience, we assume that these are terminal histories where the decision is made by a coin toss.

3.1. Differential equations

In this section we derive some useful properties that hold in any symmetric equilibrium where the uninformed concedes at flow rate $x(t) > 0$ for all t in some interval of time $[\underline{t}, \overline{t})$, while the informed types always persist. In any such equilibrium, by indifference the equilibrium expected payoff $\mathcal{U}(t)$ of an uninformed type upon reaching $t \in [\underline{t}, \overline{t})$ can be computed by assuming that the uninformed type concedes at t. Denoting as $\gamma(t)$ the belief of the uninformed at time t that the state is M, we have

$$\mathcal{U}(t) = \gamma(t)\underline{\pi}_M + (1 - \gamma(t))\overline{\pi}_D. \tag{2}$$

The above follows because by assumption $y(t) = 0$, and so even though his uninformed opponent's flow rate of concession is strictly positive, the probability that the latter concedes at the given time t is zero. Since $\mathcal{U}(t)$ depends on t only through $\gamma(t)$ in (2), we can define a payoff function

$$U(\gamma) = \gamma\underline{\pi}_M + (1 - \gamma)\overline{\pi}_D, \tag{3}$$

which is valid whenever $\gamma = \gamma(t)$ and $x(t) > 0$ for some $t \in [\underline{t}, \overline{t})$.

Given that the equilibrium continuation payoff of the uninformed type is pinned down by the belief $\gamma(t)$ for any t in the interval of time $[\underline{t}, \overline{t})$, the indifference condition between conceding and persisting on the same interval then relates the rate of change of the belief γ to its current value $\gamma(t)$ and to the equilibrium flow rate of concession $x(t)$. Using Bayes'

[7] Reverse disagreements occur with probability zero both on the path in the equilibrium constructed below and after unilateral deviations. In proving that our equilibrium is generically unique, we require only that the continuation payoffs after a reverse disagreement are feasible.

rule, we obtain a differential equation for the evolution of the belief of the uninformed in $[\underline{t}, \overline{t})$. This result is stated in Lemma 1 below, and proved in Appendix A. An immediate implication of Lemma 1 is that the equilibrium belief of the uninformed $\gamma(t)$ and the equilibrium rate of concession $x(t)$ in the time interval $(\underline{t}, \overline{t})$ are functions of the starting belief $\gamma(\underline{t})$ only.

LEMMA 1. *Let $(y(t), x(t))$ be the strategy and $\gamma(t)$ the belief of the uninformed types in a symmetric equilibrium where the informed types always persist. If $y(t) = 0$ and $x(t) > 0$ for all $t \in [\underline{t}, \overline{t})$, then*

$$-\frac{\dot{\gamma}(t)}{1-\gamma(t)} = \frac{\delta}{\overline{\pi}_M - \underline{\pi}_M}, \tag{4}$$

and

$$x(t) = \frac{1}{\gamma(t)} \frac{\delta}{\overline{\pi}_M - \underline{\pi}_M}.$$

Equation (4) represents the belief evolution for an uninformed type who continuously randomizes and whose opponent has failed to concede so far. Since the informed types persist with probability one, $\dot{\gamma}(t)$ is negative; that is, the uninformed types attach a lower probability to the disagreement state as the negotiation game continues. The indifference condition between persisting and conceding then implies that the uninformed types concede at an increasing flow rate as disagreement continues.

We can also use the equilibrium characterization of the flow rate of concession to pin down the evolution of the equilibrium continuation payoff for the informed types. For any $t \in [\underline{t}, \overline{t})$, let $\mathcal{V}(t)$ be the expected payoff of the informed types at time t. Since the informed types always persist, their payoff function satisfies the following Bellman equation:

$$\mathcal{V}(t) = x(t)\mathrm{d}t\ \overline{\pi}_F + (1 - x(t)\mathrm{d}t)(-\delta\mathrm{d}t + \mathcal{V}(t + \mathrm{d}t)).$$

This can be written as a differential equation by taking $\mathrm{d}t$ to 0:

$$\dot{\mathcal{V}}(t) = \delta - x(t)(\overline{\pi}_F - \mathcal{V}(t)). \tag{5}$$

Further, since $\gamma(t)$ is determined by an autonomous differential equation and $x(t)$ depends on t only through $\gamma(t)$ as given in Lemma 1, we can also describe the equilibrium continuation payoff of the informed as a function $V(\gamma)$. Using $\dot{\mathcal{V}}(t) = V'(\gamma(t))\dot{\gamma}(t)$, we can show that it satisfies the differential equation

$$V'(\gamma) = \frac{\overline{\pi}_F - V(\gamma)}{\gamma(1-\gamma)} - \frac{\overline{\pi}_M - \underline{\pi}_M}{1-\gamma}. \tag{6}$$

Thus, the equilibrium payoff to the informed types is a function of the belief of the uninformed, even though the former know the state and always persist in equilibrium.

3.2. Equilibrium with no deadline

When there are no deadlines to the negotiation process (i.e., $T = \infty$), the characterization result of Lemma 1 is sufficient for us to construct an equilibrium where the uninformed types concede at a strictly positive flow rate until a time when they concede with probability one.[8] The equilibrium strategy and the evolution of beliefs along the equilibrium path are entirely pinned down by the initial belief, and the atom of concession occurs when the uninformed types become entirely convinced that the state is an agreement state. Let $g(t;\gamma^0)$ be the unique solution to the differential equation (4) with the initial condition $g(0;\gamma^0) = \gamma^0$, given by

$$g(t;\gamma^0) = 1 - (1-\gamma^0)e^{\delta_* t}, \tag{7}$$

where for notational brevity we have defined

$$\delta_* \equiv \frac{\delta}{\overline{\pi}_M - \underline{\pi}_M}.$$

Define the "terminal time" $Q(\gamma^0)$ such that $g(Q(\gamma^0);\gamma^0) = 0$, given explicitly by

$$Q(\gamma^0) = -\frac{\ln(1-\gamma^0)}{\delta_*}. \tag{8}$$

[8] While the proposition below does not make a formal claim of uniqueness, the analysis to follow in Section 4.2 also applies to the case when there is no deadline to the negotiation process. The equilibrium characterized in the proposition is unique within the class of equilibria where the informed types always persist.

Proposition 1. *Let $T = \infty$. There exists a symmetric equilibrium where the informed types always persist, and where the strategy $(y(t), x(t))$ and the belief $\gamma(t)$ of the uninformed types are such that:*

$$\begin{cases} y(t) = 0,\ x(t) = \delta_*/\gamma(t),\ \text{and } \gamma(t) = g(t;\gamma^0) & \text{if } t < Q(\gamma^0), \\ y(t) = 1,\ \text{and } \gamma(t) = 0 & \text{if } t \geq Q(\gamma^0). \end{cases}$$

By construction, the uninformed types are indifferent between conceding and persisting at any time $t < Q(\gamma^0)$. Further, conceding is optimal at $t = Q(\gamma^0)$ for the uninformed because their belief that the state is M becomes zero at that point.[9] For the informed types, from the equilibrium strategies, their continuation payoff at the terminal time is the first best payoff $\overline{\pi}_F$. In Appendix A, we use this boundary condition to explicitly solve the differential equation (6) for the informed types' continuation payoff for any $t < Q(\gamma^0)$, and verify that it is optimal for them to always persist.

In equilibrium, protracted negotiations make the uninformed types increasingly convinced that the state is the agreement state supporting the rival's favorite choice, and motivate them to concede at an increasing rate. This distinctive feature of "gradually increasing concessions," unique to our model of negotiation that combines preference-driven and information-driven disagreements, has implications for the duration of the negotiation process and its hazard rate function. Denote as τ_I and τ_U the random duration of the game conditional on the state being an agreement state and the disagreement state respectively. In the former case one of the player is an informed type, while in the latter case both are uninformed. Since $x(t)\mathrm{d}t$ is the probability that the game ends in time interval $(t, t+\mathrm{d}t]$ conditional on it having survived up to time t, the hazard function of τ_I is simply $x(t)$. When the state is M, independent randomizations by the two players imply that the cumulative distribution functions $F_I(t;\gamma^0)$ of τ_I and the distribution function $F_U(t;\gamma^0)$ of τ_U satisfy $1 - F_U(t;\gamma^0) = (1 - F_I(t;\gamma^0))^2$, and thus the hazard function of τ_U is $2x(t)$. The hazard rate is therefore increasing in time in both cases. From an outside observer's point of view, however, the more interesting object is the unconditional duration of the

[9] The game ends with probability one before $t = Q(\gamma^0)$. We specify the strategy and the belief of the uninformed types after the terminal time to complete equilibrium description after unilateral deviations.

negotiation game. Let τ represent this random variable, and $F(t;\gamma^0)$ its distribution function. As the game continues, the conditional hazard rates for τ_I and τ_U both increase, but the probability that $\tau = \tau_I$, which is associated with a lower hazard rate, also increases, so it is not obvious whether the unconditional hazard rate for τ will increase over time.[10] However, from the relationship

$$1 - F(t;\gamma^0) = \frac{2(1-\gamma^0)}{2-\gamma^0}(1 - F_I(t;\gamma^0)) + \frac{\gamma^0}{2-\gamma^0}(1 - F_U(t;\gamma^0))$$

we can obtain the hazard function of τ as $2\delta_*/(g(t;\gamma^0)(2 - g(t;\gamma^0)))$, which is decreasing in $g(t;\gamma^0)$.[11] Since in equilibrium the belief of the uninformed types about M decreases as disagreements continue, the unconditional hazard rate unambiguously increases in time. Combined with the fact that the belief $g(t;\gamma^0)$ is increasing in γ^0 for any t, an increase in the initial belief, representing a greater degree of conflict, reduces the unconditional hazard rate, and hence increases the unconditional expected duration of negotiation.

4. Finite Deadlines

We use the analysis in the previous section to construct a symmetric equilibrium in which the informed types always persist, and the uninformed types generally start by continuously randomizing between conceding and persisting when the time to the deadline is sufficiently long, stop and persist until just before the deadline is reached, and play an equilibrium of the no-delay game ($T = 0$) corresponding to the stopping belief. We later argue that this equilibrium is unique subject to the restriction that the informed types always persist.

[10] This is similar to the classic problem of duration dependence versus heterogeneity in the econometric analysis of duration data. See, for example, Heckman and Singer (1984).

[11] To derive the hazard function for τ, note that the conditional density functions $f_I(t;\gamma^0)$ and $f_U(t;\gamma^0)$ and the unconditional density function $f(t;\gamma^0)$ satisfy

$$\frac{f(t;\gamma^0)}{1-F(t;\gamma^0)} = \frac{2(1-\gamma^0)f_I(t;\gamma^0) + \gamma^0 f_U(t;\gamma^0)}{2(1-\gamma^0)(1-F_I(t;\gamma^0)) + \gamma^0(1-F_U(t;\gamma^0))} = \frac{2\delta_*}{g(t;\gamma^0)(2-g(t;\gamma^0))},$$

where the last equality uses $1 - F_I(t;\gamma^0)) = \exp\{-\int_0^t x(s)\mathrm{d}s\} = g(t;\gamma^0))(1-\gamma^0)/(\gamma^0(1-g(t;\gamma^0))))$ and $f_I(t;\gamma^0)) = (1 - F_I(t;\gamma^0)))\delta_*/g(t;\gamma^0))$, and corresponding expressions for F_U and f_U.

A remarkable feature of our construction is that the equilibrium randomization strategy of the uninformed is identical to the no-deadline case ($T = \infty$). That is, when the time to the deadline is sufficiently long, the uninformed behaves as if there is no deadline. This feature is the main analytical advantage of a continuous time framework over a discrete time model. It follows from equation (3) in our preliminary analysis, because there is a unique equilibrium value function for a randomizing uninformed type that depends on the time to deadline only through the belief of the uninformed.

4.1. Construction of an equilibrium

The necessity, in equilibrium, of a persistence phase before the deadline is reached, can be easily understood as follows. At any time t when the belief of an uninformed type is $\gamma(t) = \gamma$ and he is conceding with a positive flow rate, his payoff is pinned down by the the function $U(\gamma)$ given in equation (3). For any $\gamma > 0$, this payoff is strictly lower than the payoff from the no-delay game $U^0(\gamma)$ as given in equation (1). If the time remaining to the deadline, $T - t$, is sufficiently short, persisting until the end and playing a no-delay equilibrium when the deadline arrives would constitute a profitable deviation for the uninformed. This deadline effect of having a persistence phase just before the deadline is robust with respect to our game specification. Whenever the default payoff at the deadline of a negotiation game yields an equilibrium payoff upon reaching the deadline that is larger than the payoff from concession, then in any equilibrium a period of inactivity always precedes, the arrival of the deadline.[12]

How long the persistence phase can last in equilibrium depends on the difference between the payoff from immediate concession $U(\gamma)$ and the payoff in the no-delay game $U^0(\gamma)$. To state our equilibrium characterization result in the next proposition, we define $B(\gamma)$ as the longest length of time from the deadline such that it is an equilibrium for an uninformed type with belief γ to persist until the deadline and then play an equilibrium corresponding to the no-delay game associated with γ. In other words, the value of $B(\gamma)$

[12] A similar deadline effect is present in existing models of war of attrition (e.g., Hendricks, Weiss and Wilson, 1988). The novel feature of our model as a war of attrition game is that endogenous information about the state is generated as the game continues, so that the deadline effect depends on the initial belief through the equilibrium belief evolution prior to stopping.

measures the length of the longest persistence phase that starts when the uninformed belief is γ. For any belief $\gamma \neq \gamma_*$, this is uniquely given by

$$U^0(\gamma) - \delta B(\gamma) = U(\gamma).$$

Since $U^0(\gamma_*)$ assumes a continuum of values, corresponding to the probability of conceding ranging from 0 to 1, we choose the maximal value in the above equation to define $B(\gamma_*)$. Using the expressions for $U^0(\gamma)$ and $U(\gamma)$, we have

$$B(\gamma) = \begin{cases} \gamma/(2\delta_*) & \text{if } \gamma \leq \gamma_*, \\ (\gamma - \gamma_*)/(2(1-\gamma_*)\delta_*) & \text{if } \gamma > \gamma_*. \end{cases} \tag{9}$$

Note that $B(\gamma)$ is discontinuous at γ_*. Next, for an initial belief γ^0, we describe how long it takes, in equilibrium, before the persistence phase begins. To do so, we define $J(\gamma^0)$ as the earliest calendar time t such that the time-to-deadline is shorter than $B(\gamma(t))$ given that the belief $\gamma(t)$ of the uninformed evolves according to (7) starting with γ^0. That is,

$$J(\gamma^0) = \inf_{t \geq 0}\{t : T - t \leq B(g(t;\gamma^0))\}. \tag{10}$$

Note that by definition, $J(\gamma^0) = 0$ if $T \leq B(\gamma^0)$. The functions $J(\gamma^0)$ and $T - J(\gamma^0)$ describe the length of the concession and the persistence phases respectively in our equilibrium characterization.

