Legislative Bargaining with Reconsideration

Daniel Diermeier  Pohan Fong

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2Department of Managerial Economics and Decision Sciences (MEDS) and Ford Motor Company Center for Global Citizenship, Kellogg School of Management, and Northwestern Institute on Complex Systems (NICO), Northwestern University.

3Department of Managerial Economics and Decision Sciences (MEDS), Kellogg School of Management, Northwestern University.
Abstract

We present a dynamic model of legislative bargaining with an endogenously evolving default policy and a persistent agenda setter. Policy-making proceeds until the agenda setter can no longer pass a new policy to replace an approved bill. We prove existence and necessary conditions of pure-strategy stationary equilibria for any finite policy space, any number of players and any preference profile. In equilibrium, the value of proposal power is limited compared to the case that disallows reconsideration, as voters are induced to protect each other’s benefits in order to maintain their future bargaining positions. The agenda setter, in turn, would prefer to limit his ability to reconsider. The lack of commitment due to the possibility of reconsideration, however, enhances policy efficiency.

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1 Introduction

The agenda-setting model of Romer and Rosenthal (1978; 1979) constitutes one of the central building blocks of political economy models of policy making. In their seminal paper, an agenda setter makes a policy proposal, which is then pitted against a default alternative in an up-or-down majority vote. This approach captures a prominent feature in modern democratic polities, that there is typically an authority (an executive, commission, committee and so forth) that effectively holds agenda control, yet that its power is nonetheless checked by the requirement of majority approval. This model and its applications have yielded two fundamental insights. First, the existence of the "power to propose" provides an agenda setter with the ability to bias policy outcomes in his favor even in the case where a median voter exists, as he can make a policy proposal to satisfy a bare majority of the "cheapest" voters necessary to ensure approval and extract all the additional surplus. Second, the policy outcome not only depends on voter’s preferences but also on the location of the default policy, which defines the reservation utility of the voters.

The Romer-Rosenthal model assumes that decision-making on a given issue ends when a majority approves a proposal. This is appropriate in some policy domains, for example, the school budget referenda studied by Romer and Rosenthal (1979), but not in others. In many policy areas, default policies persist until they are changed. This, for example, is true for entitlement programs such as social security. In turn this means that legislators may be able to reconsider a passed bill or existing policy during a legislative session. In other words, in a dynamic policy environment, the passage of a bill does not prevent the legislature from coming back to the same policy issue at a later date. Rather, in this context the passage of a bill merely sets the default for subsequent rounds of policy making. This feature was recently pointed
out by Bernheim, Rangel and Rayo (2006).

In this paper we combine the agenda-setting model with an institutional arrangement that allows for reconsideration. As in Romer and Rosenthal (1978), we assume a single agenda setter, who is conferred the sole power to make proposals throughout a legislative session. Proposal power is persistent as, after any proposal is approved by a majority, the same agenda setter may initiate reconsideration to the approved bill by making a new policy proposal. Policy-making proceeds until the agenda setter is either unable or unwilling to change the previously approved bill. This apparently minor modification nevertheless leads to radically different implications, as there arise incentives among the non-proposing legislators, or voters, to limit the ability of the agenda setter to expropriate others.

We are, of course, not the first to propose a dynamic version of the agenda-setting model. Indeed, the most widely applied model of legislative bargaining, due to Baron and Ferejohn (1989a), can be interpreted as such a dynamic version, in which counteroffers are allowed in case a proposal is turned down. In a subsequent paper Baron (1996) considered a dynamic framework where a policy adopted in the current period becomes the status quo policy for the next one. In these models and the growing literature that followed, agenda setters are randomly recognized in all rounds of decision-making.\textsuperscript{1} In a more abstract sense our model and the above mentioned models all constitute dynamic extensions of the Romer-Rosenthal model but differ in the institutional features of decision making after the first round of proposal making and voting. In the Baron-Ferejohn model the next proposer is drawn from the collection of voters, whereas we assume a single, persistent agenda setter in

\textsuperscript{1}See Banks and Duggan (2000, 2006) and Duggan and Kalandrakis (2007) for general specifications of legislative bargaining models with random recognition of agenda setters. Battaglini and Coate (2007; 2008) provide recent applications of this approach in public finance. Riboni (2008) is a rare exception that assumes a single, unchanged proposer over time in the context of monetary policy.
each round.

The Baron-Ferejohn approach has become so ubiquitous in models of legislative bargaining that it may seem to be the only "natural model" of dynamic agenda setting. Yet, there are various institutional contexts where a model with persistent proposal power seems like a better formal representation of the political institution. Robinson and Torvik (2008) argue that elected presidents hold persistent legislative agenda-setting power in various multiparty presidential countries in Africa and Latin America. In the U.S. Congress, committees and their chairs may effectively control proposal power over certain policy issues and be able to block amendments (Knight, 2005). Other examples include decision-making in cabinets with agenda control by the prime minister (Döring 1995). Finally, Riboni (2008) argues that a central bank is often dominated by a strong chairman, whose policy must be approved by members of the board of directors. More generally, our model captures majoritarian decision-making where proposal power rests with a single leader subject to majority approval. Arguably, this was the motivation for the original Romer-Rosenthal model.

At the very least such apparent institutional variation may by itself warrant the exploration of alternative models of policy bargaining. But perhaps more important are the theoretical questions raised. The most important insight of the Romer-Rosenthal model, preserved and elaborated by the Baron-Ferejohn model, is the "power to propose."\(^2\) This may suggest that granting more *de jure* power, here the sole power to initiate reconsideration, to the agenda setter would only enhance his *de facto* power, i.e. allow the agenda setter to pass a more favorable proposal. To the contrary, our results show that the exact opposite holds: the agenda setter’s power is weakened when he is granted more power to reconsider the approved policy. Indeed, an agenda setter would prefer to commit himself not to reconsider any policy in the future. As

\(^2\)This was the title of a lesser known follow-up paper by Baron and Ferejohn (1989b).
we show below, the possibility of reconsideration in equilibrium induces a group of voters to "defend" the benefits for one another. In particular, self-interested voters may decline any policy proposal when some other voters are substantially expropriated. In equilibrium, voters protect others as a means to prevent the agenda setter from playing off the voters against each other in the future. Intuitively, voter $J$ protects voter $K$ so that the agenda setter cannot use the low reservation value of $K$ to exploit $J$ when the policy is reconsidered. The incentive of mutual protection among the voters therefore effectively constrains the agenda setter’s ability to expropriate, resulting in a more equal allocation of the benefit. The equilibrium value of proposal power is thus substantially limited. This mechanism is illustrated in the following example.

**Example.** Consider a legislature with three players. The first player is the agenda setter throughout the legislative session. The legislature must divide 10 dollars among its members, where each dollar is assumed to be indivisible. Suppose that the initial default policy is $x = (3, 3, 4)$, where the $i$-th element of $x$ refers to the amount that goes to the $i$-th player. In the agenda-setting model where reconsideration is not allowed, the agenda setter would propose $y = (7, 3, 0)$, which would be approved by the second player, who is satisfied by her reservation value given by the default policy.

Now consider the case where the agenda setter is allowed, with a sufficiently large probability, to reconsider the policy issue and make another proposal. We argue that in this case the second player would not accept policy $y = (7, 3, 0)$ in equilibrium, even though $y$ yields exactly the same benefit as the default. To see why, consider counterfactually what would happen if this player accepted $y$. Following the acceptance, $y$ would become the new default policy. The agenda setter would then have an incentive to reconsider the policy issue and propose $z = (10, 0, 0)$, which
would not be vetoed by the indifferent third player. This implies that the second player would eventually be fully expropriated if she voted for $y$ in the first proposal round.

Applying the same logic, we conclude that the agenda setter is not able to pass any policy that offers the third player (whose vote is not necessary for policy approval) any amount less than 3 dollars, given that for a sufficiently large probability the agenda setter would get a chance to reconsider any approved policy. The equilibrium policy outcome is thus $(4, 3, 3)$, a much more egalitarian division of the benefit. In this equilibrium, the second player is induced to defend the benefit for the third player, since by doing so the second player secures her long-term bargaining position in the legislature. Whereas the agenda setter has an incentive to expropriate as much as possible provided his policy obtains majority support, he is constrained by the voters who protect each other in equilibrium. As a consequence, the value of proposal power is substantially reduced compared to the single-period case.

In this paper we generalize this example to a model with an arbitrary finite policy space, any number of voters and any preference profile. The core of the analysis is an algorithm we propose to construct a set of policy alternatives which would persist as default in equilibrium (henceforth a stable set). Since the policy converges in the long run, any policy alternative outside the equilibrium stable set cannot appear as a final policy outcome. With this algorithm we prove the existence and necessary conditions of stationary Markov perfect equilibria with pure strategies. In all such equilibria and regardless of the policy space, proposal power is endogenously limited compared to the case that disallows reconsideration. So, the agenda setter is unambiguously worse off with the power to reconsider.

