Optimal Information Disclosure in Auctions and the Handicap Auction*

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Abstract

We analyze a situation where a monopolist is selling an indivisible good to risk neutral buyers who only have an estimate of their private valuations. The seller can release, without observing, certain additional signals that affect the buyers' valuations. Our main result is that in the expected revenue maximizing mechanism, the seller makes available all the information that she can, and her expected revenue is the same as it would be if she could observe the part of the information that is "new" to the buyers.

We also show that this mechanism can be implemented by what we call a *handicap auction* in interesting applications. In the first round of this auction, each buyer picks a price premium from a menu offered by the seller (a smaller premium costs more). Then the seller releases the additional signals. In the second round, the buyers bid in a second-price auction where the winner pays the sum of his premium and the second highest non-negative bid. In the case of a single buyer, this mechanism simplifies to a menu of European call options.

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1 Introduction

In many examples of the monopolist's selling problem (optimal auctions),[1] the seller has considerable control over the accuracy of the buyers' information concerning their own valuations. Often the seller can decide whether the buyers can access information that refines their valuations; however, she either cannot observe these signals, or at least, she is unaware of their significance to the buyers. For example, the seller of an oil field or a painting can determine the number and nature of the tests the buyers can carry out privately (without the seller observing the results). Another example (due to Bergemann and Pesendorfer, 2002) is where the seller of a company has detailed information regarding the company's assets (e.g., its client list), but does not know how well these assets complement the assets of the potential buyers. Here, the seller can choose the extent to which she will disclose information about the firm's assets to the buyers. In other applications (e.g., selling broadcast rights for a future sports event) the buyers' valuations for the good become naturally more precise over time as the uncertainty resolves, and the seller can decide how long to wait with the sale.

When the buyers' information acquisition is controlled by the seller, that process can also be optimized by the mechanism designer. In the present paper we explore the revenue maximizing mechanism for the sale of an indivisible good in a model where the buyers initially only have an estimate of their private valuations. The seller can costlessly release additional private signals to the buyers that affect their valuations.[2] These additional signals are not observable to the seller (nor to any outside party), but the seller decides whether a buyer can observe them. This model captures the common theme of the motivating examples: the seller controls, but cannot learn, certain private information that the buyers care about.[3]

Our main result is that in the revenue-maximizing mechanism the seller releases all the information she can, and that her expected revenue is as high as it would be if she could observe the part of the information controlled by her that is "new" to

[1] Early seminal contributions include Myerson (1981), Harris and Raviv (1981), Riley and Samuelson (1981), and Maskin and Riley (1984).

[2] In the formal model, a given buyer's ex-post valuation will be a general function of his original type and a shock. We only impose on this function certain "non-increasing returns" conditions.

[3] Note that in some of the motivating examples the seller did observe the "message" disclosed to the potential buyer, but importantly, she did not observe its effect on the buyer's valuation. This situation is modeled by the assumption that the seller can release, without observing, private signals to the buyers.

the buyers.[4] That is, in the optimal mechanism, the buyers do not enjoy additional informational rents from learning more about their ex-post valuations when the access to additional signals is controlled by the seller. Apart from these findings, an added theoretical interest of our model is that the standard revelation principle cannot be applied: the information that buyers learn at the seller's discretion is not part of their type (à la Harsányi), moreover, the use of multi-stage mechanisms where the acquired information is reported back by the buyers is not without loss of generality. Despite this difficulty we are able to characterize the optimal mechanism.

We also exhibit a simple mechanism, dubbed the *handicap auction*, which implements the revenue-maximizing outcome in some interesting applications of the general model. The first application is one where the buyers' original information pertains solely to the expected value of their valuation. This situation is modeled so that a buyer's ex-post valuation is the sum of his original value-estimate and an independent noise whose realization is the signal that the seller can disclose to him.[5] In the second application each buyer's ex-post valuation is the realization of a normally distributed random variable. The buyers' initial estimates and the seller's signals are normally distributed, conditionally independent noisy observations of the buyers' valuations. Note that in this "sampling" application a buyer's private information and the signal controlled by the seller are strictly affiliated.[6]