Proposition 2. *Let T be finite. There exists a symmetric equilibrium in which the informed types always persist, and the strategy $(y(t), x(t))$ and the belief $\gamma(t)$ of the uninformed types are such that:*

$$\begin{cases} y(t) = 0,\ x(t) = \delta_*/\gamma(t) \text{ and } \gamma(t) = g(t;\gamma^0) & \text{if } T - t > B(g(t;\gamma^0)) \text{ and } t < Q(\gamma^0), \\ y(t) = 0,\ x(t) = 0 \text{ and } \gamma(t) = g(J(\gamma^0);\gamma^0) & \text{if } B(g(t;\gamma^0)) \geq T - t > 0 \text{ and } t < Q(\gamma^0), \\ y(t) = 1, \text{ and } \gamma(t) = 0 & \text{if } T > t \geq Q(\gamma^0); \end{cases}$$

and

$$y(T) = \begin{cases} 0 & \text{if } g(J(\gamma^0);\gamma^0) > \gamma_*, \\ 2\delta_*(T - J(\gamma^0))/\gamma_* & \text{if } g(J(\gamma^0);\gamma^0) = \gamma_*, \\ 1 & \text{if } g(J(\gamma^0);\gamma^0) < \gamma_*. \end{cases}$$

The logic of Proposition 2 is apparent from our construction of $B(\gamma)$ and $J(\gamma^0)$. For each belief γ of the uninformed, the equilibrium payoff function $U^0(\gamma)$ in the no-delay game gives a continuation equilibrium outcome at the instant when the deadline arrives, providing the starting point for backward induction. This continuation equilibrium outcome is unique if $\gamma \neq \gamma_*$, and so if the deadline T is short relative to the initial belief γ^0, i.e., if $T \leq B(\gamma^0)$, the equilibrium is for the uninformed to persist until the deadline and then play the continuation equilibrium corresponding to γ^0. By construction, when $T = B(\gamma^0)$, the equilibrium payoff to the uninformed is precisely $U(\gamma^0)$. If $\gamma^0 = \gamma_*$ and $T \leq B(\gamma_*)$, we choose a continuation equilibrium in the no-delay game, corresponding to a probability of concession $y(T) = 2\delta_* T/\gamma_*$, such that the uninformed types obtain the payoff of $U(\gamma_*)$ from this deadline play.[13] Therefore, when the deadline T is sufficiently long relative to the initial belief γ^0 so that $J(\gamma^0) > 0$, it is an equilibrium for the uninformed types to start by conceding with a flow rate $x(t)$ given in Proposition 1 for the no-deadline game until $t = J(\gamma^0)$, when the belief becomes $g(J(\gamma^0); \gamma^0)$ and the payoff reaches $U(g(J(\gamma^0); \gamma^0))$, followed by the deadline play. Finally, if the deadline T is too long with $T \geq Q(\gamma^0)$, the equilibrium is identical to the one constructed in the no-deadline game.[14] Details of the proof of Proposition 2 (including the argument that the informed types will indeed persist throughout) are presented in Appendix A.

The equilibrium behavior of the uninformed types is illustrated in Figure 1. The horizontal axis represents both the deadline T, and for a fixed T, the time remaining before the deadline is reached; the vertical axis is the belief of the uninformed. For ease of interpretation, we have shown the discontinuous function $B(\gamma)$ as the thick piecewise-linear graph. It represents the boundary in the T-γ space between the "persistence phase" when the uninformed types persist until the deadline and their belief does not change, and the "concession phase" when they concede with a positive and increasing flow rate and their belief continuously drops. If the deadline T is neither too long nor too short relative

[13] Since any $y(T)$ greater than $2\delta_* T/\gamma_*$ preserves the incentives for the uninformed to persist, there is a continuum of equilibria when $\gamma^0 = \gamma_*$ and $T < B(\gamma_*)$.

[14] In this case, (10) implies that the phase-transition time $J(\gamma^0)$ is equal to $Q(\gamma^0)$ and the corresponding belief $g(J(\gamma^0); \gamma^0)$ is zero.

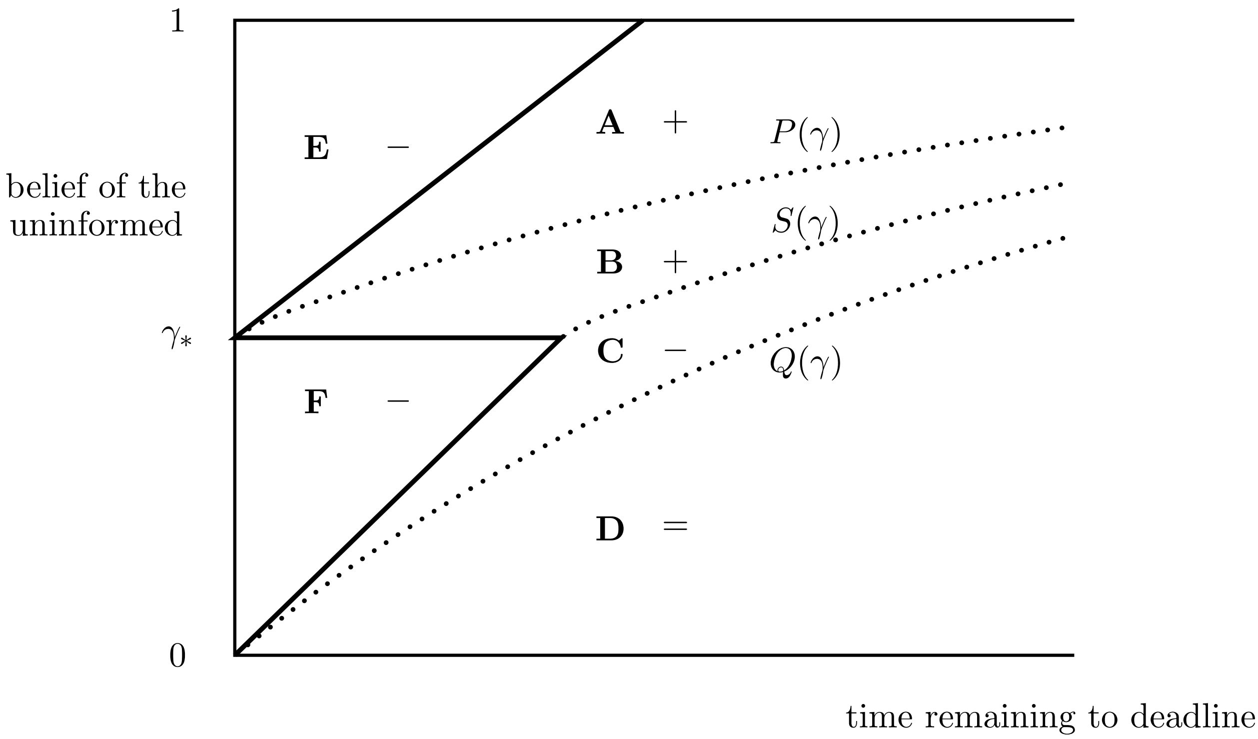

Figure 1

to the initial belief γ^0, that is, if $T \in (B(\gamma^0), Q(\gamma^0))$, there is a unique "phase-transition time" from the concession phase to the persistence phrase. This is just $J(\gamma^0)$ defined in (10), and satisfies

$$J(\gamma^0) \begin{cases} = T - B(g(J(\gamma^0);\gamma^0)) & \text{if } g(J(\gamma^0);\gamma^0) \neq \gamma_*, \\ \in [0, T - T_*] & \text{if } g(J(\gamma^0);\gamma^0) = \gamma_*, \end{cases}$$

where for convenience we have defined

$$T_* \equiv B(\gamma_*) = \frac{\gamma_*}{2\delta_*}.$$

For example, for any deadline T and initial belief γ^0 on the dotted curve S in Figure 1, the phase-transition time $J(\gamma^0)$ is exactly $T - T_*$, that is,

$$g(S(\gamma) - T_*; \gamma) = \gamma_*. \tag{11}$$

Alternatively, for a given deadline T and initial belief γ^0 such that $T = S(\gamma^0)$, the dotted curve traces the equilibrium evolution of the belief $\gamma(t)$ until the phase-transition time.

Similarly, P and Q represent two other corner cases of equilibrium belief evolution; in both cases the phase-transition time happens exactly when the deadline arrives so there is no persistence phase.[15]

Summarizing the equilibrium play of the uninformed types, we partition the T-γ space of Figure 1 into six regions by S, P and Q:[16]

Region A. The uninformed types concede with a flow rate $\delta_*/g(t;\gamma^0)$ for $t < J(\gamma^0)$, and persist for t larger.

Region B. The uninformed types concede with a flow rate $\delta_*/g(t;\gamma^0)$ for $t < J(\gamma^0)$, persist for all $t \in [J(\gamma^0), T)$, and concede with probability $2\delta_*(T - J(\gamma^0))/\gamma_*$ at $t = T$.

Region C. The uninformed types concede with a flow rate $\delta_*/g(t;\gamma^0)$ for $t < J(\gamma^0)$, persist for all $t \in [J(\gamma^0), T)$, and concede with probability one at $t = T$.

Region D. The uninformed types concede with a flow rate $\delta_*/g(t;\gamma^0)$, with the game ending with probability one by the terminal time $Q(\gamma^0)$ before the deadline expires.

Region E. The uninformed types persist for all t.

Region F. The uninformed types persist for all $t < T$ and concede with probability one at $t = T$.

Each of the six regions above has its own distinctive features. Together they provide a rich set of negotiation dynamics available in our model. In Region D, the deadline is not binding. Gradual concessions are made at an increasing rate until an agreement is reached as if there is no deadline; the dynamics of endogenous information aggregation is already described in the previous section. In all other regions, the deadline is binding, with the effect of suspending the negotiations at some point of the process in anticipation of the arrival of the deadline. When the deadline is too short, in both Regions E and F, and on the boundary between Regions F and B, this effect takes hold at the very beginning so there is no attempt at resolving the differences before the deadline. The difference

[15] Since the belief evolution does not depend on the deadline T in the concession phase, the three dotted curves in Figure 1 are the same function, shifted by the deadline. Thus, $P(\gamma) = S(\gamma) - T_*$ for $\gamma \geq \gamma_*$, and $Q(\gamma)$ is given by (8).

[16] The boundary between Regions B and F is formally part of Region B. We have $J(\gamma^0) = 0$ so there is no concession phase and the uninformed types concede at $t = T$ with probability $2\delta_* T/\gamma_*$. The assignment of other boundaries is immaterial.

between the two regions is that E represents a "deadlock" with no hope of ever reaching an agreement because the initial degree of conflict is too high, while the deadline effect in F and on the boundary between F and B describes a "cooling-off" period before an eleventh hour attempt at striking an agreement. When the deadline is sufficiently long relative to the initial degree of conflict, in Regions A, B and C, the negotiation all starts off with gradual and increasing concessions as in Region D. The difference among the three regions lies in how much time and conflict remain when the deadline effect kicks in after the unsuccessful initial attempts. In Region A, too little time is left to overcome the residual conflict, so the negotiation becomes a deadlock. The opposite happens in Region C, as there is a complete change of position in the final attempt to reconcile the difference after a cooling-off period. In between we have Region B, where more time left when the deadline effect kicks in means a greater chance of reaching an agreement at the deadline.

4.2. Uniqueness of the equilibrium

The equilibrium constructed in Proposition 2 is generically unique in the class of perfect Bayesian equilibria with the informed types always persisting. This is perhaps surprising, because the amount of endogenous information generated in equilibrium during the concession phase depends on the flow rate of concession of the uninformed types, which in turn is determined by how much the uninformed types learn in equilibrium about the state. One may wonder if it is possible to construct multiple equilibria by coordinating through calendar time the flow rate of concession of the uninformed types. For example, after trying but failing to reach an agreement by conceding with a positive flow rate, the uninformed types may persist for a fixed length of time before resuming a new concession phase. However, this and other possibilities for multiple equilibria are ruled out by the following proposition.

PROPOSITION 3. *Given any deadline T and initial belief γ^0 of the uninformed types, except for $T < T_*$ and $\gamma^0 = \gamma_*$, there is a unique equilibrium in which the informed types always persist.*

When $T < T_*$ and $\gamma^0 = \gamma_*$, there is a continuum of equilibria in which the informed types always persist and the uninformed types persist for all $t < T$ followed by any probability of concession equal to or greater than $2\delta T/\gamma_*$ at the deadline. This multiplicity of equilibria is due to the multiplicity in the no-delay game ($T = 0$) when the initial belief of the uninformed is γ_*. However, it is not generic, because for the same $T < T_*$ the equilibrium is unique when γ^0 is different from γ_*, no matter how small the difference is.[17] Moreover, since at $\gamma^0 = \gamma_*$ there is an equilibrium in the no-delay game with the first best payoffs, we argue that the optimal deadline for $\gamma^0 = \gamma_*$ is $T = 0$, and thus the particular multiplicity at γ_* does not affect our characterization of the optimal deadline.

The generic uniqueness of the equilibrium is important for our main objective in this paper, which is to characterize the ex ante optimal deadline. Moreover, the uniqueness result implies that the equilibrium strategies in the game with finite deadline T cannot be supported as part of equilibrium in a no-deadline game, which means that deadlines are more than a mere coordinating device to select among multiple equilibria. In Appendix A we formally prove Proposition 3 by establishing a series of claims about the properties of any equilibrium. Here, we give intuitive explanations for some of the properties to highlight the underlying logic of why the equilibrium is unique.

A key step in establishing the generic uniqueness of the equilibrium is to show that in any equilibrium the uninformed types cannot concede with a strictly positive probability before the deadline arrives. Intuitively, if an uninformed type concedes with probability $y(t) > 0$ at some time $t < T$, then the opposing uninformed type could persist at t and concede immediately after. The payoff gain relative to conceding would be strictly positive because $y(t) > 0$, while the loss from the extra delay would be arbitrarily small. An immediate implication is that at any time before the deadline, an uninformed type must either persist with probability one or concede with a positive flow rate. In other words, the equilibrium play of an uninformed type must either be in a persistence phase or in a concession phase.

In any equilibrium the persistence and concession phases of the two uninformed types must be synchronized. That is, if the flow rate of concession $x(t)$ for one uninformed type is positive in some interval period of time, then the same is true for his uninformed opponent.