Our paper is closely related to Bernheim, Rangel and Rayo (2006), who also
propose a distributive model of legislative bargaining with the possibility of reconsideration. They assume a finite number of proposal rounds and common knowledge of the proposer sequence. Given that a sufficient number of players can make proposals, in the unique equilibrium the last proposer has nearly dictatorial power and implements a sufficiently unequal distribution of the benefit. We complement the results of Bernheim et al. by showing that endogenous constraints on proposal power arise if a single party is persistently granted the power to propose and reconsider policy. This insight reshapes our understanding about how the allocation of proposal power shapes policy outcomes and sheds light on issues of institutional design.

Our paper is also linked to the methodological literature on the existence and characterization of stationary equilibria in dynamic games. Notice that in dynamic legislative bargaining models with an endogenous default a stationary equilibrium need not exist, and, if it does, it is usually associated with mixed strategies. This was shown in different models by Kalandrakis (2004; 2007), Fong (2006), Battaglini and Palfrey (2007) and Penn (2009). Duggan and Kalandrakis (2007) prove general existence of a pure-strategy stationary equilibrium for this class of dynamic games, but only with some suitably assumed randomness on preferences and the dynamic process of the default policy. Although our model does not satisfy the sufficient conditions of Duggan and Kalandrakis, pure-strategy stationary equilibria still exist. This property makes our analytical framework tractable for applications in dynamic policy environments or specific contexts, for example, in public finance. The proposed algorithm also provides a "toolkit" for solving such models (see Diermeier and Fong, 2008b; 2009).

The possibility of reconsideration can be interpreted as lack of commitment by the agenda setter. Whereas it has been commonly understood that lack of commitment by policymakers could be a source of policy inefficiency, the model considered here
may yield the opposite conclusion.\textsuperscript{3} As the agenda setter has an incentive to fully exploit the legislators with disadvantaged bargaining positions in the future, a majority of voters implicitly coordinate to vote against any proposal that substantially expropriates some of the other voters. Therefore, lack of commitment by the agenda setter induces the whole legislature to commit to choosing the policy from an effectively smaller set of policy alternatives. For example, sufficient unequal allocations of public resources are excluded from consideration in a model of pork-barrel politics as well as in the case of public goods production. As a consequence, the possibility of reconsideration not only leads to more equal distributions but also enhances policy efficiency in those applications.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 defines a stationary Markov perfect equilibrium with pure strategies. Section 4 proposes an algorithm to construct an equilibrium stable set, and proves the existence and necessary conditions of an equilibrium. Section 5 demonstrates the analytical technique and new insights through a distributive model with three players. Section 6 generalizes the insight of endogenous constraint on proposal power for an arbitrary policy space and discusses some additional implications. Section 7 illustrates more examples with various policy environments, including a model of public goods production, a model with the pork-barrel policy space and a spatial model. Section 8 concludes.

\textsuperscript{3}The commitment problem was first formally addressed by Kydland and Prescott (1977). More recent political economy studies of government policies include Persson and Svensson (1989), Tabellini and Alesina (1990), Besley and Coate (1998), Acemoglu and Robinson (2001), and Baron, Diermeier and Fong (2008), to name only a few. See Acemoglu (2003) for a comprehensive survey of the commitment literature in political economy.
2 The Model

Let $N = \{1, 2, ..., n\}$ be a set of $n$ players in the legislature, where $n = 2m + 1$ and $m \in \mathbb{N}$. The legislature must collectively choose a policy $x$ from a finite policy space denoted by $X$. Preferences of any player $\ell$ are represented by a von Neumann-Morgenstern utility function $u_\ell(x)$. We refer to $u = (u_1, u_2, ..., u_n)$ as a preference profile.

There is one agenda setter in the legislature. Assume this position is occupied by player 1. The agenda setter is conferred the sole power to make policy proposals from the policy space during the legislative session. All other players, $i \in N \setminus \{1\}$, are referred to as voters.

The legislature selects a policy over the course of potentially infinite rounds of proposal making and voting, where the number of rounds depends on exogenous factors and the decision made by the agenda setter.

As the legislative session commences, an initial default $x^0 \in X$ is exogenously given. The initial default is interpreted as the policy that has been enacted prior to the legislative session. Since then, activities prior to round $t$ establish a default $x^{t-1} \in X$. In round $t$, the agenda setter either chooses to make a proposal $y^t \in X$ or pass the proposal round. A "pass" means inaction by the agenda setter and, for mathematical convenience, is modeled as a proposal $y^t = x^{t-1}$. The proposal $y^t$ is then put to an immediate vote against the default $x^{t-1}$. If the proposal is approved by majority rule, it replaces $x^{t-1}$ as the new default policy and $x^t = y^t$. If the proposal is not approved, the default policy then remains the same and $x^t = x^{t-1}$. The default policy thus evolves endogenously across proposal rounds. At the point the legislative

\footnote{A discrete policy space limits the extent to which utilities are transferable among the players, and is critical to our main results. From the perspective of modeling real-life policy issues, however, this assumption seems innocuous. For example, entitlement programs usually involve a minimal spending unit, even if it is very small, say a dollar.}
session ends, the policy that survives as default is implemented.

In contrast to Bernheim et al. (2006) who assume a fixed number of proposal rounds, in the model considered here the last proposal round is not predetermined. We say the legislative session ends \textit{endogenously} after proposal round $t$, if the default $x^t$ established by the first $t$ rounds of proposal making and voting is such that the agenda setter will choose to pass any possible proposal round $t' > t$. In addition, after any proposal round the legislative session may be terminated \textit{exogenously} with probability $1 - \delta$, where $\delta \in [0, 1)$ is the probability that the agenda setter will have an opportunity to reconsider the policy that emerges from the current round.

We interpret $\delta$ as a parameter of the legislative institution, since various legislative rules, unmodeled here, may affect the likelihood of chances for reconsideration. For example, the case of $\delta = 0$ is associated with the agenda-setting model of Romer and Rosenthal (1978). In this paper we intend to focus on institutions where legislative actions are very likely to continue until the session ends endogenously. In other words, we study the case in which $\delta < 1$ with $\delta$ sufficiently close to 1.\footnote{The case of $\delta = 1$ admits a plethora of equilibria, as the bargaining position of a voter is solely determined by what she believes to happen eventually.} This assumption is maintained throughout this paper, although not repeated unless it is necessary.

\section{Equilibrium Definition}

As is customary in the legislative bargaining literature we focus the analysis on stationary Markov perfect equilibria, in which the players condition their strategies only on the prevailing default policy.\footnote{See Baron and Ferejohn (1989a) and Austen-Smith and Banks (2005) for justifications of stationary equilibria in legislative bargaining games.} From now on, we drop the subscript $t$ for the proposal round from the notations.

Let $f : X \rightarrow X$ be the (pure) proposal strategy of the agenda setter. In particular,

\begin{equation*}
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\end{equation*}
\( f(x) \) denotes the policy that the agenda setter proposes when the prevailing default is \( x \).\(^7\)

Let \( U_\ell(x) \) be the expected utility of player \( \ell \) if policy \( x \) is approved. With probability \( 1 - \delta \) the legislative session is exogenously terminated after the current proposal round and this player receives a utility of \( u_\ell(x) \). With probability \( \delta \) the agenda setter has a chance to reconsider the approved policy and make a new proposal according to strategy \( f \). In this case, player \( \ell \) receives a continuation utility of \( U_\ell(f(x)) \). Thus,

\[
U_\ell(x) = (1 - \delta) u_\ell(x) + \delta U_\ell(f(x)).
\] (1)

We refer to \( U_\ell : X \to \mathbb{R} \) as the value function of player \( \ell \).

We make two technical assumptions regarding how the players break indifference. First, any player votes against a policy proposal if and only if passage of the proposal makes him strictly worse off. Second, the agenda setter never proposes any shift in policy that is destined to be vetoed by a majority of voters. None of our qualitative results depend on the first assumption. The second one simplifies the notation, but is otherwise innocuous, since making a losing proposal is equivalent to remaining at the prevailing default.

In each proposal round, the agenda setter selects a policy proposal to maximize his expected utility. A policy is politically feasible if, as a proposal, it would be approved by a majority of players. By the second assumption above, the maximization problem of the agenda setter is subject to the constraint of political feasibility. By the first assumption above, the feasibility constraint is equivalent to an incentive

\(^7\)Diermeier and Fong (2008a) also construct mixed-strategy stationary equilibria for the game with a distributive policy. In those equilibria the sole proposer strategically designs a sequence of proposals with randomization to achieve his ideal point eventually. Diermeier and Fong argue that if the legislature needs to make a collective decision on whether to discuss a policy, i.e. to put it on the agenda, those mixed-strategy equilibria disappear and only the pure-strategy equilibria survive.
compatibility constraint that requires that a majority of players be weakly better off with the proposal policy than with the prevailing default. To sum up, given any default \( x \in X \), \( f(x) \) must solve

\[
\max_{y \in X} U_1(y) \quad \text{s.t.} \quad \left| \{ \ell \in N : U_\ell(y) \geq U_\ell(x) \} \right| \geq m + 1,
\]

(2)

where, for any finite set \( A \), \(|A|\) denotes the number of its elements. We are now ready to define an equilibrium.