The handicap auction, which implements the optimal mechanism in the two applications above, consists of two rounds. In the first round, each buyer buys a price premium from a menu provided by the seller (a smaller premium costs more). Then, without observing, the seller releases as much information as she can. In the second round, the buyers bid in a second-price auction, where the winner is required to pay his premium over the second highest non-negative bid. We call the whole mechanism a handicap auction because buyers compete under unequal conditions in the second round; a bidder with a smaller premium has an advantage.[7]

[4] In this hypothetical comparison the buyers may or may not observe (but in either case can ex post verify) the additional signals.

[5] In this application the noise must be additive and independent otherwise the buyer's original estimate would be informative not only about his valuation, but also about the precision of his original estimate.

[6] We thank Marco Ottaviani for suggesting that we develop an application along these lines.

[7] The handicap auction can also be implemented as a mechanism where, in the first round, each bidder buys a discount (larger discounts cost more), and then participates in a second price auction with a positive reservation price, where the winner's discount is applied towards his payment.

When there is only a single buyer, the handicap auction simplifies to a mechanism that can be thought of as a menu of European call options offered by the seller. These types of options are widely used in financial forward markets. In our model, as in reality, an option with a higher strike price costs more to the buyer. Buyers with different initial estimates regarding the good's value sort themselves and choose different options.

Our model nests the classical (independent private values) auction design problem as a special case, where the additional signals are identically zero. In this case, the handicap auction implements the outcome of the optimal auction of Myerson (1981) and Riley and Samuelson (1981).

Several papers have studied issues related to how buyers learn their valuations in auctions, and what consequences that bears on the seller's revenue. One strand of the literature, see Persico (2000), Compte and Jehiel (2001) and the references therein, focuses on the buyers' incentives to acquire information in different auction formats. Our approach is different in that we want to *design* a revenue-maximizing mechanism in which the seller has the opportunity to costlessly release (without observing) information to the buyers. In our model, it is the seller (not the buyers) who controls how much information the buyers acquire.

In Baron and Besanko (1984), Riordan and Sappington (1987), and Courty and Li (2000), a principal and an agent are contracting over two periods. Independently of the contract, the agent learns payoff-relevant private information in both periods. These papers analyze the optimal two-stage revelation mechanism where the contract is signed in the first period, when the agent only knows his first-period type. In contrast, in our paper, we solve for the optimal mechanism in a multi-agent auction environment where the seller can *decide* whether or not the buyers receive additional private signals.

Information disclosure by the seller in an auction has been studied in the context of the winner's curse and the linkage principle by Milgrom and Weber (1982). They investigate whether in traditional auctions the seller should commit to disclose public signals that are affiliated with the buyers' valuations. They find that the seller gains from committing to full disclosure, because that reduces the buyers' fear of overbidding, thereby increasing their bids and hence the seller's revenue. Our problem differs from this classic one in many aspects. Most importantly, in our setting, the signals that the seller can release are private (not public) signals, in the sense that each signal affects the valuation of a single buyer and can be disclosed to that buyer only. The seller will gain from the release of information not because of the linkage principle, but because the information can improve efficiency. As we show, some of the potential efficiency gain is appropriated by the seller.

Our motivation is closer to that of Bergemann and Pesendorfer (2002) and Ganuza (2003), where the seller decides how much *private information* the buyers may learn prior to participating in an auction. Ganuza (2003) focuses on the incentives of the auctioneer to release signals to the buyers that refine their private valuations before a second-price auction.[8] He finds that, in order to maximize the revenue, the seller should not reveal all her information. In our model, we find just the opposite: the seller releases everything she can. The reason for the discrepancy is that Ganuza (2003) fixes the selling mechanism as a second-price auction, while we allow the seller to design that, too. Bergemann and Pesendorfer's (2002) problem is also different from ours: In that paper, the seller can design but cannot commit to the selling mechanism when announcing the information disclosure policy. (Their model also differs from ours in that the buyers do not have private information at the beginning of the game.) Under these assumptions, Bergemann and Pesendorfer (2002) show that the information structure that allows the seller to subsequently design the auction with the largest expected revenue is necessarily imperfect. In this structure, buyers are only allowed to learn which element of a finite partition their valuation falls into.