[17] In addition, the multiplicity of equilibria for $T < T_*$ and $\gamma^0 = \gamma_*$ is not robust with respect to the specification of the default payoffs in the no-delay game. In the model of Section 5.2 where we introduce a penalty that the players incur if they fail to reach an agreement when the deadline expires, the same argument for Proposition 3 can be used to establish that the equilibrium is unique for all T and γ^0.

This is because if an uninformed player is continuously indifferent between conceding and persisting in an interval period of time, his belief that the state is a disagreement state must change over time. Otherwise, conceding at the beginning of the interval would give the player the same expected payoff from the outcome but with a smaller delay cost. Synchronization then follows because the belief of the player changes only if his opponent's flow rate of concession is positive. Conversely, if an uninformed player persists in some time interval, then so does the opposing uninformed player. Thus, the belief of neither player changes in a synchronized persistence phase. Since the payoff to an uninformed type $U(\gamma(t))$ in a concession phase is pinned down by the corresponding belief $\gamma(t)$ and is computed with the opposing uninformed type persisting at t, a persistence phase cannot be followed by a concession phase. Otherwise, each uninformed type would strictly prefer to concede with probability 1 during the persistence phase to avoid the payoff loss from the delay, which we already know cannot happen in an equilibrium.

Our game is symmetric. In any equilibrium the two uninformed types not only synchronize their persistence and concession phases with the same phase-transition time, they also adopt identical strategies in the concession phase and in the deadline play. At the phase-transition time, both uninformed types must be indifferent between an immediate concession and the deadline play. Since their expected payoffs from both options are functions of their individual beliefs only, for the indifference conditions to hold at the same time, their beliefs must coincide. The symmetry of the equilibrium strategy of the uninformed in the concession phase then follows, because the uniqueness of the solution of the differential equation (4) implies that the beliefs of the two players coincide at the phase-transition time only if they are identical throughout the concession phase. Given the symmetry, the construction of the boundary $B(\gamma)$ and the phase-transition time $J(\gamma^0)$ is unique due to the indifference of the uninformed types between an immediate concession with the payoff $U(g(J(\gamma^0);\gamma^0)$ and the deadline play with the payoff $U^0(g(J(\gamma^0);\gamma^0)) - \delta B(g(J(\gamma^0);\gamma^0))$, yielding the uniqueness of the equilibrium.

4.3. Optimal deadline

In this subsection we characterize the ex ante optimal deadline for the repeated proposal game. We start by studying the effects of marginally extending the deadline T on the equilibrium payoffs of the informed and the uninformed in the different regions of the T-γ^0 space. Refer to Figure 1.

In Regions E and F of Figure 1, where $T < B(\gamma^0)$ and $\gamma^0 \neq \gamma_*$, the deadline is too short relative to the initial belief to allow a concession phase. The welfare effect of the deadline is clearly negative. Extending the deadline just makes the uninformed types persist for a longer period of time without changing their behavior at the deadline. Consequently, both the uninformed and the informed are hurt by a longer deadline.

In Region D, where $T \geq Q(\gamma^0)$, the deadline is too long to allow a persistence phase. There is no welfare effect. Since the negotiation ends before the deadline with probability one, extending it further will not affect the equilibrium behavior or payoffs.

In Region B, where $T \in [P(\gamma^0), S(\gamma^0))$, the effect of lengthening the deadline is to make the uninformed persist longer after the phase transition, but concede with a larger probability when the deadline arrives. Since the behavior of the players during the concession phase does not depend on T, the phase-transition time $J(\gamma^0)$ is also independent of T. Once the negotiation enters the persistence phase, the uninformed types persist from time $J(\gamma^0)$ through T, and then concede with probability $2\delta_*(T - J(\gamma^0))/\gamma_*$. Lengthening the deadline increases the delay for the informed types, but also increases their chance of getting their favorite decision (with payoff $\overline{\pi}_F$) rather than a coin toss (with payoff $\frac{1}{2}(\overline{\pi}_F + \underline{\pi}_F)$). The net effect on the welfare of the informed types is

$$\frac{\partial V(\gamma^0)}{\partial T} = -\delta + \frac{2\delta_*}{\gamma_*}\frac{\overline{\pi}_F - \underline{\pi}_F}{2}, \tag{12}$$

which is positive by Assumption 1. There is no effect on the welfare of the uninformed, because their payoff is pinned down by $U(\gamma^0)$, which is independent of T. In sum, a longer deadline is beneficial for the ex ante welfare of the players in this region.[18]

Finally, consider Region A where $T \in [B(\gamma^0), P(\gamma^0))$, and Region C where $T \in [B(\gamma^0), Q(\gamma^0))$ for $\gamma^0 < \gamma_*$ or $T \in [S(\gamma^0), Q(\gamma^0))$ for $\gamma^0 \geq \gamma_*$. As in Region B, the equilibrium play of the uninformed types in A or C also consists of both a concession phase and a persistence phase. However, unlike in B, increasing the deadline in A or C

[18] Under the selection of the continuation equilibrium given in Proposition 2, the same analysis and conclusion hold on the horizontal segment of the boundary B, with $\gamma^0 = \gamma_*$ and $T \leq T_*$.

lengthens the concession phase while shortening the persistence phase, with no change in equilibrium play at the deadline ($y(T) = 0$ in Region A or $y(T) = 1$ in Region C). The welfare effect on the uninformed is again nil, since their payoff is fixed at $U(\gamma^0)$. The welfare effect on the informed can be studied by solving the differential equation (5) (or equivalently, 6) with appropriate boundary conditions obtained from the equilibrium deadline play of the uninformed types.

Take Region A for example. The game enters the persistence phase from the concession phase at time $J(\gamma^0)$. From the deadline play of the uninformed types, the payoff to the informed types at $t = J(\gamma^0)$ is

$$\mathcal{V}(J(\gamma^0)) = \frac{1}{2}(\overline{\pi}_F + \underline{\pi}_F) - \delta(T - J(\gamma^0)).$$

Their payoff at the beginning of the game is

$$V(\gamma^0) = \mathcal{V}(0) = \mathcal{V}(J(\gamma^0)) - \int_0^{J(\gamma^0)} \dot{\mathcal{V}}(t)\mathrm{d}t,$$

where $\dot{\mathcal{V}}(t)$ is given by equation (5). Lengthening the deadline affects the welfare of the informed by changing the boundary value $\mathcal{V}(J(\gamma^0))$ directly and by prolonging the concession phase through increasing $J(\gamma^0)$. The overall effect is

$$\frac{\partial V(\gamma^0)}{\partial T} = -\delta + x(J(\gamma^0))(\overline{\pi}_F - \mathcal{V}(J(\gamma^0)))\frac{\partial J(\gamma^0)}{\partial T}. \tag{13}$$

The cost of a longer deadline is δ, while the benefit to the informed is the increased length of the concession phase times the flow rate of concession times the value of the resulting improvement in the decision. The analysis for Region C is similar, except that the boundary value becomes

$$\mathcal{V}(J(\gamma^0)) = \overline{\pi}_F - \delta(T - J(\gamma^0)).$$

The welfare effect on the informed is given by the same expression (13).

Crucial to our characterization of optimal deadline, we establish in the proof of Proposition 4 below that the welfare effect (13) is positive in Region A but negative in Region C. The intuition behind this result is quite simple. In Region A, if the game survives to the deadline, the uninformed will persist and the quality of the resulting decision is bad for the informed. Therefore a longer concession phase that allows more information aggregation in the beginning of the negotiation is highly valuable. In Region C, on the other hand, the uninformed will concede if the game survives to the deadline. Since the informed will eventually obtain his favorite decision, a longer concession phase in the beginning is of less value. This explains the contrasting welfare effects for these two cases.

The welfare effects of a marginal extension of the deadline are summarized in Figure 1. A "+" sign indicates that a longer deadline improves the welfare of the informed types, with no effect on the uninformed; a "−" sign indicates a negative welfare effect on the informed, together with either a negative effect (in Regions E and F) or no effect (in Region C) on the uninformed; and a "=" sign indicates that the welfare effect is nil for both the informed and the uninformed. For $\gamma^0 \geq \gamma_*$, we can see that as the deadline T increases, the welfare effect is first negative in Region E, then positive in Regions A and B, and finally turning negative in Region C. Therefore the optimal deadline must be either 0, or $S(\gamma^0)$, which is the boundary between Regions B and C. For $\gamma^0 < \gamma_*$, we see that the welfare effect is negative so long as the deadline is binding, and is nil when the deadline is too long. Therefore the optimal deadline must be $T = 0$. To state our main result on the optimal deadline, let

$$W(\gamma^0) = \frac{1}{2-\gamma^0} U(\gamma^0) + \frac{1-\gamma^0}{2-\gamma^0} V(\gamma^0) \tag{14}$$

denote the ex ante welfare of a player before he knows his type. Define T^{opt} to be the length of the deadline T that maximizes $W(\gamma^0)$.

Proposition 4. *There exists a* $\overline{\gamma} \in (\gamma_*, 1)$ *such that*

$$T^{\text{opt}} = \begin{cases} 0 & \text{if } \gamma^0 \in [0, \gamma_*] \text{ or } \gamma^0 \in (\overline{\gamma}, 1), \\ S(\gamma^0) & \text{if } \gamma^0 \in (\gamma_*, \overline{\gamma}), \\ 0 \text{ or } S(\gamma^0) & \text{if } \gamma^0 = \overline{\gamma}. \end{cases}$$

The proof of this proposition involves showing that the welfare effect (13) is positive in Region A and negative in Region C. Together with the result that the welfare effect (12) is positive in Region B, we establish that the local maxima of $W(\gamma^0)$ are at $T = 0$

and $T = S(\gamma^0)$ when $\gamma > \gamma_*$. The remainder of the proof consists of comparing the values of $W(\gamma^0)$ at the two local maxima. The details are in Appendix A.

Proposition 4 shows that the optimal deadline is zero when γ^0 is either sufficiently small or sufficiently large. When $\gamma^0 \leq \gamma_*$, the equilibrium in the no-delay game is efficient, so that extending allowing the players to negotiate in a continuous-time game will only introduce unnecessary delay. At the other end, when γ^0 is sufficiently close to 1, under a sufficiently long deadline the uninformed types concede at a low rate and revise their belief slowly. Although the welfare effect of the deadline is locally positive, making the decision immediately by flipping a coin is even better from the ex ante perspective because the long delay is avoided in the first place.

For intermediate levels of γ^0, Proposition 4 shows that the optimal deadline is both finite and bounded away from zero. These two properties follow from the characterization of the optimal deadline by the condition that the remaining time for negotiation is T_* when the belief of the uninformed types drops to γ_* after an unsuccessful concession phase. Alternatively, since the uninformed types in equilibrium concede with probability one if and only if the stopping belief is γ_* and the time remaining to the deadline is T_*, or the stopping belief is strictly lower than γ_*, the optimal deadline for the intermediate levels of initial belief γ^0 is such that the concession phase is the shortest, and correspondingly the persistence phase is longest, for there to be efficient information aggregation at the deadline. Thus, the optimal deadline is finite for $\gamma^0 \in (\gamma_*, \overline{\gamma})$, not because too long a deadline eventually becomes non-binding with no welfare effect, but because conditional on achieving efficient information aggregation at the deadline, the optimal deadline minimizes the length of the concession phase. That it is bounded away from zero implies that there are discontinuities in the optimal deadline, both at $\gamma^0 = \gamma_*$ and at $\gamma^0 = \overline{\gamma}$. These discontinuities are not a consequence of the equilibrium payoff discontinuity in the no-delay game.[19] Rather, they are due to the deadline effect: for deadlines sufficiently short, the uninformed types will simply persist from the start all through the deadline, which means that the welfare effect is always negative for short deadlines. Put differently, when positive

[19] In Section 5.2, where we modify the non-delay game to eliminate the payoff discontinuity, the optimal deadline remains discontinuous.

the optimal deadline is bounded away from zero because it has to be sufficiently long to give incentives for the uninformed to change their deadline behavior and achieve efficient information aggregation.

Using the definition of S in equation (11), we can obtain an explicit formula for T^{opt} when it is positive:

$$T^{\text{opt}} = \frac{1}{\delta_*}\Big(\frac{\gamma_*}{2} + \ln\frac{1-\gamma_*}{1-\gamma^0}\Big).$$

The above formula immediately reveals that the optimal deadline, when positive, is an increasing function of γ^0. This makes sense, because starting from a higher initial belief γ^0 it takes a longer time for the revised belief to reach γ_*. It is also straightforward to verify using the formula that T^{opt} is decreasing in γ_* for any δ_*. Thus, the optimal deadline for negotiation, when positive, is longer when the flow delay cost δ is lower, when the payoff difference in the disagreement state $\overline{\pi}_M - \underline{\pi}_M$ is larger, or when the payoff difference in the agreement state supporting the rival's favorite choice, $\overline{\pi}_D - \underline{\pi}_D$, is smaller. These comparative statics results are intuitive, as they all point to changes in the underlying parameter values that make it more difficult and costly in terms of longer delay to achieve efficient information aggregation endogenously through gradual concessions.

5. Extensions

In setting up the model we have abstracted from any detail in the deadline implementation to focus on the welfare effect of the deadline. In this section we briefly present two extensions of the model, both of which add greater detail and some degree of realism. However, this is not the main objective of these two extensions. Rather, we use them to gain more insight about the source of the welfare effect, and to demonstrate its robustness.

5.1. Stochastic deadlines

Our analysis so far is confined to the case of pre-committed deterministic deadlines. We now study the repeated proposal game with exogenous but stochastic breakdowns, interpreted as stochastic deadlines. Let $\epsilon > 0$ be the constant rate of exogenous exit, so that upon reaching time t, the probability that the game ends exogenously in the next time interval $\mathrm{d}t$ is $\epsilon \mathrm{d}t$. In this event, we assume that the decision is made by a fair coin flip. For simplicity we assume that $T = \infty$. A smaller value of ϵ corresponds to a longer stochastic deadline, with $\epsilon = \infty$ corresponding to the no-delay game analyzed in Section 2, and $\epsilon = 0$ equivalent to the no deadline game analyzed in Section 3.

Following the same steps in deriving the differential equation for $\gamma(t)$ in the case of $\epsilon = 0$, we have

$$-\frac{\dot{\gamma}(t)}{1-\gamma(t)} = \delta_* \frac{\alpha - \gamma(t)}{\alpha - \gamma_*}, \tag{15}$$

where we have defined

$$\alpha \equiv \gamma_* + (1-\gamma_*)\frac{2\delta_*}{\epsilon}.$$

The derivation of the above differential equation is in the proof of Proposition B1 in Appendix B. There are two cases to consider.