**Definition 1** A stationary Markov perfect equilibrium is a proposal strategy \( f \) and a set of value functions \( \{U_\ell\}_{\ell=1}^n \) such that:

1. Given \( f \), \( \{U_\ell(x)\}_{\ell=1}^n \) solve the equation system defined by (1).

2. Given \( \{U_\ell\}_{\ell=1}^n \), \( f(x) \) solves problem (2) of the agenda setter for any default \( x \in X \).

For any proposal strategy \( f \) and any \( x \in X \), let \( f^0(x) \equiv x \) and \( f^t(x) \equiv f(f^{t-1}(x)) \) for all \( t \in \mathbb{N} \). A policy path, \( \{f^t(x)\}_{t=0}^\infty \), thus traces the evolution of default along an equilibrium path that starts with an initial default \( x \in X \). We further restrict attention to equilibria where any policy path converges. Given that we assume a single persistent agenda setter, it is natural to rule out the equilibria where the policy oscillates until the legislative session is exogenously terminated.\(^8\)

**Definition 2** A stationary Markov perfect equilibrium \((f, \{U_\ell\}_{\ell=1}^n)\) is well-behaved if and only if, for all \( x \in X \), there exists \( T \in \mathbb{N} \) such that \( f(f^T(x)) = f^T(x) \).

In this paper we characterize any well-behaved, pure-strategy stationary Markov perfect equilibrium and simply call it an equilibrium.

\(^8\)See Diermeier and Fong (2008a) for an example of such equilibrium.
4 Analysis

4.1 An Algorithm

For any equilibrium \((f, \{U_t\}_{t=1}^n)\), let

\[ S_f = \{ x \in X : f(x) = x \} \]

be the equilibrium stable set and refer to any \(x \in S_f\) as a stable policy. In words, a stable policy persists as default. In principle, the boundary of a stable set depends on the equilibrium proposal strategy \(f\), so two different equilibria may imply two distinct stable sets.

Let \(z_1 \in X\) be an ideal point of the agenda setter. As the agenda setter would never make a new proposal to replace his ideal point as default, any equilibrium stable set is nonempty since \(z_1 \in S_f\).

Our characterization of an equilibrium rests on an algorithm that could be applied to construct an equilibrium stable set. For a more concise presentation, we write \(y \succeq x\) and say \(y\) dominates \(x\) if two conditions are met: (A) \(u_1(y) \geq u_1(x)\); and (B) there exists \(M \subset N \setminus \{1\}\) such that \(|M| = m\) and \(u_i(y) \geq u_i(x)\) for all \(i \in M\).

Algorithm 1 Construct a policy set \(\hat{S} \subset X\) through the following steps:

1. Let \(Y_0 = X\) and \(K = 0\).
2. Let \(k = K\).
3. Let \(C_k\) be any nonempty subset of

\[ C^*_k \equiv \arg\max_{x \in Y_k} u_1(x) \, . \tag{3} \]
4. Let

\[ D_k \equiv \{ x \in Y_k \setminus C_k : \exists y \in C_k \text{ s.t. } y \succeq x \}, \quad (4) \]

and

\[ Y_{k+1} \equiv Y_k \setminus (C_k \cup D_k). \quad (5) \]

5. If \( Y_{k+1} = \emptyset \), then let

\[ \hat{S} \equiv \bigcup_{k'=0}^{k} C_{k'}. \quad (6) \]

Otherwise let \( K = k + 1 \) and repeat Steps 2-5.

Let \( S \) be the collection of all policy sets that can be constructed by the algorithm. This algorithm has three technical features. First, \( \hat{S} \neq \emptyset \) for all \( \hat{S} \in S \). Any policy set constructed by the algorithm must contain at least an ideal policy of the agenda setter. Second, the iteration must end in finite rounds given that the policy space is finite. Third, multiple policy sets may be constructed by the algorithm, due to the degree of freedom in constructing \( C_k \) when \( C_k^* \) is not a singleton. If the maximization problem defined by (3) has a unique solution in every round of the iteration, a unique policy set is constructed by the algorithm.

### 4.2 Equilibrium Existence

Theorem 1 applies the algorithm to characterize a class of equilibria in which reconsideration does not actually occur.

**Theorem 1** For any \( \hat{S} \in S \), there exists an equilibrium \( (f, \{U_\ell\}_{\ell=1}^n) \) such that:

1. for all \( x \in X \) and all \( \ell \in N \),

\[ U_\ell (x) = (1 - \delta) u_\ell (x) + \delta u_\ell (f (x)); \quad (7) \]
2. for all $x \in \hat{S}$, $f(x) = x$;

3. for all $x \notin \hat{S}$, $f(x)$ is an element of

$$F\left(x; \hat{S}\right) \equiv \left\{ y \in \hat{S} : y \succeq x \right\}.$$  \hfill (8)

**Proof of Theorem 1.** Consider a proposal strategy $f$ and a set of value functions $\{U_\ell\}_{\ell=1}^n$ that satisfy conditions 1-3 for some $\hat{S}$ constructed by the algorithm along with $\{C_k, C_k^*, D_k, Y_k\}$. Through a series of claims we prove that $(f, \{U_\ell\}_{\ell=1}^n)$ constitute an equilibrium. Claim 1 shows that $F\left(x; \hat{S}\right) \neq \emptyset$ for all $x \notin \hat{S}$ and therefore $f(x)$ is well-defined. Claims 2 and 5 provide instrumental results useful for the rest of the proof. Claim 3 shows that $\{U_\ell(x)\}_{\ell=1}^n$ solve the equation system defined by (1), so Condition 1 of Definition 1 is satisfied. Claims 4 and 6 jointly show that $f(x)$ solves problem (2) of the agenda setter for any default $x \in X$, so Condition 2 of Definition 1 is satisfied. Respectively, Claims 4 and 6 prove that $f(x)$ is politically feasible and that no other politically feasible policy can do strictly better than $f(x)$ for the agenda setter.

**CLAIM 1.** For all $x \notin \hat{S}$, $F\left(x; \hat{S}\right) \neq \emptyset$.

**PROOF.** Take any $x \notin \hat{S}$. Without loss of generality, assume that $x \in D_k$ for some $k \in \mathbb{Z}_+$. Note that $C_k \neq \emptyset$ since $D_k \neq \emptyset$. Then take any $y \in C_k$. By (4) and (6), $y \succeq x$ and $y \in \hat{S}$. Therefore $y \in F\left(x; \hat{S}\right)$.

**CLAIM 2.** For all $x \in X$ and $\ell \in N$, (a) $U_\ell(f(x)) = u_\ell(f(x))$; and (b) $u_\ell(x) > u_\ell(f(x))$ if and only if $U_\ell(x) > U_\ell(f(x))$.

**PROOF.** These directly follow (7) and the fact that $f(f(x)) = f(x)$ for all $x$.

**CLAIM 3.** For all $\ell \in N$, $U_\ell$ satisfies equation (1).
PROOF. This directly follows (7) and Claim 2.

CLAIM 4. For all \( x \in X \), (a) \( U_1 ( f ( x )) \geq U_1 ( x ) \); and (b) there exists \( M \subseteq N \setminus \{1\} \) such that \( |M| = m \) and \( U_i ( f ( x )) \geq U_i ( x ) \) for all \( i \in M \).

PROOF. The claim is obviously true for all \( x \in \hat{S} \), so take any \( x \notin \hat{S} \). By (8), \( u_1 ( f ( x )) \geq u_1 ( x ) \) and there exists \( M \subseteq N \setminus \{1\} \) such that \( |M| = m \) and \( u_i ( f ( x )) \geq u_i ( x ) \) for all \( i \in M \). Then by Claim 2, for all \( j \in M \cup \{1\} \), \( U_j ( f ( x )) \geq U_j ((x)) \).

CLAIM 5. For all \( x, y \in X \) and \( \ell \in N \), if \( u_1 ( f ( x )) > u_1 ( f ( y )) \) then \( U_\ell ( x ) > U_\ell ( y ) \), \( U_\ell ( x ) > U_\ell ( f ( y )) \) and \( U_\ell ( f ( x )) > U_\ell ( y ) \).

PROOF. By (7) and given that \( \delta < 1 \) is sufficiently large, \( U_1 ( y ) \) and \( U_1 ( x ) \) are sufficiently close to \( u_1 ( f ( y )) \) and \( u_1 ( f ( x )) \), respectively. The rest directly follows.

CLAIM 6. For all \( x, y \in X \), either \( U_1 ( f ( x )) \geq U_1 ( y ) \), or there exists \( M_+ \subset N \) such that \( |M_+| \geq m + 1 \) and \( U_i ( x ) > U_i ( y ) \) for all \( i \in M_+ \).