In the first case, $\gamma^0 < \min\{1, \alpha\}$, and the differential equation (15) gives the belief evolution of an equilibrium in which the informed types always persist and the uninformed types with belief γ concede with a flow rate $\epsilon(\alpha - \gamma)/(2(1-\gamma_*)\gamma)$.[20] In this case, the exogenous exit rate ϵ is sufficiently small, or equivalently the stochastic deadline is sufficiently long, relative to the initial belief γ^0 of the uninformed. Qualitatively, this case is similar to the no-deadline game of Section 3, or the non-binding deadline case of Section 4 (Region D in Figure 1).

In the second case, with $\gamma^0 \in [\min\{1, \alpha\}, 1)$, in equilibrium the uninformed types persist with probability one at any time t just as the informed types, with the game ending by an exogenous exit. This case occurs when the exit rate ϵ is great and the initial belief γ^0 is high. Since flipping a coin gives a higher payoff to the uninformed than $U(\gamma^0)$, and since the expected wait for the stochastic exit to occur is short when ϵ is large, the uninformed types have no incentive to deviate to conceding. This case is qualitatively similar to short deadline case in Section 4 (Region E in Figure 1).

We are interested in the effect of the stochastic exit rate ϵ on players' welfare. The question we want to answer is whether in a game with no deterministic deadline with $T = \infty$, the exogenous stochastic exit can be used to improve the ex ante welfare of the

[20] If $\epsilon \leq 2\delta_*$, this is the only possible case. Note that α approaches infinity as ϵ approaches 0, in which case (15) reduces to (4) for the no-deadline case.

players in a way similar to the optimal finite deadline analyzed in Section 4. Since the equilibrium in the no-delay game ($\epsilon = \infty$, or equivalently $T = 0$) is efficient for any initial belief γ^0 of the uninformed below γ_*, we are only interested in the question of the optimal exogenous exit rate for $\gamma^0 > \gamma_*$.

For the first case of $\gamma^0 < \min\{1, \alpha\}$, the payoff function for the uninformed $U_\epsilon(\gamma^0)$ is identical to $U(\gamma^0)$ given by in (3), and thus does not depend on ϵ. This is because an uninformed type conceding with a positive rate is indifferent between persisting and conceding, and his payoff from conceding is computed with both the opposing uninformed type conceding and the exogenous exit occurring at the instant with probability zero. For the informed, we show in the proof of Proposition B2 in Appendix B that the payoff function $V_\epsilon(\gamma^0)$ is decreasing in ϵ so long as $\gamma^0 > \gamma_*$. The intuition behind this critical result is that an increase in the exogenous exit rate directly reduces the probability that the informed types receive their first best payoffs, which occurs only when the uninformed concede. Although an increase in ϵ generally has ambiguous effects on the equilibrium belief evolution and hence the equilibrium flow rate of concession by the uninformed, the negative direct effect dominates. The welfare effect of an increase in ϵ is negative in this case.

In the second case of $\gamma^0 \in [\min\{1, \alpha\}, 1)$. Both $V_\epsilon(\gamma^0)$ and $U_\epsilon(\gamma^0)$ are increasing in ϵ, because a greater exogenous rate of exit reduces the expected duration of the equilibrium play without affecting the decision, which is always a coin flip. The welfare effect of an increase in ϵ is positive in this case.

Thus, for any initial belief $\gamma^0 > \gamma_*$, as the exogenous exit rate ϵ increases, starting from $\epsilon = 0$ and α arbitrarily large, the welfare effect is negative for all ϵ such that $\alpha > \gamma^0$, and then positive for all greater ϵ. It follows that the optimal exogenous exit rate is either zero, which makes the game equivalent to the no-deadline game of $T = \infty$, or infinity, which is equivalent to ending the game by flipping a coin as in the equilibrium of the no-delay game of $T = 0$. In either case, we conclude that stochastic deadlines cannot be used to improve the ex ante welfare of the players. This failure of stochastic deadlines illustrates the crucial role of the deadline effect, or the deadline play, in improving the ex ante welfare of the players. Since the exogenous exit motivates the uninformed types to either always concede with a positive flow rate, or always persist, stochastic deadlines cannot generate the kind of deadline effect under a finite deadline where the equilibrium play of the uninformed types transits from an unsuccessful concession phase to a persistence phase when the time-to-deadline and the belief jointly reach some critical time horizon. The absence of such deadline effect under stochastic deadlines is the reason for its ineffectiveness in improving the ex ante welfare of the players.

5.2. Deadline penalties

A notable feature of the no-delay game with $T = 0$ in our model is that the equilibrium behavior of the uninformed, and the payoffs of both the uninformed and the informed change discontinuously as γ increases from below γ_* to above. Corresponding to this discontinuity, there is a continuum of equilibria at $\gamma = \gamma_*$ when $T = 0$. This particular feature is not critical for our results. We demonstrate this robustness by modifying the model of Section 2 and introducing an additional payoff loss $\lambda > 0$ for the two players when they fail to reach an agreement by the end of the deadline. We assume that λ is small, with $\lambda \leq \frac{1}{2}(\overline{\pi}_M - \underline{\pi}_M)$.[21]

The deadline penalty eliminates the payoff discontinuity and the multiplicity of equilibria at γ_* in the no-delay game, and redefines the boundary in the T-γ space that separates the concession and persistence phases. Define

$$\gamma_- \equiv \frac{\overline{\pi}_D - \underline{\pi}_D + 2\lambda}{\overline{\pi}_D - \underline{\pi}_D + \overline{\pi}_M - \underline{\pi}_M + 4\lambda},$$

and

$$\gamma_+ \equiv \frac{\overline{\pi}_D - \underline{\pi}_D + 2\lambda}{\overline{\pi}_D - \underline{\pi}_D + \overline{\pi}_M - \underline{\pi}_M}.$$

There is now a unique equilibrium in the no-delay game ($T = 0$) for any belief γ of the uninformed. The informed types always persist. The probability that the uninformed types concede is zero for any $\gamma \geq \gamma_+$, one for $\gamma \leq \gamma_-$, and given by

$$Y(\gamma) = \frac{\overline{\pi}_D - \underline{\pi}_D + 2\lambda}{4\lambda\gamma} - \frac{\overline{\pi}_D - \underline{\pi}_D + \overline{\pi}_M - \underline{\pi}_M}{4\lambda} \tag{16}$$

[21] This assumption says that if the state is known to be M the uninformed types still prefer the disagreement outcome of flipping a coin and paying the penalty λ to conceding to the other side and avoiding the penalty.

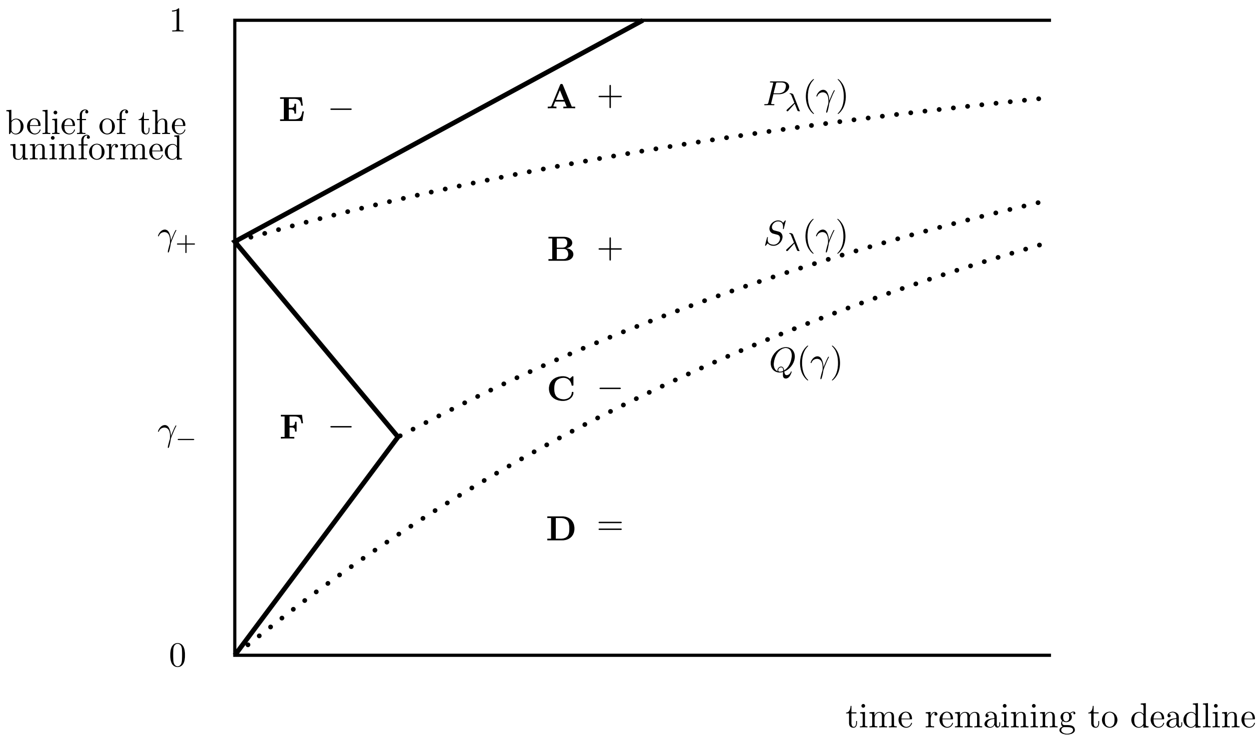

Figure 2

for $\gamma \in (\gamma_-, \gamma_+)$. The new boundary $B_\lambda(\gamma)$ is continuous, determined by the same indifference condition of the uninformed types between an immediate concession and the deadline play

$$U(\gamma) = -\delta B_\lambda(\gamma) + U^0_\lambda(\gamma),$$

where $U(\gamma)$ remains the same as before and is given by (3), and $U^0_\lambda(\gamma)$ is the unique continuation payoff in the no-delay game. The new boundary is shown as the thick piecewise linear graph in Figure 2. The main difference is that the horizontal segment corresponding to γ_* in Figure 1 is replaced by the downward sloping segment between γ_+ and γ_- in Figure 2 when $\lambda > 0$.

Both the equilibrium characterization and the welfare analysis are quite similar to those for the case $\lambda = 0$, as can be seen in Figure 2 with two of the dotted curves indexed by λ. They are formally stated as Propositions C1 and C2 and proved in Appendix C. Here, we highlight the main difference that arises in this extension, which is the welfare analysis of the deadline in Region B in Figure 2. Let $J_\lambda(\gamma^0)$ represent the phase-transition time when the updated belief hits the downward-sloping segment of the boundary B_λ. The payoff $\mathcal{V}_\lambda(J_\lambda(\gamma^0))$ to the informed types at $J_\lambda(\gamma^0)$ is:

$$-\delta(T - J_\lambda(\gamma^0)) + Y(g(J_\lambda(\gamma^0);\gamma^0))\overline{\pi}_F + \Big(1 - Y(g(J_\lambda(\gamma^0);\gamma^0))\Big)\Big(\frac{\overline{\pi}_F + \underline{\pi}_F}{2} - \lambda\Big).$$

This is the boundary condition that determines the equilibrium payoff to the informed through the differential equation (5). Using the same argument as in the case without deadline penalty, we can decompose the welfare effect of the deadline $\partial V_\lambda(\gamma^0)/\partial T$ in three terms as follows:

$$-\delta + Y'(g(J_\lambda;\gamma^0))\dot{g}(J_\lambda;\gamma^0)\Big(\frac{\overline{\pi}_F - \underline{\pi}_F}{2} + \lambda\Big)\frac{\partial J_\lambda}{\partial T} + x_\lambda(J_\lambda)(\overline{\pi}_F - \mathcal{V}_\lambda(J_\lambda))\frac{\partial J_\lambda}{\partial T}. \qquad (17)$$

Lengthening the deadline prolongs the concession phase ($\partial J_\lambda/\partial T > 0$). The cost is the additional delay, represented by the first term above, but there are two benefits, represented by the second and third terms. The second term results because a prolonged concession phase means that the updated belief is lower when it hits the boundary ($\dot{g} < 0$), and thus the uninformed types concede with a higher probability at the deadline ($Y' < 0$ for $g(J_\lambda;\gamma^0)$ between γ_+ and γ_-), reducing the chance of making the wrong decision and incurring the penalty. This term generalizes the second expression in (12) for Region B in the case of $\lambda = 0$. The third term is proportional to the flow rate of concession $x_\lambda(J_\lambda)$ by the uninformed times the relative benefit of reaching an agreement during the concession phase. This term takes the form as in (13) for Regions A and C in the case of $\lambda = 0$, but is absent from (12) because the horizontal segment in Figure 1 means that $\partial J/\partial T = 0$ when $\lambda = 0$ in Region B. In spite of the differences, in the proof of Proposition C2 in Appendix C we show that the overall effect (17) is positive, as in Region B of Figure 1.

As in Section 4 where $\lambda = 0$, the optimal deadline is 0 for $\gamma^0 \leq \gamma_-$, and is either 0 or $S_\lambda(\gamma^0)$ for $\gamma^0 > \gamma_-$, where $S_\lambda(\gamma^0)$ is such that when the belief of the uninformed as determined by $g(t;\gamma^0)$ reaches γ_- the time remaining is $B_\lambda(\gamma_-)$. That is,

$$g(S_\lambda(\gamma^0) - B_\lambda(\gamma_-);\gamma^0) = \gamma_-.$$

In the proof of Proposition C2 in Appendix C we compare the ex ante welfare at these two local maxima, and show that there exists an intermediate range of beliefs γ^0 above

λ_- for which the optimal deadline is $S_\lambda(\gamma^0)$. Thus, the optimal deadline, when positive, is still characterized by the shortest concession phase that achieves efficient information aggregation at the deadline. The main properties of the optimal deadline established for the case of $\lambda = 0$, that it is finite, bounded away from zero and increasing in the degree of conflict, are all robust to the introduction of the deadline penalty.

6. Concluding Remarks

This paper is an outgrowth of our earlier paper (Damiano, Li and Suen, 2009). In that paper we use a discrete time model with more restrictive preference assumptions to show that costly delay can improve strategic information aggregation and hence ex ante welfare in a variety of environments with regard to deadlines. See Damiano, Li and Suen (2009) for a more comprehensive list of references to related papers on dynamic games with asymmetric information. However, the discrete time framework is not suitable for studying the issue of optimal deadlines in strategic information aggregation, because an explicit characterization of equilibrium play is difficult to obtain.

In our model the positive welfare effects of extending the deadline are directly related to the deadline behavior of the uninformed stopping the concessions at some point and then conceding with a positive probability upon reaching the deadline. A longer deadline is beneficial for the informed even though the uninformed types persist for a longer period of time during the deadline play, because the latter concede with a greater probability when the deadline is reached. We have argued that the failure in inducing this deadline behavior is the reason that stochastic deadlines, or exogenous negotiation breakdowns, are ineffective in raising the ex ante welfare for the informed. However, an implicit assumption we have made in modeling stochastic deadlines is that exogenous breakdowns occur at a constant flow rate. We have not investigated either time-varying flow rates, or atoms in the flow rate. The latter case is perhaps more natural way of modeling stochastic deadlines, and is likely to generate some deadline behavior and positive welfare effects of increasing the breakdown rate. A related point is that we have assumed throughout that the two parties incur payoff losses from delay at a constant flow rate. It is possible that delay cost exhibits atoms in the flow rate that correspond to temporary suspensions of the negotiation process. We expect such atoms to generate some kind of deadline behavior and positive welfare effects. All these issues are left for future research.

In our framework of negotiation with a finite deadline, we have shown that there is a boundary in the space of the belief of the uninformed and the time remaining to the deadline, which separates the region where the uninformed types concede at the same flow rate as when there is no deadline, and the region where the uninformed types stop the concessions and the evolution of the belief until the deadline and then concede with the same probability as when there is no delay. Modifications to the no-delay game change the equilibrium play only through changing the shape of the boundary. Although we have chosen the most natural no-delay game in our setup, it would be interesting to decouple the no-delay game and the no-deadline game by considering the no-delay payoffs in other reduced forms. Doing so may provide more general insights about the deadline effects and the optimal deadlines than the present model.

Our repeated proposal game is symmetric, and we have shown that there is a unique equilibrium and it is symmetric. Games with asymmetric preferences and delay costs are worth future research because asymmetry adds an interesting element to equilibrium dynamics of information aggregation. The restriction to equilibria in which the informed types always persist is natural in our setup because the informed types know what the mutually preferred choice is. Our Assumption 1, which states that the payoff loss from making the wrong choice is greater for the informed types than the payoff loss from conceding in the disagreement state for the uninformed, is shown to be sufficient for us to focus on equilibrium play of the uninformed and turn to the informed only for welfare analysis. In a more general setup, we may have one type better but not perfectly informed about the mutually preferred choice than other types. This would be more challenging as there is no longer the dichotomy between strategic analysis and welfare analysis, but the present paper may still provide a good starting point.

Our result that the optimal deadline is positive and increasing for intermediate levels of initial conflicts hinges on two implicit assumptions about the game that may be questioned in practice. First, the two parties in the joint decision situation are assumed to be able to commit to a precise deadline at the start of the negotiation process. According to our characterization of equilibrium play, before the process reaches the critical point when the parties are supposed to become inactive until the deadline arrives, they have no incentive to renegotiate the deadline. However, as soon as the critical point is reached, they would want to jump to the end-game play immediately. Of course if such renegotiation of the deadline is anticipated the equilibrium play before this critical point would be changed. It is potentially interesting to formalize this commitment issue and reexamine the optimal deadline. The other implicit assumption we have made is that the initial belief of the uninformed is common knowledge between the two parties when setting the deadline. We hasten to emphasize that our result that extending the deadline can have positive welfare effects is robust to slight perturbations to the initial belief of the uninformed. However, a perhaps more interesting issue is whether the two parties may find some way to communicate their knowledge about the initial degrees of conflict before jointly setting the deadline for negotiation. Such communication raises strategic issues that are worth further research in the future.

Appendix

A. Proofs

Proof of Lemma 1.

For all time interval $[t, t+\mathrm{d}t)$ in $[\underline{t}, \bar{t})$, an uninformed type is indifferent between conceding, with the payoff $\mathcal{U}(t)$ given in (2), and persisting. Therefore,

$$\mathcal{U}(t) = \gamma(t)x(t)\mathrm{d}t\ \overline{\pi}_M + \Big(\gamma(t)(1 - x(t)\mathrm{d}t) + (1-\gamma(t))\Big)(-\delta \mathrm{d}t + \mathcal{U}(t+\mathrm{d}t)).$$

Subtracting $\mathcal{U}(t + \mathrm{d}t)$ from both sides of the equation, dividing by $\mathrm{d}t$, and taking the limit as $\mathrm{d}t$ goes to 0, we have a differential equation for the value function $\mathcal{U}(t)$. Using equation (2) for the value function, we can transform this differential equation for $\mathcal{U}(t)$ into a differential equation for $\gamma(t)$, given by

$$\dot{\gamma}(t) = \gamma(t)x(t)\left(\gamma(t) + \frac{\overline{\pi}_M - \overline{\pi}_D}{\overline{\pi}_D - \underline{\pi}_M}\right) - \frac{\delta}{\overline{\pi}_D - \underline{\pi}_M}.$$

By Bayes' rule, given the uninformed opponent is using the strategy represented by $x(t)$, the updated belief after persisting for the time interval $[t, t + \mathrm{d}t)$ is

$$\gamma(t+\mathrm{d}t) = \frac{\gamma(t)(1 - x(t)\mathrm{d}t)}{\gamma(t)(1-x(t)\mathrm{d}t) + (1-\gamma(t))}.$$

As $\mathrm{d}t$ goes to 0, the updating formula can be written as:

$$\dot{\gamma}(t) = -\gamma(t)(1-\gamma(t))x(t).$$

The two equations for $\dot{\gamma}(t)$ and $x(t)$ reduce to (4). Using (4) and Bayes' rule, we also get

$$x(t) = \frac{1}{\gamma(t)}\frac{\delta}{\overline{\pi}_M - \underline{\pi}_M}.$$

Proof of Proposition 1.

It suffices to show that it is optimal for the informed types to always persist. This is clearly the case at $t = Q(\gamma^0))$, implying that the continuation payoff for the informed is $\overline{\pi}_F$ when the belief of the uninformed becomes 0. Using $V(0) = \overline{\pi}_F$ as the boundary condition for the differential equation (6), and solving it, we have

$$V(\gamma) = \overline{\pi}_F - \left(1 + \frac{1-\gamma}{\gamma} \ln(1-\gamma)\right)(\overline{\pi}_M - \underline{\pi}_M).$$

The above gives the equilibrium payoff of the informed types for any $t < Q(\gamma^0)$ so that $\gamma > 0$. It is immediate from the solution that it is greater than or equal to $\overline{\pi}_F - (\overline{\pi}_M - \underline{\pi}_M)$ for all γ, which by Assumption 1 is greater than or equal to $\underline{\pi}_F$. Thus it is optimal for the informed to persist for any $t < Q(\gamma^0)$.

Proof of Proposition 2.

Using the expressions (8) and (9), we can easily verify that $B(\gamma) \leq Q(\gamma)$, with equality if and only if $\gamma = 0$. Thus, for T and γ^0 such that $T < Q(\gamma^0)$, there is a unique phase-transition time $J(\gamma^0)$ given by (10). Further, $J(\gamma^0) > 0$ if and only if $T > B(\gamma^0)$. Finally, for T and γ^0 such that $T \in (B(\gamma^0), Q(\gamma^0))$, by construction we have

$$U(g(J(\gamma^0);\gamma^0)) = U^0(g(J(\gamma^0);\gamma^0)) - \delta B(g(J(\gamma^0);\gamma^0)),$$

so that the equilibrium payoff of the uninformed is continuous at $t = J(\gamma^0)$. We discuss three cases separately.

Case (i): $T \leq B(\gamma^0)$. The construction of B implies that it is optimal for the uninformed to persist for all $t < T$ and then concede with probability y at $t = T$, with $y = 1$ if $\gamma^0 < \gamma_*$, $y = 2\delta_* T/\gamma_*$ if $\gamma^0 = \gamma_*$, and $y = 0$ if $\gamma^0 > \gamma_*$. For the informed types, at any $t \leq T$, persisting all through the deadline yields

$$y\overline{\pi}_F + (1-y)\frac{\overline{\pi}_F + \underline{\pi}_F}{2} - \delta(T-t).$$

Conceding at any $t < T$ yields $\underline{\pi}_F$, which by Assumption 1 is smaller than the above because

$$T - t < B(1) = \frac{1}{2\delta_*} = \frac{\overline{\pi}_M - \underline{\pi}_M}{2\delta}.$$

Conceding at $t = T$ cannot be optimal either because it is not part of any equilibrium of the no-delay game.

Case (ii): $T \in (B(\gamma^0), Q(\gamma^0))$. Case (i) already establishes that there is no incentive for any player to deviate at any $t \geq J(\gamma^0)$. Since the equilibrium payoff of the uninformed is continuous at $t = J(\gamma^0)$, there is no incentive for the uninformed to deviate at any $t < J(\gamma^0)$ either. For the informed types, at any $t < J(\gamma^0)$ and corresponding belief $\gamma = g(t;\gamma^0)$ of the uninformed, the equilibrium payoff $V(\gamma)$ is given by the following solution to the differential equation (6)

$$V(\gamma) = \overline{\pi}_F - \frac{1-\gamma}{\gamma}\ln(1-\gamma)(\overline{\pi}_M - \underline{\pi}_M) + \frac{1}{\gamma}\Big((1-\gamma)(C+\overline{\pi}_F) - (\overline{\pi}_M - \underline{\pi}_M)\Big),$$

where C is a constant determined by the boundary condition

$$V(g(J(\gamma^0);\gamma^0)) = y\overline{\pi}_F + (1-y)\frac{\overline{\pi}_F + \underline{\pi}_F}{2} - \delta(T - J(\gamma^0)).$$

We already know from case (i) that $V(g(J(\gamma^0);\gamma^0) \geq \underline{\pi}_F$. For any $\gamma > g(J(\gamma^0);\gamma^0)$, we have $V(\gamma) \geq \underline{\pi}_F$ if

$$\frac{(\overline{\pi}_F - \underline{\pi}_F) - (\overline{\pi}_M - \underline{\pi}_M)}{1-\gamma} - \ln(1-\gamma)(\overline{\pi}_M - \underline{\pi}_M) + C \geq -\underline{\pi}_F,$$

which is true because the left-hand-side is increasing in γ by Assumption 1. Thus, it is optimal for the informed types to persist for all $t < J(\gamma^0)$.

Case (iii): $T \geq Q(\gamma^0)$. The strategy and the belief given in the proposition form an equilibrium identical to the one in Proposition 1.

Proof of Proposition 3.

We first establish a series of claims. The proposition follows immediately after Claim 4 below.

Claim 1. In any equilibrium where the informed types always persist, $y(t) = 0$ for all $t \in (0, T)$.

Proof. First, we show that $y(t) < 1$ for both uninformed types at any time $t < T$. Suppose there is an equilibrium where an uninformed player concedes for sure at some $t < T$. Then, for any $\eta > 0$, his uninformed opponent must concede with probability one before $t + \eta$. This is because, if the game continues past t, the opponent believes that the state is the agreement state with probability one, and thus conceding immediately is optimal. Then, for η sufficiently small, persisting in the interval $[t, t+\eta)$ and then conceding at $t+\eta$ yields a strictly larger payoff than conceding with probability one at t for the initial player.

Next, we show that for any $t < T$ it cannot be the case that both uninformed types concede with strictly positive probabilities. If $\gamma(t)$ is the belief of an uninformed player upon reaching t, his equilibrium payoff is

$$\gamma(t)y(t)U_r + \gamma(t)(1-y(t))\underline{\pi}_M + (1-\gamma(t))\overline{\pi}_D,$$

where $y(t)$ is the probability that the opponent concedes at t and U_r the player's payoff in the continuation equilibrium after reverse disagreement. For any small and positive η, the payoff to the player from persisting in the interval $[t, t+\eta)$ and then conceding at $t+\eta$, is at least as large as

$$\gamma(t)y(t)\overline{\pi}_M + \gamma(t)(1-y(t))\underline{\pi}_M + (1-\gamma(t))\overline{\pi}_D - \eta\delta.$$

Because the sum of the two players' payoffs in the continuation game after reverse disagreement cannot exceed $\overline{\pi}_M + \underline{\pi}_M$, for η sufficiently small at least one of the two uninformed types has a profitable deviation.

Suppose now that one uninformed player concedes with positive probability at some $t \in (0, T)$. His equilibrium payoff upon reaching t is $U(\gamma(t))$. Further, an argument similar to the above can be used to establish that for all η sufficiently small, his uninformed opponent must persist in the interval of time $[t-\eta, t]$. This implies that the player's belief γ does not change during the same interval. Then, conceding with probability one at $t-\eta$ is uniquely optimal because the player gets the same payoff from the decision but with a smaller delay cost. This is a contradiction because we have shown that conceding with probability one at any time but the deadline cannot be part of equilibrium strategies.

Claim 2. If in equilibrium the flow concession rate for one of the uninformed players is $x(t) = 0$ for all t in an open interval $(\underline{t}, \overline{t})$, then the same is true for the other uninformed player. Further, $x(t) = 0$ for all $t \in (\underline{t}, T)$.

Proof. For the first part of the claim, suppose $x(t) = 0$ for an uninformed player in an interval $(\underline{t}, \bar{t})$. Then, his uninformed opponent's belief does not change over the same period. By the same argument as in the proof of Claim 1, if the opponent's equilibrium strategy is such that $\tilde{x}(t') > 0$ for some $t' \in (\underline{t}, \bar{t})$, it must also be the case that $\tilde{y}(t) = 1$ for all $t \in (\underline{t}, t')$, a contradiction to Claim 1. For the second part of the claim, the same argument as above implies that if $t' = \inf_{t \geq \bar{t}}\{t : x(t) > 0\} < T$, then the equilibrium strategy of this uninformed player must prescribe that $y(t) = 1$ for all $t \in (\underline{t}, t')$, again a contradiction.

Claim 3. All equilibria where the informed always persist are symmetric.

Proof. First, we show that in any equilibrium if $t' \equiv \inf\{t : x(t) = 0\} > 0$, then the beliefs of the uninformed types upon reaching t' are identical. By Claim 2, both uninformed types stop conceding at the same time t' and persist until the deadline is reached. Thus, the belief of each uninformed type upon reaching the deadline is the same as his belief upon reaching t'. Since t' is interior, Claim 1 implies that at t' both uninformed types are indifferent between conceding and persisting until the deadline and then playing the equilibrium strategy in the no-delay game associated with their own belief at t'. Suppose that upon reaching t', both the belief γ_L of the uninformed player L and the belief γ_R of the uninformed player R are above γ_*. If B is the time left to the deadline at t' the indifference conditions of the two uninformed players are

$$\gamma_L \underline{\pi}_M + (1-\gamma_L)\overline{\pi}_D = \gamma_L \frac{\overline{\pi}_M + \underline{\pi}_M}{2} + (1-\gamma_L)\frac{\overline{\pi}_D + \underline{\pi}_D}{2} - \delta B$$
$$\gamma_R \underline{\pi}_M + (1-\gamma_R)\overline{\pi}_D = \gamma_R \frac{\overline{\pi}_M + \underline{\pi}_M}{2} + (1-\gamma_R)\frac{\overline{\pi}_D + \underline{\pi}_D}{2} - \delta B.$$

The above can be satisfied for the same B only if $\gamma_L = \gamma_R$. If $\gamma_L = \gamma_R = \gamma_*$, for the two indifference conditions to hold for the same B, the equilibrium strategy upon reaching the deadline must also be symmetric. The other cases are similar.