PROOF. Let \( k (x), k(y) \in \text{Z}_+ \) be such that \( f ( x ) \in C_{k(x)} \) and \( f ( y ) \in C_{k(y)} \). We discuss the three cases below. Case 1. Suppose that \( u_1 ( f ( x )) > u_1 ( f ( y )) \). Then by Claim 5, \( U_1 ( f ( x )) > U_1 ( y ) \). Case 2. Suppose that \( u_1 ( f ( x )) < u_1 ( f ( y )) \). Then \( k (x) > k (y) \). This implies that \( f ( x ) \in Y_{k(y)} \setminus (C_{k(y)} \cup D_{k(y)}) \). By definition of \( D_{k(y)} \), there exists \( M_+ \subset N \) such that \( |M_+| = m + 1 \) and \( u_i ( f ( x )) > u_i ( f ( y )) \) for all \( i \in M_+ \). Then by Claims 5, \( U_i ( x ) > U_i ( y ) \) for all \( i \in M_+ \). Case 3. Suppose that \( u_1 ( f ( x )) = u_1 ( f ( y )) \). If \( u_1 ( y ) > u_1 ( f ( y )) \), then by Claim 5, \( U_1 ( y ) > U_1 ( f ( y )) \). This contradicts the optimality of \( f ( y ) \) for the agenda setter. Therefore, it must be the case that \( u_1 ( y ) \leq u_1 ( f ( y )) = u_1 ( f ( x )) \). Then by Claim 2, \( U_1 ( f ( x )) = U_1 ( f ( y )) \geq U_1 ( y ) \). ■

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*Suppose to the contrary that \( k (x) \leq k (y) \), then \( f ( y ) \in Y_{k(x)} \). Since \( f ( x ) \in C_{k(x)} \), \( u_1 ( f ( x )) \geq u_1 ( f ( y )) \). This contradicts the condition that \( u_1 ( f ( x )) < u_1 ( f ( y )) \).
Theorem 1 has several notable implications. First, reconsideration of a passed bill does not actually occur in any equilibrium characterized by Theorem 1. However, the possibility of reconsideration changes the nature of the policy-making game, as the agenda setter is endogenously constrained to select a policy proposal from some \( \hat{S} \in \mathcal{S} \) instead of the whole policy space \( X \).

Second, in dynamic legislative bargaining games existence of a pure-strategy stationary equilibria is usually not assured. Duggan and Kalandrakis (2007) prove the existence of a pure-strategy stationary equilibrium for a general class of dynamic legislative bargaining games in which the default policy endogenously evolves, but only with some suitably assumed randomness on preferences and the dynamic process of the default. While our model does not satisfy the sufficient conditions of Duggan and Kalandrakis, Theorem 1 shows that a class of pure-strategy equilibria still exists given that the policy space is finite and the probability of a chance to reconsider is sufficiently high.

Third, for any \( \hat{S} \in \mathcal{S} \), there exists at least one equilibrium \( (f, \{U_t\}_{t=1}^\infty) \) in which the equilibrium stable set is exactly the constructed policy set, i.e. \( S_f = \hat{S} \).

Fourth, there may exist multiple equilibria. Technically, multiplicity arises due to the degrees of freedom to construct \( C_k \) by (3), when \( C^*_k \) is more than a singleton, and to construct \( f(x) \) by (8) for all \( x \notin \hat{S} \), when \( F(x; \hat{S}) \) is more than a singleton.

Intuitively, different equilibria result from self-fulfilling expectations of the players. In every proposal round all players anticipate the equilibrium strategies to be played in all subsequent rounds, and based on this common expectation they calculate their reservation values that determine the strategies for the current proposal round. Therefore, players’ expectations determine their current play. Stationarity requires that the expectations on future strategies be consistent with the strategies carried out in the current round. As we focus on institutions in which \( \delta < 1 \) is sufficiently large,
the players’ future becomes disproportionally important as the players are mainly concerned about the stable policies that they would eventually reach. Multiple equilibria thus result from the existence of multiple pairs of expectation and strategy that are consistent with each other.

Some additional assumptions could be made to reduce the set of equilibria. In these cases the proposed algorithm could be modified accordingly. For example, Diermeier and Fong (2008a) characterize equilibria in which the stable set is symmetric with respect to positions of the voters and pin down a unique equilibrium stable set. Diermeier and Fong (2009) assume that in the legislature there is not only an agenda setter, who is granted proposal power by the legislative procedures, but also a coordinating legislator who ensures common beliefs among the players. Whereas the agenda setter controls *de jure* power to submit proposals, the coordinator controls *de facto* power to direct players’ beliefs and actions. In principle, the coordinator could be the same as or different from the agenda setter, and this may depend on various features in a legislature including norms or conventions. A general discussion of equilibrium selection or equilibrium refinement, however, is beyond the scope of this paper.

Given the possibility of multiple equilibria, it is important to characterize a set of necessary conditions so as to capture common properties that prevail in all equilibria. We do this in the next section.

### 4.3 Necessary Conditions

Given the requirement of well-behavedness, any policy path induced by an equilibrium proposal strategy $f$ must converge. For any $x \in X$, let $f^\infty (x) \equiv \lim_{t \to \infty} f^t (x)$. Given that the policy space is finite, $f^\infty (x) = f^t (x)$ for all $t$ sufficiently large. Therefore $f (f^\infty (x)) = f^\infty (x)$ and $f^\infty (x) \in S_f$ for all $x \in X$. For any initial default $x$, we
will call \( f^\infty(x) \) the \textit{final policy outcome} if the legislative session lasts for sufficiently many proposal rounds so that in realization the stable set \( S_f \) is reached. This section focuses on general properties of an equilibrium stable set and the bounds on final policy outcomes.

Lemma 1 first provides conditional inequalities useful for development of the main results.

**Lemma 1** Consider any equilibrium \((f, \{U_\ell\}_{\ell=1}^n)\). For all \( x \in X \) and \( \ell \in N \):

1. \( U_\ell(f(x)) \geq U_\ell(x) \iff U_\ell(f(x)) \geq u_\ell(x) \).

2. \( u_\ell(f^\infty(x)) < u_\ell(x) \Rightarrow U_\ell(f(x)) < u_\ell(x) \).

**Proof.** Part 1 directly follows (1). To prove Part 2, take any \( x \in X \) and \( \ell \in N \) such that \( u_\ell(f^\infty(x)) < u_\ell(x) \). Let \( T \in \mathbb{N} \) be such that (a) \( f^t(x) = f^\infty(x) \) for all \( t \geq T \), and (b) either \( T = 1 \) or \( f^{t+1}(x) \neq f^t(x) \) for all \( t \leq T - 1 \). Then

\[
U_\ell(f(x)) = (1 - \delta) \left( \sum_{t=1}^{T-1} \delta^{t-1} u_\ell(f^t(x)) \right) + \delta^{T-1} u_\ell(f^\infty(x)).
\]

Given that \( \delta < 1 \) is sufficiently large, \( U_\ell(f(x)) \) is sufficiently close to \( u_\ell(f^\infty(x)) \) so that \( U_\ell(f(x)) < u_\ell(x) \).

Theorem 2 shows that, in any equilibrium, the final policy outcome must dominate the initial default.

**Theorem 2** For any equilibrium \((f, \{U_\ell\}_{\ell=1}^n)\) and for any \( x \in X \),

\[
f^\infty(x) \succeq x;
\]
i.e. (A) \( u_1(f^\infty(x)) \geq u_1(x) \), and (B) there exists \( M \subset N \setminus \{1\} \) such that \( |M| = m \) and \( u_i(f^\infty(x)) \geq u_i(x) \) for all \( i \in M \).

**Proof of Theorem 2.**  Part A. Suppose that \( u_1(x) > u_1(f^\infty(x)) \). Then by Lemma 1, \( u_1(x) > U_1(f(x)) \) and \( U_1(x) > U_1(f(x)) \). This contradicts the optimality of \( f(x) \) for the agenda setter. Part B. Suppose to the contrary that there exists \( M_+ \subset N \setminus \{1\} \) such that \( |M_+| = m + 1 \) and \( u_i(x) > u_i(f^\infty(x)) \) for all \( i \in M_+ \). Then by Lemma 1, for all \( i \in M_+ \), \( u_i(x) > U_i(f(x)) \) and \( U_i(x) > U_i(f(x)) \). This contradicts political feasibility of \( f(x) \).

Intuitively, as we assume that the agenda setter is sufficiently likely to have a chance to reconsider any passed bill, players are mainly concerned about how their proposal making and voting will lead to the final *stable* policy outcome. In other words, if it takes more than one proposal round to reach the final policy outcome, any policy approved in a transitional proposal round only contributes insignificantly to the calculation of expected utilities by the players. What the players do care about concerning the path of transitional policies is where it leads to – the eventual stable policy. The agenda setter, for example, would avoid proposing any policy that would eventually transition to a stable policy that makes him strictly worse off. Similarly, no policy is politically feasible if it would lead to some stable policy where a majority of voters would be strictly worse off.

Regardless of the initial default, the final policy outcome must be a stable policy. Therefore, we need to characterize the equilibrium stable set. Theorem 3 shows that any equilibrium stable set is constructible by the algorithm.