Claim 2 above has already established the symmetry of the equilibrium strategy for uninformed types when $t' = 0$. If instead $t' > 0$, the same claim also implies that both uninformed players concede at a strictly positive flow rate in the interval $(0, t')$. Since the belief upon reaching t' is the same in equilibrium for the two uninformed types, the uniqueness of the solution to the differential equation for the equilibrium evolution of the belief implies that both the beliefs and the flow rate of concession are identical for the two players for any $t \in (0, t')$. Since the initial belief γ_0 is identical for the two players, neither player can concede with a positive probability at $t = 0$ either, thus the equilibrium strategies are symmetric for all t.

Claim 4. For any initial belief γ^0 of the uninformed types, equation (9) gives the unique value of $B(\gamma^0)$ such that in any equilibrium $x(0) = 0$ if $T < B(\gamma^0)$ and $x(0) > 0$ if $T > B(\gamma^0)$.

Proof. Fix any initial belief γ^0. Suppose that in some equilibrium $x(0) = 0$ for some deadline $T > B(\gamma^0)$. By Claim 2, we have $x(t) = 0$ for all $t \in [0, T)$. The payoff to the uninformed in this posited equilibrium is then $-T\delta + U^0(\gamma^0)$ where U^0 is the payoff function of the no-delay game (with the best equilibrium payoff corresponding to the uninformed conceding with probability one in the case of $\gamma^0 = \gamma_*$). By the construction of $B(\gamma^0)$, this payoff is strictly less than the payoff from conceding immediately at $t = 0$, a contradiction.

Now, suppose that in some equilibrium $x(0) > 0$ for some deadline $T < B(\gamma^0)$. The expected payoff to the uninformed from this posited equilibrium is equal to the payoff from conceding immediately, which is $U(\gamma^0)$. Consider the following deviation strategy for an uninformed type: persist until the deadline, and then play the unique equilibrium strategy in the no-delay game corresponding to γ^0 if $\gamma^0 \neq \gamma_*$ and concede with probability one if $\gamma^0 = \gamma_*$. For $\gamma^0 \neq \gamma_*$, since the payoff to the uninformed increases whenever the uninformed opponent concedes, and in the no-delay game the equilibrium probability of concession is decreasing in the belief of the uninformed, the payoff from this deviation is at least as large as when the opposing uninformed type follows the same deviation strategy. The same is true for $\gamma^0 = \gamma_*$, because if the uninformed opponent initially concedes with a positive flow rate for any arbitrarily small interval of time, his belief falls below γ_* in the posited equilibrium. It follows then from the construction of B that this is a profitable deviation, a contradiction.

Proof of Proposition 4.

First, we show that the welfare effect (13) is positive in Region A of Figure 1. The phase-transition time $J(\gamma^0)$ is defined by the indifference condition:

$$\delta(T - J(\gamma^0)) = U^0(g(J(\gamma^0);\gamma^0)) - U(g(J(\gamma^0);\gamma^0)) = \frac{g(J(\gamma^0);\gamma^0) - \gamma_*}{1-\gamma_*}\frac{\overline{\pi}_M - \underline{\pi}_M}{2}.$$

Taking derivative respect to T, and using the fact that $\dot{g} = -(1-g)\delta_*$, we obtain:

$$\frac{\partial J(\gamma^0)}{\partial T} = \frac{2(1-\gamma_*)}{1 - 2\gamma_* + g(J(\gamma^0);\gamma^0))}.$$

Furthermore, by Assumption 1,

$$\overline{\pi}_F - \mathcal{V}(J(\gamma^0)) = \frac{\overline{\pi}_F - \underline{\pi}_F}{2} + \delta(T - J(\gamma^0)) > \frac{\overline{\pi}_M - \underline{\pi}_M}{2}\left(1 + \frac{g(J(\gamma^0);\gamma^0) - \gamma_*}{1-\gamma_*}\right).$$

Finally, since $x(J(\gamma^0)) = \delta_*/g(J(\gamma^0);\gamma^0)$, we have

$$x(J(\gamma^0))(\overline{\pi}_F - \mathcal{V}(J(\gamma^0)))\frac{\partial J(\gamma^0)}{\partial T} > \frac{\delta}{g(J(\gamma^0);\gamma^0)} > \delta.$$

Next, we show that the welfare effect (13) is negative in Region C. The phase-transition time $J(\gamma^0)$ is defined by:

$$\delta(T - J(\gamma^0)) = g(J(\gamma^0);\gamma^0)\frac{\overline{\pi}_M - \underline{\pi}_M}{2}.$$

Take derivative respect to T to get $\partial J(\gamma^0)/\partial T = 2/(1 + g(J(\gamma^0);\gamma^0))$. Furthermore,

$$\overline{\pi}_F - \mathcal{V}(J(\gamma^0)) = \delta(T - J(\gamma^0)) = g(J(\gamma^0);\gamma^0)\frac{\overline{\pi}_M - \underline{\pi}_M}{2}.$$

Therefore,

$$x(J(\gamma^0))(\overline{\pi}_F - \mathcal{V}(J(\gamma^0)))\frac{\partial J(\gamma^0)}{\partial T} = \frac{\delta}{1 + g(J(\gamma^0);\gamma^0)} < \delta.$$

The final part of the proof is to compare the value of $W(\gamma^0)$ at the two local maxima $T = 0$ and $T = S(\gamma^0)$ for $\gamma^0 > \gamma_*$. The ex ante welfare $W^0(\gamma^0)$ for $T = 0$ is given by (14). Let $U^S(\gamma^0)$ and $V^S(\gamma^0)$ be the welfare of the uninformed and the informed when $T = S(\gamma^0)$. We have $U^S(\gamma^0) = \gamma^0\underline{\pi}_M + (1-\gamma^0)\overline{\pi}_D$ as given by (3). Solving the differential equation (6) for the payoff to the informed with the boundary condition $V(\gamma_*) = \overline{\pi}_F - \delta T_*$, we obtain

$$V^S(\gamma^0) = \overline{\pi}_F - \frac{1-\gamma^0}{\gamma^0}\left(\ln\left(\frac{1-\gamma^0}{1-\gamma_*}\right) + \frac{1}{1-\gamma^0} - \frac{2-\gamma_*^2}{2(1-\gamma_*)}\right)(\overline{\pi}_M - \underline{\pi}_M).$$

The difference in ex ante welfare $W^S(\gamma^0) - W^0(\gamma^0)$ is equal to $\Delta(\gamma^0)/(2(2-\gamma^0))$, where

$$\Delta(\gamma^0) = (1-\gamma^0)(\overline{\pi}_F - \underline{\pi}_F + \overline{\pi}_D - \underline{\pi}_D) - \gamma^0(\overline{\pi}_M - \underline{\pi}_M) \\ - \frac{2(1-\gamma^0)^2}{\gamma^0}\left(\ln\left(\frac{1-\gamma^0}{1-\gamma_*}\right) + \frac{1}{1-\gamma^0} - \frac{2-\gamma_*^2}{2(1-\gamma_*)}\right)(\overline{\pi}_M - \underline{\pi}_M).$$

Take derivative of Δ with respect to γ^0 to obtain:

$$\Delta'(\gamma^0) = -(\overline{\pi}_F - \underline{\pi}_F + \overline{\pi}_D - \underline{\pi}_D) - 3(\overline{\pi}_M - \underline{\pi}_M) \\ + \frac{2(1-(\gamma^0)^2)}{(\gamma^0)^2}\left(\ln\left(\frac{1-\gamma^0}{1-\gamma_*}\right) + \frac{1}{1-\gamma^0} - \frac{2-\gamma_*^2}{2(1-\gamma_*)}\right)(\overline{\pi}_M - \underline{\pi}_M).$$

The limit of the last term as γ^0 goes to 1 is equal to $4(\overline{\pi}_M - \underline{\pi}_M)$. Further, it is increasing for all $\gamma^0 > \gamma_*$: the derivative has the same sign as

$$-1 - (1+\gamma^0)^2 - 2\ln\left(\frac{1-\gamma^0}{1-\gamma_*}\right) + \frac{2-\gamma_*^2}{1-\gamma_*},$$

which is an increasing function of γ^0; at $\gamma^0 = \gamma_*$, this derivative is equal to $\gamma_*^3/(1-\gamma_*)$, which is positive. Thus, $\Delta'(\gamma^0) \leq -(\overline{\pi}_F - \underline{\pi}_F + \overline{\pi}_D - \underline{\pi}_D) + \overline{\pi}_M - \underline{\pi}_M$, which is negative by Assumption 1. We have proved that $\Delta(\gamma^0) = 0$ implies $\Delta'(\gamma^0) < 0$ for all $\gamma^0 > \gamma_*$. Note that $\lim_{\gamma^0 \downarrow \gamma_*} \Delta(\gamma_*) = (1-\gamma_*)(\overline{\pi}_F - \underline{\pi}_F - \gamma_*(\overline{\pi}_M - \underline{\pi}_M))$, which is positive by Assumption 1. Also, $\lim_{\gamma^0 \to 1} \Delta(\gamma^0) = -(\overline{\pi}_M - \underline{\pi}_M) < 0$. It follows from the intermediate value theorem that there exists a $\overline{\gamma} \in (\gamma_*, 1)$ such that $\Delta(\overline{\gamma}) = 0$. Moreover, the single-crossing property of Δ implies that such $\overline{\gamma}$ is unique, with $W^S(\gamma^0) > W^0(\gamma^0)$ if and only if $\gamma^0 \in (\gamma_*, \overline{\gamma})$.

B. Stochastic deadlines

Proposition B1. *Suppose that $T = \infty$ and $\epsilon > 0$. There exists a symmetric equilibrium in which the informed types always persist; the uninformed types with belief γ concede with a flow rate equal to $\epsilon(\alpha-\gamma)/(2(1-\gamma_*)\gamma)$ if $\gamma(t) \in (0, \min\{1,\alpha\})$, concede with probability one if $\gamma = 0$ and persist if $\gamma \in [\min\{1,\alpha\}, 1)$; and the belief $\gamma(t)$ of the uninformed solves (15) with the initial value γ^0 if $\gamma^0 < \min\{1,\alpha\}$, and is equal to γ^0 if $\gamma^0 \in [\min\{1,\alpha\}, 1)$.*

Proof. First, we derive the differential equation (15) for the equilibrium belief evolution. Note that the expected payoff of the uninformed from conceding is still given by (2). The payoff from persisting becomes

$$\gamma(t)x_\epsilon(t)\mathrm{d}t\,\overline{\pi}_M + \Big(\gamma(t)(1-x_\epsilon(t)\mathrm{d}t) + (1-\gamma(t))\Big)(1-\epsilon\mathrm{d}t)(-\delta\mathrm{d}t + \mathcal{U}(t+\mathrm{d}t)) \\ + \epsilon\mathrm{d}t\left((1-\gamma(t))\frac{\overline{\pi}_D + \underline{\pi}_D}{2} + \gamma(t)(1-x_\epsilon(t)\mathrm{d}t)\frac{\overline{\pi}_M + \underline{\pi}_M}{2}\right),$$

where $x_\epsilon(t)$ denotes the flow rate of concession by the uninformed. Equating the two payoff expressions and using the same Bayes' rule as in the proof of Lemma 1 immediately give us (15). The corresponding flow rate of concession is

$$x_\epsilon(t) = \frac{\epsilon(\alpha - \gamma(t))}{2(1-\gamma_*)\gamma(t)}.$$

For the case of $\gamma^0 \in (0, \min\{1, \alpha\})$, it suffices to verify that the equilibrium payoff of the informed is at least as large as the payoff from deviating to conceding, which is equal to $\underline{\pi}_F$ regardless of ϵ. The differential equation for the value function of the informed is

$$V_\epsilon'(\gamma) = -\frac{(\alpha-\gamma_*)(\overline{\pi}_M - \underline{\pi}_M) + (1-\gamma_*)(\overline{\pi}_F - \underline{\pi}_F)}{(1-\gamma)(\alpha-\gamma)} + \frac{\alpha - \gamma + 2(1-\gamma_*)\gamma}{\gamma(1-\gamma)(\alpha-\gamma)}(\overline{\pi}_F - V_\epsilon(\gamma)),$$

with the boundary condition $V_\epsilon(0) = \overline{\pi}_F$. The solution to this differential equation is

$$V_\epsilon(\gamma) = \overline{\pi}_F - \left(1 - \frac{1-\gamma}{\gamma}\frac{H(\gamma)}{2(1-\gamma_*)}\right)\frac{(\alpha-\gamma_*)(\overline{\pi}_M - \underline{\pi}_M) + (1-\gamma_*)(\overline{\pi}_F - \underline{\pi}_F)}{(\alpha-\gamma_*) + (1-\gamma_*)},$$

where

$$H(\gamma) \equiv \alpha - \alpha\left(\frac{\alpha(1-\gamma)}{\alpha-\gamma}\right)^{2\epsilon/(2\delta_* - \epsilon)}.$$

Note that $H(\gamma) > 0$ for all $\gamma \in (0, \alpha)$, regardless of whether α is greater or less than 1. Since

$$\frac{(\alpha-\gamma_*)(\overline{\pi}_M - \underline{\pi}_M) + (1-\gamma_*)(\overline{\pi}_F - \underline{\pi}_F)}{(\alpha-\gamma_*) + (1-\gamma_*)} \le \overline{\pi}_F - \underline{\pi}_F,$$

it follows immediately from Assumption 1 that $V_\epsilon(\gamma) \ge \underline{\pi}_F$ for all γ.