**Theorem 3** For any equilibrium \( (f, \{U_i\}_{i=1}^n) \),

\[
S_f \in \mathcal{S}.
\]
In other words, there exists $\hat{S} \in \mathcal{S}$ such that $f^\infty (x) \in \hat{S}$ for all $x \in X$, i.e.,

$$f (x) = x \leftrightarrow x \in \hat{S}.$$ 

**Proof of Theorem 3.** Take any equilibrium $(f, \{U_\ell\}_{\ell=1}^n)$. The proof proceeds by math induction through Claims 1-5.

**CLAIM 1.** For any $\{C_k, C^*_k, D_k, Y_k\}$ constructed by the algorithm, $C^*_0 \cap S_f \neq \emptyset$.

**PROOF.** Suppose that $C^*_0 \cap S_f = \emptyset$. Note that $C^*_0 \neq \emptyset$ so take any $x \in C^*_0$. Since $f^\infty (x) \in S_f$, $f^\infty (x) \notin C^*_0$ by supposition. Then $u_1 (x) > u_1 (f^\infty (x))$ by (3) for $k = 0$. By Lemma 1, $u_1 (x) > U_1 (f (x))$ and $U_1 (x) > U_1 (f (x))$. This contradicts the optimality of $f (x)$ for the agenda setter.

**CLAIM 2.** Take any $K \in \mathbb{N}_+$ and let $\{C_k, C^*_k, D_k, Y_k\}$ be constructed by the algorithm such that $Y_K \neq \emptyset$ and $C_K \subseteq S_f$. Then $(D_K \setminus C^*_K) \cap S_f = \emptyset$.

**PROOF.** Suppose that $(D_K \setminus C^*_K) \cap S_f \neq \emptyset$ and take any $x \in (D_K \setminus C^*_K) \cap S_f$. Also take any $y \in C_K$. Since $C_K \subseteq S_f$, $y \in S_f$. By (3) and (4) for $k = K$, (a) $u_1 (y) > u_1 (x)$ and (b) there exists $M \subset N \setminus \{1\}$ such that $|M| = m$ and $u_i (y) \geq u_i (x)$ for all $i \in M$. Since $x, y \in S_f$, $U_\ell (x) = u_\ell (x)$ and $U_\ell (y) = u_\ell (y)$ for all $\ell \in N$. Therefore, $U_1 (y) > U_1 (x)$ and $U_i (y) \geq U_i (x)$ for all $i \in M$. This implies that $f (x) \neq x$ and $x \notin S_f$, which is a contradiction.

**CLAIM 3.** There exists $\{C'_k, C''_k, D'_k, Y'_k\}$ constructible by the algorithm such that $C'_1 \subseteq S_f$ and $D'_1 \cap S_f = \emptyset$.

**PROOF.** Let $\{C'_k, C''_k, D'_k, X'_k\}$ be constructed by the algorithm such that $C'_1 = C''_1 \cap S_f$. By Claim 1, $C'_1 \neq \emptyset$. By construction, $C'_1 \subseteq S_f$ and $(C''_1 \setminus C'_1) \cap S_f = \emptyset$. By Claim 2, $(D'_1 \setminus C''_1) \cap S_f = \emptyset$. Note that $D'_1 = (D'_1 \setminus C''_1) \cup (C''_1 \setminus C'_1)$. Therefore $D'_1 \cap S_f = \emptyset$. 

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CLAIM 4. Take any \( K \in \mathbb{N} \) and let \( \{C_k, C_k^*, D_k, Y_k\} \) be constructed by the algorithm such that, for all \( k \leq K \), \( Y_K \neq \emptyset \), \( C_k \subseteq S_f \) and \( D_k \cap S_f = \emptyset \). If \( Y_{K+1} \neq \emptyset \), then (A) \( x \in Y_{K+1} \Rightarrow f^{\infty}(x) \in Y_{K+1} \), and (B) \( C_{K+1}^* \cap S_f \neq \emptyset \).

PROOF. Part A. Take any \( x \in Y_{K+1} \) and suppose that \( f^{\infty}(x) \notin Y_{K+1} \). Since \( f^{\infty}(x) \in S_f \), \( f^{\infty}(x) \notin \bigcup_{k=0}^{K} D_k \). Then \( f^{\infty}(x) \in \bigcup_{k=0}^{K} C_k \) by (5) and (6). Without loss of generality assume that \( f^{\infty}(x) \in C_k \) for some \( k \leq K \). Since \( x \in Y_{K+1} \subseteq Y_k \) and \( x \notin (C_k \cup D_k) \), \( u_1(f^{\infty}(x)) \geq u_1(x) \) and \( f^{\infty}(x) \notin x \). This implies that there exists \( M_+ < N \setminus \{1\} \) such that \( |M_+| = m + 1 \) and \( u_i(x) > u_i(f^{\infty}(x)) \) for all \( i \in M_+ \). By Lemma 1, for all \( i \in M_+ \), \( u_1(x) > U_1(f(x)) \) and \( U_1(x) > U_1(f(x)) \). This contradicts political feasibility of \( f(x) \).

Part B. The argument is in parallel to that for Claim 1. Suppose that \( C_{K+1}^* \cap S_f = \emptyset \). Note that \( C_{K+1}^* \neq \emptyset \) since \( Y_{K+1} \neq \emptyset \). So take any \( x \in C_{K+1}^* \). Note that \( x \in Y_{K+1} \) and therefore \( f^{\infty}(x) \in Y_{K+1} \) by Part A of the claim. Since \( f^{\infty}(x) \in S_f \), \( f^{\infty}(x) \notin C_{K+1}^* \) by supposition. Since \( x \in C_{K+1}^* \) and \( f^{\infty}(x) \in Y_{K+1} \), \( u_1(x) > u_1(f^{\infty}(x)) \) by (3) for \( k = K + 1 \). By Lemma 1, \( u_1(x) > U_1(f(x)) \) and \( U_1(x) > U_1(f(x)) \). This contradicts the optimality of \( f(x) \) for the agenda setter.

CLAIM 5. Suppose that, for some \( K \in \mathbb{N} \), \( \{C_k, C_k^*, D_k, Y_k\} \) is constructed by the algorithm such that, for all \( k \leq K \), \( C_k \subseteq S_f \) and \( D_k \cap S_f = \emptyset \). If \( Y_{K+1} \neq \emptyset \), then there exists \( \{C'_k, C'_k^*, D'_k, Y'_k\} \) constructible by the algorithm such that, for all \( k \leq K + 1 \), \( C'_k \subseteq S_f \) and \( D'_k \cap S_f = \emptyset \).

PROOF. The argument is in parallel to that for Claim 3. Let \( \{C'_k, C'_k^*, D'_k, Y'_k\} \) be constructed by the algorithm such that \( C'_k = C_k \) for all \( k \leq K \) and \( C'_{K+1} = C'_{K+1}^* \cap S_f \). By Claim 4, \( C'_{K+1} \neq \emptyset \). By construction, \( C'_{K+1} \subseteq S_f \) and \( (C'_{K+1} \setminus C'_{K+1}^*) \cap S_f = \emptyset \). Since \( C'_{K+1} \subseteq S_f \), \( (D'_{K+1} \setminus C'_{K+1}) \cap S_f = \emptyset \) by Claim 2. Note that \( D'_{K+1} = (D'_{K+1} \setminus C'_{K+1}) \cup (C'_{K+1} \setminus C'_{K+1}) \). Therefore \( D'_{K+1} \cap S_f = \emptyset \).
Whereas Theorem 1 shows that any policy set constructed by the algorithm is the stable set in some equilibrium, Theorem 3 shows that the stable set in any equilibrium is constructible by the algorithm. These theorems thus jointly show that the collection of all possible equilibrium stable sets is identical to the collection of all policy sets constructible by the algorithm.

Theorems 2 and 3 also jointly imply that, given any initial default \( x \in X \), the final policy outcome could be any policy in \( F \left( x; \hat{S} \right) \) for some \( \hat{S} \in \mathcal{S} \), where \( F \) is defined by (8). Recall Part 3 of Theorem 1 and we can see that restricting the focus to equilibria in which no reconsideration occurs does not reduce the set of possible policy outcomes, given that the legislative session lasts for sufficiently many rounds so that a stable policy is reached. Therefore, if we only focus on policy outcomes we miss nothing by ignoring equilibria in which reconsideration actually does occur.\(^{10}\)

5 Example: A Distributive Model with Three Players

This section fully characterizes a distributive model with three players, indexed by \( \ell \in \{1, 2, 3\} \), where player 1 is assumed to be the sole agenda setter. This example illustrates how the algorithm can be applied and identifies the key mechanism at work. The policy is to divide \( \pi \in \mathbb{N} \) units of fixed benefits among the three players. The policy space is therefore \( X = \Delta^3_\pi \equiv \{ x \in \mathbb{Z}^3_+ : \sum_{\ell=1}^3 x_\ell = \pi \} \). Given any policy \( x = (x_1, x_2, x_3) \in X \), player \( \ell \) receives \( x_\ell \) units of benefits and derives a utility of \( u_\ell (x) = x_\ell. \(^{11}\)

\(^{10}\)See Diermeier and Fong (2008a) for an example of a pure-strategy stationary equilibrium in which a passed bill is actually reconsidered in some contingency.