For the case of $\gamma^0 \in [\min\{1, \alpha\}, 1)$, in equilibrium the game ends with exogenous exit, with a terminal payoff of $\frac{1}{2}(\overline{\pi}_F + \underline{\pi}_F)$ to the informed and

$$\gamma\frac{\overline{\pi}_M + \underline{\pi}_M}{2} + (1-\gamma)\frac{\overline{\pi}_D + \underline{\pi}_D}{2}$$

to the uninformed. Further, the exogenous exit time follows an exponential distribution with parameter ϵ, and hence the expected duration of the game is $1/\epsilon$. Thus, the equilibrium expected payoff loss from delay is δ/ϵ for both the informed and the uninformed. If the uninformed types deviate to conceding, the expected payoff is

$$\gamma\underline{\pi}_M + (1-\gamma)\overline{\pi}_D < \gamma\frac{\overline{\pi}_M + \underline{\pi}_M}{2} + (1-\gamma)\frac{\overline{\pi}_D + \underline{\pi}_D}{2} - \frac{\delta}{\epsilon},$$

because $\gamma < \alpha$. For the informed, the expected payoff from concession is $\underline{\pi}_F$, which is lower than the equilibrium payoff because $\overline{\pi}_F - \underline{\pi}_F \geq 2\delta/\epsilon$, by Assumption 1 and by the assumption that $\alpha < 1$.

Proposition B2. *Suppose that $T = \infty$. For any $\gamma^0 > \gamma_*$, the optimal exogenous exit rate is either zero or infinity.*

Proof. We only need to establish that the payoff function $V_\epsilon(\gamma^0)$ for the case $\gamma^0 < \min\{1, \alpha\}$ is indeed decreasing in ϵ for $\gamma^0 > \gamma_*$.

It is convenient to use the fact that $\lim_{\gamma^0 \to 0} H(\gamma^0) = 0$ to write

$$H(\gamma^0) = \int_0^{\gamma^0} h(\gamma)\ \mathrm{d}\gamma,$$

where

$$h(\gamma) = \frac{2(1-\gamma_*)}{(1-\gamma)^2} \left(\frac{\alpha(1-\gamma)}{\alpha-\gamma}\right)^{(2\delta_*+\epsilon)/(2\delta_*-\epsilon)}.$$

The term $H(\gamma^0)(1-\gamma^0)/\gamma^0$ is decreasing in γ^0 because its derivative is

$$\begin{aligned}
&\frac{1-\gamma^0}{\gamma^0} h(\gamma^0) - \frac{1}{(\gamma^0)^2} H(\gamma^0) \\
&= -\frac{\alpha}{(\gamma^0)^2}\left(1 - \left(\frac{\alpha(1-\gamma^0)}{\alpha-\gamma^0}\right)^{2\epsilon/(2\delta_*-\epsilon)} \left(\frac{2(1-\gamma_*)\gamma^0}{\alpha-\gamma^0} + 1\right)\right) \\
&= -\frac{\alpha}{(\gamma^0)^2}\int_0^{\gamma^0} 2(1-\gamma_*)\left(\frac{\alpha(1-\gamma)}{\alpha-\gamma}\right)^{2\epsilon/(2\delta_*-\epsilon)} \frac{\gamma((\alpha-\gamma_*)+(1-\gamma_*))}{(\alpha-\gamma)^2(1-\gamma)}\mathrm{d}\gamma,
\end{aligned}$$

which is negative as $\alpha > \gamma_*$. Now, since $\lim_{\gamma^0 \to 0} H(\gamma^0) = 0$, and thus

$$\lim_{\gamma^0 \to 0} \frac{H(\gamma^0)}{\gamma^0} = \lim_{\gamma^0 \to 0} h(\gamma^0) = 2(1-\gamma_*),$$

we have

$$\frac{1-\gamma^0}{\gamma^0}\frac{H(\gamma^0)}{2(1-\gamma_*)} < 1$$

for all $\gamma^0 > 0$. Because the coefficient on $H(\gamma^0)$ in the $V_\epsilon(\gamma^0)$ function is increasing in ϵ, a sufficient condition for $V_\epsilon(\gamma^0)$ to be decreasing in ϵ is that $H(\gamma^0)$ is increasing in α. A sufficient condition for the latter is that $\ln h(\gamma^0)$ is increasing in α, or

$$-\ln\left(\frac{\alpha(1-\gamma^0)}{\alpha-\gamma^0}\right) + \frac{(\alpha-1)\gamma^0}{\alpha(1-\gamma^0)}\frac{(\alpha-\gamma_*)+(1-\gamma_*)}{2(1-\gamma_*)} > 0.$$

Since the above is equal to 0 at $\gamma^0 = 0$, it is sufficient if its derivative with respect to γ^0 is strictly positive. This derivative is given by

$$\left(\frac{\alpha-1}{\alpha-\gamma^0}\right)^2\left(\frac{1}{1-\gamma^0}-\frac{1}{2(1-\gamma_*)}\right).$$

Therefore, $V_\epsilon(\gamma^0)$ decreases with ϵ so long as $\gamma^0 > \gamma_*$.

C. Deadline penalties

PROPOSITION C1. *Suppose that $T < \infty$, and $\lambda \in (0, (\overline{\pi}_M - \underline{\pi}_M)/2]$. There is a symmetric equilibrium in which the informed types always persist; the strategy of the uninformed types at time t with any belief γ is such that: (i) if $t = T$, concede with probability one if $\gamma \le \gamma_-$, with probability 0 if $\gamma \ge \gamma_+$, and with probability $Y(\gamma)$ if $\gamma \in (\gamma_-, \gamma_+)$; (ii) if $T - t \in (0, B_\lambda(\gamma)]$, persist; and (iii) if $T - t > B_\lambda(\gamma)$, concede with a flow rate δ_*/γ if $\gamma > 0$ and with probability one if $\gamma = 0$.*

PROOF. For case (i), we show that there is a unique equilibrium in the game without delay ($T = 0$). Fix a belief γ that the state is M for an uninformed player. Suppose that the opposing uninformed type concedes with probability y. Then the difference between an uninformed player's payoff from conceding and his payoff from persisting is

$$-\gamma\Big(\frac{\overline{\pi}_M-\underline{\pi}_M}{2}-\lambda\Big)+(1-\gamma)\Big(\frac{\overline{\pi}_D-\underline{\pi}_D}{2}+\lambda\Big)-2\gamma\lambda y.$$

The above is strictly decreasing in y, and therefore there is a unique equilibrium for any γ, given as follows. If $\gamma \le \gamma_-$, then the difference in payoffs is always non-negative, and thus the unique equilibrium is $y = 1$; if $\gamma \ge \gamma_+$, the difference in payoffs is always non-positive and thus the unique equilibrium is $y = 0$; and if $\gamma \in (\gamma_-, \gamma_+)$, the unique equilibrium is $y = Y(\gamma)$, where $Y(\gamma)$ is given by (16).

For case (ii), the equilibrium payoff to the uninformed at any time $t' \in [t, T)$ from persisting throughout the game is given by

$$\gamma\Big(\tilde{Y}(\gamma)\overline{\pi}_M+(1-\tilde{Y}(\gamma))\Big(\frac{\overline{\pi}_M+\underline{\pi}_M}{2}-\lambda\Big)\Big)+(1-\gamma)\Big(\frac{\overline{\pi}_D+\underline{\pi}_D}{2}-\lambda\Big)-\delta(T-t'),$$

where

$$\tilde{Y}(\gamma) = \begin{cases} 1 & \text{if } \gamma \le \gamma_- \\ Y(\gamma) & \text{if } \gamma^0 \in (\gamma_-, \gamma_+) \\ 0 & \text{if } \gamma \ge \gamma_+. \end{cases}$$

It is straightforward to show that if $t' = t$ and $T - t = B_\lambda(\gamma)$, the above is equal to $U(\gamma)$, the deviation payoff to the uninformed from conceding at time t' given the equilibrium strategy of the uninformed opponent. Thus, there is no incentive for the uninformed to deviate for any time $t' \in [t, T)$. For the informed types, at any $t' \in [t, T]$ the equilibrium payoff from persisting is

$$\tilde{Y}(\gamma)\overline{\pi}_F + (1 - \tilde{Y}(\gamma))\Big(\frac{\overline{\pi}_F + \underline{\pi}_F}{2} - \lambda\Big) - \delta(T - t').$$

The payoff from conceding right away is $\underline{\pi}_F$. It is optimal for the informed to persist if

$$\tilde{Y}(\gamma)(\overline{\pi}_F - \underline{\pi}_F) + (1 - \tilde{Y}(\gamma))\Big(\frac{\overline{\pi}_F - \underline{\pi}_F}{2} - \lambda\Big) \ge \delta T.$$

We have just argued that the uninformed type weakly prefers persisting until the deadline followed by conceding with probability $\tilde{Y}(\gamma)$ to conceding immediately. Since $\tilde{Y}(\gamma) > 0$ for $\gamma < \gamma_+$, the equilibrium condition of the uninformed implies that

$$\gamma\tilde{Y}(\gamma)\Big(\frac{\overline{\pi}_M + \underline{\pi}_M}{2} - \lambda\Big) + \gamma(1 - \tilde{Y}(\gamma))\underline{\pi}_M + (1 - \gamma)\overline{\pi}_D - \delta(T - t) \ge U(\gamma),$$

or

$$\gamma\tilde{Y}(\gamma)\Big(\frac{\overline{\pi}_M - \underline{\pi}_M}{2} - \lambda\Big) \ge \delta(T - t).$$

By Assumption 1 and the assumption that $\lambda \le \frac{1}{2}(\overline{\pi}_M - \underline{\pi}_M)$, we have

$$\tilde{Y}(\gamma)(\overline{\pi}_F - \underline{\pi}_F) + (1 - \tilde{Y}(\gamma))\Big(\frac{\overline{\pi}_F - \underline{\pi}_F}{2} - \lambda\Big) > \frac{\overline{\pi}_F - \underline{\pi}_F}{2} - \lambda > \gamma\tilde{Y}(\gamma)\Big(\frac{\overline{\pi}_M - \underline{\pi}_M}{2} - \lambda\Big),$$

and thus the equilibrium condition of the informed is satisfied. For the case of $\gamma \ge \gamma_+$ we have $\tilde{Y}(\gamma) = 0$, and the equilibrium condition of the uninformed is

$$\gamma\Big(\frac{\overline{\pi}_M + \underline{\pi}_M}{2} - \lambda\Big) + (1 - \gamma)\Big(\frac{\overline{\pi}_D + \underline{\pi}_D}{2} - \lambda\Big) - \delta(T - t) \ge \gamma\underline{\pi}_M + (1 - \gamma)\overline{\pi}_D,$$

which implies

$$\gamma\Big(\frac{\overline{\pi}_M - \underline{\pi}_M}{2} - \lambda\Big) > \delta(T - t).$$

Thus, the equilibrium condition of the informed is satisfied.

For case (iii), for any initial belief of the uninformed γ^0, either $T > Q(\gamma^0)$, in which case the proof is the same as the case of no deadlines in Section 3, or otherwise on the equilibrium path there is a unique time $t = J_\lambda(\gamma^0)$ satisfying

$$T - J_\lambda(\gamma^0) = B_\lambda(g(J_\lambda(\gamma^0); \gamma^0)).$$

By construction, the uninformed types are indifferent between conceding and persisting for all $t \in [0, J_\lambda(\gamma^0))$, so there is no profitable deviation before $t = J_\lambda(\gamma^0)$. Further, by construction, the equilibrium payoff to the uninformed at $t = J_\lambda(\gamma^0)$ is

$$\begin{aligned}\mathcal{U}(J_\lambda(\gamma^0)) =& g(J_\lambda(\gamma^0); \gamma^0)\Big(\tilde{Y}(\gamma^0)\overline{\pi}_M + (1 - \tilde{Y}(\gamma^0))\frac{\overline{\pi}_M + \underline{\pi}_M}{2} - \lambda\Big) \\ &+ (1 - g(J_\lambda(\gamma^0); \gamma^0))\Big(\frac{\overline{\pi}_D + \underline{\pi}_D}{2} - \lambda\Big) - \delta(T - J_\lambda(\gamma^0)).\end{aligned}$$

Thus, by the argument for cases (i) and (ii) above, there is no profitable deviation for the uninformed after $t = J(\gamma^0)$ either. For the informed, given the arguments for cases (i) and (ii), it suffices to show that there is no profitable deviation before $t = J_\lambda(\gamma^0)$. The equilibrium payoff function $V_\lambda(\gamma)$ at any $\gamma = g(t; \gamma^0)$ for $t < J_\lambda(\gamma^0)$ is given by the solution to the differential equation (6) with the boundary condition that $V_\lambda(g(J_\lambda(\gamma^0); \gamma^0))$ is

$$\tilde{Y}(g(J_\lambda(\gamma^0); \gamma^0))\overline{\pi}_F + \Big(1 - \tilde{Y}(g(J_\lambda(\gamma^0); \gamma^0))\Big)\Big(\frac{\overline{\pi}_F + \underline{\pi}_F}{2} - \lambda\Big) \quad - \delta(T - J_\lambda(\gamma^0)).$$

The claim that it is optimal for the informed to persist at all $t < J_\lambda(\gamma^0)$ follows from the identical arguments in the proof of Proposition 2.

PROPOSITION C2. *Suppose that* $\lambda \in (0, (\overline{\pi}_M - \underline{\pi}_M)/2]$. *There exist thresholds* $\underline{\gamma}_\lambda$ *and* $\overline{\gamma}_\lambda$, *with* $\gamma_- < \underline{\gamma}_\lambda < \gamma_+ < \overline{\gamma}_\lambda < 1$, *such that the optimal deadline for any initial belief* γ^0 *of the uninformed is* $S_\lambda(\gamma^0)$ *if* $\gamma^0 \in (\underline{\gamma}_\lambda, \overline{\gamma}_\lambda)$, *and is* 0 *otherwise.*

PROOF. We first verify that the welfare effects are positive in Regions A and B but negative in Region C in Figure 2.