\(^{11}\)The same analysis applies if \( u_\ell (x) = v_\ell (x_\ell) \), where \( v_\ell (x_\ell) \) is a strictly increasing function for all \( \ell \in \{1, 2, 3\} \).
By the algorithm, for all $k \in [0, \frac{1}{2}\pi]$,

$$Y_k = \{ x \in X : \min \{x_2, x_3\} \geq k \},$$

$$C_k = C_k^* = \{ x \in X : x_2 = x_3 = k \},$$

$$D_k = \{ x \in X : \max \{x_2, x_3\} > \min \{x_2, x_3\} = k \},$$

and a unique policy set

$$\hat{S} = \{ x \in X : x_2 = x_3 \}$$

is constructed. By Theorem 3, this is the unique stable set in any equilibrium. By Theorems 2 and 3, for any initial default $x \in X$, the final policy outcome $f^\infty(x)$ is such that $f_2^\infty(x) = f_3^\infty(x) \geq \min \{x_2, x_3\}$ and $f_1^\infty(x) = \pi - 2 \min \{x_2, x_3\}$. Intuitively, reconsideration leads to more egalitarian allocations as the possibility of reconsideration induces the two voters to "defend" the benefits for each other. In particular, a voter may decline a policy proposal if the other voter is substantially expropriated, as this prevents the agenda setter from playing off the voters against each other in the future. In equilibrium the agenda setter must allocate the same amount of benefits to both voters. As a consequence, the agenda setter receives strictly less than what he would do if reconsideration was not allowed, i.e. $\pi - \min \{x_2, x_3\}$.

Theorem 1 can also be applied to construct equilibria in which reconsideration does not actually occur. In particular $f(x) = x$ for any default $x \in X$ such that $x_2 = x_3$; otherwise, $f(x)$ can be an arbitrary element of a subset of policies $F(x, \hat{S})$, where by (8),

$$F(x, \hat{S}) = \{ (\pi - 2e, e, e) \in X : \min \{x_2, x_3\} \leq e \leq \frac{1}{2} (x_2 + x_3) \}.$$

Proposition 1 summarizes the analysis.
Proposition 1 Assume $n = 3$, $X = \Delta_3^\rho$, and $\delta < 1$ sufficiently large. (Necessity)

Any legislative equilibrium with policy rule $f$ must be such that, for all $x \in X$,

$$f_2^\infty (x) = f_3^\infty (x) \geq \min \{x_2, x_3\} \quad \text{and} \quad x_1 \leq f_1^\infty (x) \leq x_1 + \max \{x_2, x_3\} - \min \{x_2, x_3\}.$$ 

(Sufficiency) For all functions $e : X \to \mathbb{Z}_+$ such that, for all $x \in X$, $\min \{x_2, x_3\} \leq e (x) \leq \frac{1}{2} (x_2 + x_3)$, there exists a legislative equilibrium with policy rule $f$ such that, for all $x \in X$, $f_1 (x) = \pi - 2e (x)$ and $f_2 (x) = f_3 (x) = e (x)$.

Below we discuss key implications derived from the second half of the proposition. In those constructed equilibria, the agenda setter always proposes a stable policy immediately. In other words, reconsideration never occurs on an equilibrium path. The possibility of reconsideration, however, changes the nature of proposal making and voting.

In equilibrium the agenda setter receives no less than his benefit entitlement by default; i.e. $f_1 (x) \geq x_1$. On the other hand, the two voters receive the same amount of benefits in equilibrium, and this amount is no less than the default benefit entitlement of the voter who receives less benefits from the default among the two voters; i.e. $f_2 (x) = f_3 (x) \geq \min \{x_2, x_3\}$. Crucially, this implies that the voter whose vote is not needed to pass the new policy is not fully expropriated by the agenda setter. In fact, the agenda setter can gain at most $\max \{x_2, x_3\} - \min \{x_2, x_3\}$ units of the benefits. As a consequence, in equilibrium the benefit level received by the agenda setter is bounded above by $\pi - 2\min \{x_2, x_3\}$, and this is smaller than $\pi - \min \{x_2, x_3\}$, the agenda setter’s benefits in the case of legislative bargaining where reconsideration is not allowed.

The equilibrium proposal strategy can intuitively be described as follows: the agenda setter seeks voting support from the voter with a lower reservation value and expropriates the other voter to the extent that the two voters receive the same
amount of benefits. In a dynamic setup with an evolving default, however, it is not necessarily trivial to assess which voter is the cheaper one to satisfy. In our case the critical voter is the one who receives less from the default. To see why, consider any $x \in X$ such that $x_2 < x_3$. Since $x_2 \leq e(x) < x_3$, the reservation values of players 2 and 3 are calculated as $U_2(x) = (1 - \delta) x_2 + \delta e(x) \in (e(x) - 1, e(x)]$ and $U_3(x) = (1 - \delta) x_3 + \delta e(x) > e(x)$, given that $\delta < 1$ is sufficiently large. In equilibrium, the agenda setter offers $e(x)$ units of benefits to both voters. Among all stable policies, this is the one that just satisfies player 2 by her reservation value and maximizes the expected utility of the agenda setter.

Why does the agenda setter have to offer both voters an equal amount of benefits? Suppose that the agenda setter offers $e(x)$ units of benefits to player 2 but only some $k < e(x)$ units to player 3. It is obvious that player 3 will vote against the proposal since her benefit level is reduced. But so will player 2. To understand why, consider counterfactually, what would happen if player 2 approved the policy $y = (\pi - e(x) - k, e(x), k)$. With probability $1 - \delta$ the legislative session would end immediately and $y$ would be implemented. With probability $\delta$, however, the agenda setter would have a chance to reconsider the policy issue and propose a new policy $f(y) = (\pi - 2e(y), e(y), e(y))$ according to his equilibrium strategy. This policy makes player 3 at least as well off as with $y$ and therefore would be approved by majority voting. Since $e(y) \leq \frac{1}{2} (e(x) + k) < e(x)$, by voting for policy $y$ player 2 would be worse off when the policy is reconsidered. Anticipating such an adverse consequence, player 2 will always vote against the proposal of $y$, even though she receives $e(x) \geq x_2$ units of benefits from this policy. By this argument, player 2 will not allow the agenda setter to expropriate the other voter too much so that, in the

\[12\] For example, Bernheim, Rangel and Rayo (2006) use backward induction to identify the set of voters from whom an agenda setter optimally seeks voting support.
subsequent proposal rounds, the other voter will have a lower reservation value than hers and look more attractive for the agenda setter to ally with. As a consequence, the best the agenda setter can achieve is to offer both voters equal amount of benefits and just satisfy the voter who is given less by the default.

Although the voters derive utilities only from the benefits they receive, in equilibrium they have indirect preferences over the distribution of benefits. In the above example, player 2 strictly prefers \((\pi - 2e(x), e(x), e(x))\) to \((\pi - e(x) - k, e(x), k)\), where \(k < e(x)\), even though either policy, if realized, gives her \(e(x)\) units of benefits. Through the dynamic link of an evolving default, the allocation of benefits affects the distribution of bargaining power in the rest of the legislative session.

Therefore, the two voters effectively demand a more egalitarian allocation of resources between them. In particular, any voter does not allow the other voter to be sufficiently expropriated by the agenda setter. This demand for "fairer" allocations results from self-interested voters who want to improve their long-term bargaining positions. It does not depend on primitive preferences for fair allocations or risk aversion.

On the other hand, the agenda setter has an incentive to expropriate as much as possible. The agenda setter proposes less benefits for himself compared to the case without reconsideration because mutual protection between the voters imposes endogenous constraints on the set of policies that can be approved by majority voting in equilibrium. As a consequence, the agenda setter has limited ability to expropriate the voter whose vote is not needed.

Notice also that, compared to the case without reconsideration, granting the agenda setter the power to reconsider reduces his equilibrium payoff. Therefore, an agenda setter would have an incentive to commit not to reconsider a passed bill. In the model considered here such a promise would not be credible.
6 Endogenous Limits on Proposal Power

The previous section has hinted the additional endogenous constraints the sole agenda setter faces when he is granted power to reconsider an approved policy. In this section we fully develop this idea for the general model, with an arbitrary finite policy space $X$, any number of players and any preference profile.

Assume any initial default $x \in X$. We first consider a standard agenda-setting institution, in which there is only one round of proposal making and voting, i.e. $\delta = 0$ (Romer and Rosenthal, 1978). Let $g(x)$ denote any policy outcome from this institution. Then $g(x)$ must be politically feasible, i.e.

$$g(x) \in G(x) \equiv \{y \in X : y \succeq x\},$$

(9)

and maximizing the agenda setter’s utility, i.e.

$$u_1(g(x)) \geq u_1(y) \text{ for all } y \in G(x).$$

(10)

We then consider the institution that allows reconsideration with $\delta < 1$ sufficiently large. By Theorems 2 and 3, the final policy outcome $f^\infty(x)$ in any equilibrium must satisfy

$$f^\infty(x) \in F\left(x; \hat{S}\right) = G(x) \cap \hat{S}$$

(11)

for some $\hat{S} \in \mathcal{S}$.

A comparison of (9) and (11) shows that the agenda setter effectively faces a more stringent constraint when he is granted power to reconsider passed bills than when he is not allowed to do so. With $\delta < 1$ sufficiently large, the agenda setter must make a policy choice such that eventually the policy converges to some $\hat{S} \in \mathcal{S}$. With this...
additional constraint, the value of proposal power is in general more limited than if reconsideration was not allowed. The next theorem formalizes this insight.

**Theorem 4** Assume, as elsewhere in this paper, that $\delta < 1$ is sufficiently large. For any equilibrium $(f, \{U_t\}_{t=1}^\infty)$ and any default $x \in X,$

$$U_1(f(x)) \leq U_1(f^\infty(x)) \leq u_1(g(x)),$$

where $u_1(g(x))$ would be the equilibrium utility of the agenda setter if reconsider was not allowed, i.e. $\delta = 0.$

**Proof of Theorem 4.** Take any $x \in X.$ First note that, for any $t \in Z_+,$ the agenda setter cannot be strictly worse off by choosing $f^{t+1}(x)$ when the default is $f^t(x).$ Therefore,

$$U_1(f^t(x)) \leq U_1(f^{t+1}(x))$$

and as a consequence $U_1(f(x)) \leq U_1(f^\infty(x)).$ Also note that $f^\infty(x) \in G(x)$ by (11). Then by (10), $U_1(f^\infty(x)) = u_1(f^\infty(x)) \leq u_1(g(x))$. ■

Theorem 4 implies that, granting the sole agenda setter to initiate reconsideration only limits the value of his proposal power. In other words, if the sole agenda setter could choose, he would have committed to the institution in which he was restricted to making a proposal once and for all with no possibility for reconsideration. Counterintuitively, more power granted by the legislative procedure in this case leads to less valuable power in practice.

A critical prerequisite for this somewhat counterintuitive statement to be true is the institution of majoritarian voting. Knowing that the agenda setter may use his power to exploit the voters in the future, a majority of voters implicitly coordinates to constrain the agenda setter, which provides an endogenous commitment device that
benefits the voters yet harms the agenda setter. Our theory thus provides a novel explanation for a recent finding by Knight (2005) that empirically estimated values of proposal power are smaller than predicted by the Baron-Ferejohn model.

The possibility of reconsideration can be interpreted as lack of commitment by the agenda setter. It has been commonly understood that lack of commitment by policymakers could be a source of policy inefficiency (Kydland and Prescott, 1977; Persson and Svensson, 1989; Aghion and Bolton, 1990; Tabellini and Alesina, 1990; Besley and Coate, 1998; Acemoglu and Robinson, 2001; Acemoglu, 2003). However, our model illustrates a mechanism that works in the opposite direction in majoritarian environments. In fact, lack of commitment by an agenda setter with persistent power may lead to a less unequal allocation of public resources and more efficient policy outcome. This is because the possibility of reconsideration induces the legislature to "commit" to choosing a final policy outcome that falls in the stable set, which is typically smaller than the whole policy space. Our study thus suggests the importance of understanding the interaction of collective decision rules with commitment technologies.

7 Policy Environments

In this section, we discuss a few other commonly studied policy environments to explain how the algorithm can be applied to solve specific models and to illustrate the key insights obtained in the previous general analysis. We assume that player 1 is the sole agenda setter throughout the analysis.

\[^{13}\text{See the next section for examples with various policy environments.}\]
7.1 Public Goods Production

Assume that the three players must jointly produce benefits that they can divide and consume. In this case a policy \( x = (x_1, x_2, x_3) \) specifies not only allocation but also size of the total benefits. The policy space is therefore \( X = \mathbb{Z}_+^3 \). Public production is costly. The cost function is assumed to be quadratic and given by

\[
\kappa(x) = \frac{1}{2} \phi \cdot (x_1 + x_2 + x_3)^2,
\]

where \( \phi \) affects the marginal cost of production. Each player \( \ell \) is assumed to share equally the production cost, and for any policy \( x \in X \), derive a utility of

\[
u_\ell(x) = \mu x_\ell - \frac{1}{2} \kappa(x),
\]

where \( \mu \) is a common marginal utility of benefits consumption.\(^{14}\)

This example can be interpreted as a model with distortionary taxation and the provision of local public goods (Diermeier and Fong, 2008b, 2009). In particular, \( x_\ell \) could be the local public good for the geographical district or the socioeconomic group that legislator \( \ell \) represents, and the production cost \( \kappa(x) \) of public goods include the forgone private consumption of the individuals and the deadweight loss that any distortionary tax, e.g. a proportional labor income tax, may incur.

The initial default is assumed to be \( x_0 = (0, 0, 0) \). That is, if no agreement is made in the legislature, there will be no production and no consumption of the benefits. If the policy was chosen by a benevolent dictator, the size of total benefits would be \( \pi^* \equiv \frac{\mu}{\phi} \), at which level marginal social cost of production is equal to marginal utility of benefits consumption. Here, however, a policy is made through the political

\(^{14}\)For technical convenience, assume that the values of \( \mu \) and \( \phi \) are such that \( \frac{\mu}{\phi} \) is an integer.
process of legislative bargaining.

In the agenda-setting model of Romer and Rosenthal (1978), i.e. with \( \delta = 0 \), the agenda setter needs to satisfy one voter, for example \( j \), at his reservation value \( U_j(x^0) = 0 \) and can fully expropriate the other voter. By proposing any policy \( x \) associated with \( \pi_x \equiv x_1 + x_2 + x_3 \) units of total benefits, the agenda setter then must offer \( j \) at least \( \frac{1}{3 \mu} \kappa(\pi_x) \) units of the benefits to compensate her for the production cost, and can take at most \( \left[ \pi_x - \frac{1}{3 \mu} \kappa(\pi_x) \right] \) units for himself. The agenda setter thus selects a policy \( x \) to maximize effectively

\[
\mu \left[ \pi_x - \left( \frac{1}{3 \mu} \kappa(\pi_x) \right) \right] - \frac{1}{3} \kappa(\pi_x) = \mu \pi_x - \frac{2}{3} \kappa(\pi_x).
\]

Since the agenda setter only internalizes the costs paid by himself and voter \( j \), in equilibrium there is generally overproduction of the benefits.

With possible reconsideration and for \( \delta < 1 \) sufficiently large, an application of the algorithm shows that for any policy in the unique equilibrium stable set, the two voters must receive an equal amount of benefits for each level of total benefits production. This follows because each voter is induced to protect the benefits of the other voter in order to secure her own long-term bargaining position in the legislature. Therefore, by proposing any policy \( x \) associated with \( \pi_x \) units of total benefits, the agenda setter must offer both voters at least \( \frac{1}{3 \mu} \kappa(\pi_x) \) units of the benefits to compensate their production costs and therefore can take no more than \( \left[ \pi_x - 2 \left( \frac{1}{3 \mu} \kappa(\pi_x) \right) \right] \) units for himself. Otherwise neither voter would accept the policy proposal. In this case, the agenda setter selects a policy to maximize effectively

\[
\mu \left[ \pi_x - 2 \left( \frac{1}{3 \mu} \kappa(\pi_x) \right) \right] - \frac{1}{3} \kappa(\pi_x) = \mu \pi_x - \kappa(\pi_x).
\]
Note that any politically feasible policy $x$ thus requires the agenda setter to internalize fully all costs and gains of benefits production. As a consequence, in equilibrium the size of benefits production is socially efficient. With the possibility of reconsideration, social welfare defined by aggregate utility is unambiguously improved.

### 7.2 Pork-Barrel Politics

Consider a legislature with five players and the pork-barrel policy space formalized by Bernheim, Rangel and Rayo (2006). Each player is associated with a single project.

Let $W = \{1, 2, 3, 4, 5\}$ denote the set of all projects. Each project $\ell \in W$ produces a highly concentrated benefit $b_\ell > 0$ to player $\ell$ and incurs a cost $c_\ell > 0$ universal for everyone. A policy $x$ consists of a list of projects, so that policy space $X$ is the collection of all subsets of $W$ including the empty set $\emptyset$. Given any policy $x \in X$, player $\ell$ derives a utility of

$$u_\ell (x) = - \sum_{i \in x} c_i + \begin{cases} b_\ell, & \text{if } \ell \in x; \\ 0, & \text{otherwise.} \end{cases}$$

For illustrative purpose, we assume that every project is socially efficient, in the sense that the aggregate benefit $b_\ell$ of project $\ell$ is greater than the aggregate cost $5c_\ell$. Therefore, policy efficiency increases in the number of projects adopted. Without loss of generality, label the players such that $c_i < c_j$ for any $i, j \in \{2, 3, 4, 5\}$. The initial default is assumed to be $\emptyset$. That is, no project will be implemented with no agreement in the legislature.

In the agenda-setting model of Romer and Rosenthal (1978), i.e. with $\delta = 0$, the agenda setter seeks voting support from a bare majority of the cheapest voters and thus proposes the policy consisting projects for himself (player 1) as well as players 2
and 3. Player 1 as agenda setter then obtain a utility of \( b_1 - \sum_{i=1}^{3} c_i \).