In Region B, the phase-transition time $J_\lambda(\gamma^0)$ is defined by the indifference condition for the uninformed at the boundary B_λ:

$$\delta(T - J_\lambda) = g(J_\lambda; \gamma^0)Y(g(J_\lambda; \gamma^0))\Big(\frac{\overline{\pi}_M - \underline{\pi}_M}{2} - \lambda\Big).$$

Taking derivative with respect to T, and using the definition of Y in equation (16), we obtain:

$$\frac{\partial J_\lambda}{\partial T} = \frac{8\lambda(\overline{\pi}_M - \underline{\pi}_M)}{8\lambda(\overline{\pi}_M - \underline{\pi}_M) + (1 - g(J_\lambda;\gamma^0))(\overline{\pi}_D - \underline{\pi}_D + \overline{\pi}_M - \underline{\pi}_M)(\overline{\pi}_M - \underline{\pi}_M - 2\lambda)}.$$

Now, an explicit calculation of $\partial V_\lambda(\gamma^0)/\partial T$ given in equation (17) yields:

$$\frac{\delta}{8\lambda(\overline{\pi}_M - \underline{\pi}_M)g(J_\lambda;\gamma^0)}\Big((\overline{\pi}_M - \underline{\pi}_M + 2\lambda)(\overline{\pi}_F - \underline{\pi}_F + 2\lambda) \\ + (\overline{\pi}_D - \underline{\pi}_D + \overline{\pi}_M - \underline{\pi}_M)(\overline{\pi}_M - \underline{\pi}_M - 2\lambda)(\gamma_+ - g(J_\lambda;\gamma^0))\Big)\frac{\partial J_\lambda}{\partial T} - \delta.$$

Since $\partial J_\lambda/\partial T > 0$, by Assumption 1 the above expression is greater than:

$$\delta\frac{(\overline{\pi}_M - \underline{\pi}_M + 2\lambda)^2 + (\overline{\pi}_D - \underline{\pi}_D + \overline{\pi}_M - \underline{\pi}_M)(\overline{\pi}_M - \underline{\pi}_M - 2\lambda)(\gamma_+ - g(J_\lambda;\gamma^0))}{g(J_\lambda;\gamma^0)\big(8\lambda(\overline{\pi}_M - \underline{\pi}_M) + (1 - g(J_\lambda;\gamma^0))(\overline{\pi}_D - \underline{\pi}_D + \overline{\pi}_M - \underline{\pi}_M)(\overline{\pi}_M - \underline{\pi}_M - 2\lambda)\big)} - \delta,$$

which is equal to $\delta/g(J_\lambda;\gamma^0) - \delta > 0$.

In Region A, the phase-transition time $J_\lambda(\gamma^0)$ is defined by the indifference condition:

$$\delta(T - J_\lambda) = \frac{g(J_\lambda;\gamma^0) - \gamma_*}{2(1-\gamma_*)}(\overline{\pi}_M - \underline{\pi}_M) - \lambda.$$

Take derivative respect to T to get

$$\frac{\partial J_\lambda}{\partial T} = \frac{2(1-\gamma_*)}{1 - 2\gamma_* + g(J_\lambda;\gamma^0))}.$$

Furthermore, by Assumption 1,

$$\overline{\pi}_F - \mathcal{V}(J_\lambda) = \frac{\overline{\pi}_F - \underline{\pi}_F}{2} + \delta(T - J_\lambda) + \lambda > \frac{\overline{\pi}_M - \underline{\pi}_M}{2}\frac{1 - 2\gamma_* + g(J_\lambda;\gamma^0)}{1-\gamma_*}.$$

Finally, since $x(J_\lambda) = \delta_*/g(J_\lambda;\gamma^0)$, we have

$$\frac{\partial V_\lambda(\gamma^0)}{\partial T} = -\delta + x(J_\lambda)(\overline{\pi}_F - \mathcal{V}(J_\lambda))\frac{\partial J_\lambda}{\partial T} > 0.$$

In Region C, the phase-transition time $J_\lambda(\gamma^0)$ is defined by:

$$\delta(T - J_\lambda) = g(J_\lambda;\gamma^0)\Big(\frac{\overline{\pi}_M - \underline{\pi}_M}{2} - \lambda\Big).$$

Take derivative respect to T to get

$$\frac{\partial J_\lambda}{\partial T} = \frac{2(1-\gamma_*)}{2(1-\gamma_*) - (1-g(J_\lambda;\gamma^0))(1-\gamma_+)}.$$

Furthermore,

$$\overline{\pi}_F - \mathcal{V}(J_\lambda) = \delta(T - J_\lambda) = \frac{g(J_\lambda;\gamma^0)}{2}\frac{1-\gamma_+}{1-\gamma_*}(\overline{\pi}_M - \underline{\pi}_M).$$

Therefore,

$$\begin{aligned}\frac{\partial V_\lambda(\gamma^0)}{\partial T} &= -\delta + x(J_\lambda)(\overline{\pi}_F - \mathcal{V}(J_\lambda))\frac{\partial J_\lambda}{\partial T} = -\delta + \frac{\delta(1-\gamma_+)}{2(1-\gamma_*) - (1-g(J_\lambda;\gamma^0))(1-\gamma_+)}\\ &\le \frac{2\delta(\gamma_* - \gamma_+)}{2(1-\gamma_*) - (1-\gamma_+)} < 0.\end{aligned}$$

The remainder of the proof is to compare the value of ex ante welfare $W_\lambda(\gamma^0)$ at the two local maxima of 0 and $S_\lambda(\gamma^0)$ for $\gamma^0 > \gamma_-$.

The equilibrium payoff functions for the informed and uninformed in the no-delay game are given by

$$U^0_\lambda(\gamma^0) = \begin{cases} \frac{1}{2}\gamma^0(\overline{\pi}_M + \underline{\pi}_M - 2\lambda) + (1-\gamma^0)\overline{\pi}_D & \text{if } \gamma^0 \in [0, \gamma_-),\\ \gamma^0\underline{\pi}_M + (1-\gamma^0)\overline{\pi}_D + \frac{1}{2}\gamma^0 Y(\gamma^0)(\overline{\pi}_M - \underline{\pi}_M - 2\lambda) & \text{if } \gamma^0 \in [\gamma_-, \gamma_+],\\ \frac{1}{2}\gamma^0(\overline{\pi}_M + \underline{\pi}_M - 2\lambda) + \frac{1}{2}(1-\gamma^0)(\overline{\pi}_D + \underline{\pi}_D - 2\lambda) & \text{if } \gamma^0 \in (\gamma_+, 1); \end{cases}$$

and

$$V^0_\lambda(\gamma^0) = \begin{cases} \overline{\pi}_F & \text{if } \gamma^0 \in [0, \gamma_-),\\ Y(\gamma^0)\overline{\pi}_F + \frac{1}{2}(1 - Y(\gamma^0))(\overline{\pi}_F + \underline{\pi}_F - 2\lambda) & \text{if } \gamma^0 \in [\gamma_-, \gamma_+],\\ \frac{1}{2}(\overline{\pi}_F + \underline{\pi}_F) - \lambda & \text{if } \gamma^0 \in (\gamma_+, 1). \end{cases}$$

Under the deadline $T = S_\lambda(\gamma^0)$, the payoff to the uninformed is simply $U^S_\lambda(\gamma^0) = U(\gamma^0)$ as in (3). To compute the payoff to the informed, we solve the differential equation (6) with the boundary condition

$$V_\lambda(\gamma_-) = \overline{\pi}_F - \delta B_\lambda(\gamma_-).$$

This gives the payoff to the informed when the deadline is $T = S_\lambda(\gamma^0)$:

$$\begin{aligned}V^S_\lambda(\gamma^0) = \overline{\pi}_F &- \frac{1-\gamma^0}{\gamma^0}\left(\ln\Big(\frac{1-\gamma^0}{1-\gamma_-}\Big) + \frac{\gamma^0 - \gamma_-}{(1-\gamma^0)(1-\gamma_-)}\right)(\overline{\pi}_M - \underline{\pi}_M)\\ &- \frac{1-\gamma^0}{\gamma^0}\frac{\gamma_-^2}{1-\gamma_-}\Big(\frac{\overline{\pi}_M - \underline{\pi}_M}{2} - \lambda\Big).\end{aligned}$$

The difference in ex ante welfare $W_\lambda^S(\gamma^0) - W_\lambda^0(\gamma^0)$ is

$$\frac{1}{2-\gamma^0}(U_\lambda^S(\gamma^0) - U_\lambda^0(\gamma^0)) + \frac{1-\gamma^0}{2-\gamma^0}(V_\lambda^S(\gamma^0) - V_\lambda^0(\gamma^0)) \equiv \frac{1}{2(2-\gamma^0)}\Delta_\lambda(\gamma^0).$$

Since $Y(\gamma_-) = 1$, we have

$$\Delta_\lambda(\gamma_-) = -\gamma_-(\overline{\pi}_M - \underline{\pi}_M - 2\lambda) - \gamma_-(1-\gamma_-)(\overline{\pi}_M - \underline{\pi}_M - 2\lambda) < 0.$$

Since $Y(\gamma_+) = 0$, we have

$$\begin{aligned}\Delta_\lambda(\gamma_+) = (1-\gamma_+)(\overline{\pi}_F - \underline{\pi}_F - 2\lambda) - \frac{2(1-\gamma_+)^2}{\gamma_+}\frac{\gamma_-^2}{1-\gamma_-}(\overline{\pi}_M - \underline{\pi}_M - 2\lambda) \\ - \frac{2(1-\gamma_+)^2}{\gamma_+}\left(\ln\left(\frac{1-\gamma_+}{1-\gamma_-}\right) + \frac{\gamma_+ - \gamma_-}{(1-\gamma_+)(1-\gamma_-)}\right)(\overline{\pi}_M - \underline{\pi}_M).\end{aligned}$$

Using Assumption 1, we can show that

$$\Delta_\lambda(\gamma_+) \geq \frac{1-\gamma_+}{\overline{\pi}_M - \underline{\pi}_M + 2\lambda}\left((1-\gamma_-)(\overline{\pi}_M - \underline{\pi}_M - 2\lambda)^2 + 8(1-\gamma_+)\lambda(\overline{\pi}_M - \underline{\pi}_M)\right) > 0.$$

Thus, there exists a $\underline{\gamma}_\lambda \in (\gamma_-, \gamma_+)$ such that $\Delta_\lambda(\underline{\gamma}_\lambda) = 0$. Taking derivatives of $\Delta_\lambda(\gamma^0)$ with respect to $\gamma^0 \in (\gamma_-, \gamma_+)$ and evaluating at $\underline{\gamma}_\lambda$ using $\Delta_\lambda(\underline{\gamma}_\lambda) = 0$ yield

$$\begin{aligned}&\frac{\gamma_+(1-\gamma_-)}{\underline{\gamma}_\lambda(\gamma_+ - \gamma_-)}(\overline{\pi}_F - \underline{\pi}_F + 2\lambda) + \frac{\gamma_-(2\underline{\gamma}_\lambda - \gamma_+(1+\underline{\gamma}_\lambda))}{\underline{\gamma}_\lambda(1-\underline{\gamma}_\lambda)(\gamma_+ - \gamma_-)}(\overline{\pi}_M - \underline{\pi}_M - 2\lambda) - 2(\overline{\pi}_M - \underline{\pi}_M) \\ >&\frac{1-\gamma_-}{\gamma_+ - \gamma_-}(\overline{\pi}_F - \underline{\pi}_F + 2\lambda) + \frac{2\gamma_- - \gamma_+(1+\gamma_-)}{(1-\gamma_-)(\gamma_+ - \gamma_-)}(\overline{\pi}_M - \underline{\pi}_M - 2\lambda) - 2(\overline{\pi}_M - \underline{\pi}_M) \\ >&\frac{(1-\gamma_-)\gamma_+}{\gamma_+ - \gamma_-}(\overline{\pi}_M - \underline{\pi}_M + 2\lambda) + \frac{2\gamma_- - \gamma_+(1+\gamma_-)}{(1-\gamma_-)(\gamma_+ - \gamma_-)}(\overline{\pi}_M - \underline{\pi}_M - 2\lambda) - 2(\overline{\pi}_M - \underline{\pi}_M),\end{aligned}$$

where the first inequality follows because the first term in the expression is decreasing in $\underline{\gamma}_\lambda$ while the second term is increasing in $\underline{\gamma}_\lambda$, and the second inequality uses Assumption 1 and the assumption that $\lambda \leq \frac{1}{2}(\overline{\pi}_M - \underline{\pi}_M)$. The above can be shown to be equal to

$$\left(\frac{\overline{\pi}_M - \underline{\pi}_M}{2} - \lambda\right)\left(\frac{\overline{\pi}_D - \underline{\pi}_D}{\lambda}\left(\frac{\overline{\pi}_D - \underline{\pi}_D}{\overline{\pi}_M - \underline{\pi}_M + 2\lambda} + \frac{3}{2}\right) + \frac{\overline{\pi}_M - \underline{\pi}_M - 2\lambda}{\overline{\pi}_M - \underline{\pi}_M + 2\lambda} + \frac{\overline{\pi}_M - \underline{\pi}_M}{\lambda} - 2\right),$$

which is positive because $\lambda \leq \frac{1}{2}(\overline{\pi}_M - \underline{\pi}_M)$. As a result, $\underline{\gamma}_\lambda$ is unique, with $\Delta_\lambda(\gamma^0) > 0$ if $\gamma^0 \in (\underline{\gamma}_\lambda, \gamma_+)$, and the opposite holding if $\gamma^0 \in (\gamma_-, \underline{\gamma}_\lambda)$.

At the other end, we have

$$\lim_{\gamma^0 \to 1} \Delta_\lambda(\gamma^0) = -(\overline{\pi}_M - \underline{\pi}_M - 2\lambda) < 0.$$

Thus, there exists a $\overline{\gamma}_\lambda \in (\gamma_+, 1)$ such that $\Delta_\lambda(\overline{\gamma}_\lambda) = 0$. The derivative of $\Delta_\lambda(\gamma^0)$ with respect to $\gamma^0 \in (\gamma_+, 1)$ is given by

$$\begin{aligned} &-(\overline{\pi}_F - \underline{\pi}_F + \overline{\pi}_D - \underline{\pi}_D + 2\lambda) - 3(\overline{\pi}_M - \underline{\pi}_M) + \frac{(1-(\gamma^0)^2)\gamma_-^2}{(\gamma^0)^2(1-\gamma_-)}(\overline{\pi}_M - \underline{\pi}_M - 2\lambda) \\ &+ \frac{2(1-(\gamma^0)^2)}{(\gamma^0)^2}\left(\ln\left(\frac{1-\gamma^0}{1-\gamma_-}\right) + \frac{\gamma^0 - \gamma_-}{(1-\gamma^0)(1-\gamma_-)}\right)(\overline{\pi}_M - \underline{\pi}_M). \end{aligned}$$

As in the case of $\lambda = 0$, the sum of the last two terms in the above expression is increasing in γ^0 and approaches $4(\overline{\pi}_M - \underline{\pi}_M)$ as γ^0 approaches 1. Thus,

$$\Delta'_\lambda(\gamma^0) < -(\overline{\pi}_F - \underline{\pi}_F + \overline{\pi}_D - \underline{\pi}_D + 2\lambda) + (\overline{\pi}_M - \underline{\pi}_M) < 0,$$

because $\lambda \leq \frac{1}{2}(\overline{\pi}_M - \underline{\pi}_M)$. It follows that $\overline{\gamma}_\lambda$ is uniquely defined in $(\gamma_+, 1)$, and $\Delta_\lambda(\gamma^0) > 0$ for $\gamma^0 \in (\gamma_+, \overline{\gamma}_\lambda)$ and the opposite holds for $\gamma^0 \in (\overline{\gamma}_\lambda, 1)$.

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