With possible reconsideration and for \( \delta < 1 \) sufficiently large, an iteration of the algorithm leads to

\[
\begin{align*}
C_1 & = \{ \{1\} \}, \\
C_2 & = \{ \{1, 2, 3, 4\} \}, \\
C_3 & = \{ \{2, 3, 4, 5\} \},
\end{align*}
\]

and a unique policy set

\[
\hat{S} = \{ \{1\}, \{1, 2, 3, 4\}, \{2, 3, 4, 5\} \}.
\]

Then the unique outcome is the policy consisting of projects for all but player 5. Whereas policy efficiency improves from the case of no reconsideration, the utility of the agenda setter drops to \( b_1 - \sum_{i=1}^{4} c_i \).

Finally, consider the legislative institution with predetermined finite rounds of proposal making and voting and assume that a sufficient number of players can make proposals (Bernheim, Rangel and Rayo, 2006). Then the unique outcome is the policy that consists of only the project for the last agenda setter. In this case the value of proposal power is maximized for the last agenda setter. Paradoxically, a concentrated proposal protocol in our model, in the form with an unchanged agenda setter, leads to a smaller value of the proposal power and enhances social welfare.

### 7.3 A Spatial Model

Spatial models are usually set in a continuous policy space, but their main insights can also be captured in a discrete policy as in the following setup based on Fong
Here assume that three players must collectively choose a policy from a multi-dimensional policy space that includes 7 policy alternatives in three categories:

1. **Ideal Point.** For all $\ell \in \{1, 2, 3\}$, let $z_\ell$ be the ideal point of player $\ell$. In other words, $u_\ell(z_\ell) = a > u_\ell(x)$ for all $x \neq z_\ell$. Without loss of generality, we normalize the utility functions so that $a > 0$ and $u_\ell(z_i) = 0$ for all $i \neq \ell$.

2. **Bilateral Compromise.** For any distinct $i, j \in \{1, 2, 3\}$, let $z_{ij}$ be a policy that equally benefits players $i$ and $j$ but disadvantages the third player $\ell$. Assume that $0 < u_i(z_{ij}) = u_j(z_{ij}) = b < a$, $u_\ell(z_{ij}) = d$, and $2b > a$. We call $z_{ij}$ a bilateral compromise for players $i$ and $j$ because it attains a higher joint utility for the two players than any of their ideal points. For expositional purpose, in this section we only consider the case in which $d \leq 0$. That is, whenever players $i$ and $j$ reach their bilateral compromise, the third party $\ell$ is worse off than if $z_i$ or $z_j$ is chosen.

3. **Centrist Policy.** Let $z_{123}$ be the centrist policy in the policy space. Assume that $0 < u_\ell(z_{123}) = c < b$ for all $\ell$, and $3c > \max\{a, 2b + d\}$. That is, the centrist policy attains the maximal joint utility of all three players and delivers the same utility to each player. However, for any player $\ell$, the centrist policy is worse than his ideal point and any bilateral compromise between himself and some of the other players.

To sum up, $X = \{z_1, z_2, z_3, z_{12}, z_{13}, z_{23}, z_{123}\}$. For any player $\ell$ and any other distinct $j$ and $k$, we have $u_\ell(z_\ell) > u_\ell(z_{ij}) = u_\ell(x_{\ell k}) > u_\ell(z_{123}) > u_\ell(x_j) = u_\ell(x_k) \geq u_\ell(x_{jk})$.

\(\text{---}^{15}\)See Diermeier and Merlo (2000) for a similar model in the context of coalition bargaining.
By the algorithm,

\[ C_1 = C_1^* = \{z_1\}, \]
\[ D_1 = \{z_2, z_3, z_{12}, z_{13}\}, \]
\[ C_2 = C_2^* = \{z_{123}\}, \]
\[ C_3 = C_3^* = \{z_{23}\}, \]

and the unique policy set

\[ \hat{S} = \{z_1, z_{123}, z_{23}\} \]

is constructed. By Theorem 3, \(\hat{S}\) is the unique equilibrium stable set. Below we highlight three noteworthy features of this model.

First, although \(z_{23}\) is the policy the agenda setter dislikes the most, in any equilibrium he is not able to change it if \(z_{23}\) is the initial default. If \(\delta = 0\), the agenda setter would be able to pass \(z_{12}\), for example, to increase his utility, as player 2 would be indifferent. However, given that \(\delta < 1\) is sufficiently large, player 2 would not accept such proposal because she anticipates that with \(z_{12}\) as the new default in the next proposal round the agenda setter would ally with the cheaper player 3 and pass his ideal point, \(z_1\), which delivers less utility to player 2 than \(z_{23}\). By this logic, player 2 will not approve any policy change that could benefit the agenda setter. Similarly player 3 will do the same. As a consequence, the policy cannot be moved and the agenda setter is not able to use his power to exploit the voters. Moreover, the agenda setter ends up with the least utility among all players. This happens when the agenda setter is sufficiently disadvantaged by the initial default.

Second, by a similar argument the socially efficient policy, \(z_{123}\), is sustainable. If \(\delta = 0\), the agenda setter would be able to move the policy to either \(z_{12}\) or \(z_{12}\) and
policy efficiency measured as the aggregate utility would decrease. Here with $\delta < 1$ sufficiently large, mutual protection by the voters constrain the agenda setter and force him to retain the efficient default. This case, again, illustrates our earlier point that lack of commitment by the agenda setter serves as a commitment device for the legislature as a whole to sustain more efficient policies.

Third, with $z_2$ (or $z_3$) as default, two different final policy outcomes are possible in equilibrium. In one equilibrium, the agenda setter seeks voting support from player 3 and moves the policy from $z_2$ to his ideal point $z_1$, i.e. $f(z_2) = z_1$. In this case player 3 is indifferent and the agenda setter’s utility is maximized. This is also the equilibrium outcome in the case of no reconsideration. In the other equilibrium, the agenda setter still seeks voting support from player 3 but he is only able to move the policy from $z_2$ to the centrist policy $z_{123}$, given the constraint imposed by 3. This equilibrium emerges due to self-fulfilling expectations in the case with $\delta < 1$ sufficiently large. If players anticipate that the policy will transition to and stabilize at the centrist policy, the voters’ reservation values would be so high that they would not approve the proposal of $z_1$. The possibility of reconsideration thus supports a more efficient equilibrium outcome than what would have been chosen in the case of no reconsideration.

8 Concluding Remarks

This paper proposes a new analytical framework of legislative bargaining with a single, persistent agenda setter and the possibility of reconsideration. Policy-making is finalized only after the agenda setter has no more incentive to replace the default policy. We show that in any pure-strategy stationary equilibrium, the possibility of reconsideration limits the power of the agenda setter. This result holds for any finite
policy space and therefore can be applied in various policy environments. Our analy-
isis implies that the agenda setter would be better off by being able to commit not to
reconsider any approved policy in the future, yet such lack of commitment may lead
to efficiency gains.

How the allocation of proposal power affects equality and efficiency of legislative
outcomes is a nontrivial question. In legislative institutions that allows for recon-
sideration, Bernheim, Rangel and Rayo (2006) show that a sufficiently inclusive pro-
posal protocol effectively leads to "dictatorship" of the last agenda setter, whereas
the monopoly of proposal power in our model induces additional constraints on the
agenda setter due to voters’ mutual protection. For future work we thus suggest a
systematic investigation of the role of proposal power allocation in policy-making,
both theoretically and empirically. This could potentially complement the existing
literature on how voting rules and electoral systems shape policy outcomes.\textsuperscript{16}

In fact, different political systems can be understood as different combinations
of agenda control, voting rights and veto power in the legislature. For example, the
majoritarian institution with a single, persistent agenda setter in this paper may
be reminiscent of the legislature in a multiparty presidential system (Robinson and
Torvik, 2008), whereas the U.S. Congress may be approximated by a system with
dispersed proposal power yet with the presence of a gatekeeper, i.e. the president,
who is conferred the veto right to block any policy proposal made by some others
and at the same time able to propose a new policy in some situations. On the
other hand, in a parliamentary system the survival of the government depends on
maintaining continuous majority support in the parliament (Diermeier and Feddersen,
1998; Persson, Roland and Tabellini, 2000). This in turn gives the cabinet broad
proposal powers. A unifying framework based on the idea of power allocation can

\textsuperscript{16}For example, see Aghion, Alesina and Trebbi (2004; 2008).
potentially broaden and deepen our understanding of these institutional variations.\textsuperscript{17} This is one next step in our research agenda.

Finally, the analytical framework developed in this paper is tractable and, with the proposed algorithm, could be applied to models of public finance and macroeconomic policy choice.\textsuperscript{18} As recent empirical studies on political economy and comparative constitutions have established various stylized facts and raised new questions about how political institutions shape the dynamics of government policies (Persson and Tabellini, 2003), we expect fruitful insights from such an approach.

\textsuperscript{17}See Diermeier and Myerson (1994, 1999) for some early development of comparative models in this direction.

\textsuperscript{18}See Diermeier and Fong (2008b) for an example.
References