Compared to these two papers, the difference of our approach is that we consider the design of the information structure and the transaction rules as one mechanism design problem, as a whole. This difference may first seem subtle, nevertheless, it is important. In our model, the seller can integrate the rules of information acquisition into the mechanism used for the sale itself. For example, the seller can charge the buyers for getting more and more accurate signals (perhaps in several rounds); the buyers may even be asked to bid for obtaining more information. In contrast to the two papers cited above, we show that the seller maximizes expected revenue by designing a mechanism in which she allows the buyers to learn their valuations as precisely as they can.[9] In other words, the tradeoff between allowing the buyers to obtain more private information and generating more revenue disappears in the optimal auction.

[8] By allowing the buyers to have more private information, the seller faces a trade-off. More information increases efficiency and potentially the revenue, but it also increases the buyers' information rents in the second-price auction.

[9] Note that we, just like the existing literature, require the participation and incentive constraints to hold at all interim phases (not just ex ante), i.e., the buyers want to go ahead with the mechanism at every point in the game.

The idea that "selling" the access to information may be advantageous for the seller can be easily illustrated by an example. Suppose that there are two buyers who are unaware of their private valuations, which the seller can allow them to learn. Consider the following mechanism: The seller charges both buyers an entry fee. Then, she allows the buyers to observe their valuations, and makes them play a second-price auction with zero reservation price. The second-price auction will be efficient. If the entry fees equal to the buyers' ex-ante expected profits then the seller appropriates the entire surplus.[10]

This simple solution—the seller committing to the efficient allocation, releasing the additional signals, and charging an entry fee equal to the expected efficiency gains—only works when the buyers do not have private information to start with. Otherwise (for example, if the buyers privately observe signals, but their valuations also depend on other signals that they may see at the seller's discretion), the auctioneer, as we will show, does not want to commit to an efficient auction in the continuation, so the previously proposed mechanism does not work. One has to find a more sophisticated auction, and this is exactly what we will do in the remainder of the paper.

The paper is structured as follows. In the next section we outline the model. In Section 3 we preview the results for the single buyer case and provide intuitive derivations. In Section 4 we characterize the revenue maximizing mechanism in the general model. In Section 5 a simple implementation (via a "handicap auction") is discussed for a family of important special cases. Section 6 concludes. Some of the proofs are collected in the Appendix.

2 The Model

2.1 The Environment

There are n potential buyers for a single indivisible good sold by a seller (she). All parties are risk neutral. The seller's valuation for the good is normalized to zero; her objective is to maximize the expected revenue from the sale of the good. The value of the buyers' outside option is zero. Therefore, each buyer's payoff is the negative of his

[10] This example has been studied independently by Gershkov (2002), who also obtained the same result. In the context of a two-period principal-agent problem, Baron and Besanko (1984) make the same point: If the agent's type in the second period is independent of his type in the first period then a period one (constant and efficient) contract gives the whole period-two surplus to the principal.

payment to the seller, plus, in case he wins, the value of the object.

Buyer i's ($i = 1, ..., n$) initial estimate of his valuation for the object is v_i, which is his private information (type). The value-estimate, v_i, is distributed on the interval $[\underline{v}_i, \bar{v}_i]$ according to cdf F_i with a positive density f_i. The type distributions satisfy the monotone hazard rate condition, that is, $f_i/(1-F_i)$ is weakly increasing. The seller has the ability to disclose, *without observing*, additional signal(s) to the buyers. If she releases all the information she has control over, then buyer i's privately known posterior of his valuation becomes V_i. The reader can think of the additional signal revealed by the seller (without observing) as V_i itself. Alternatively, V_i may be the result of Bayesian updating on the part of buyer i given v_i and the additional information.

One of our main results is that in the optimal mechanism, the seller can do as well as if she could observe the part of the information controlled by her that is "new" to the buyers. In order to state this result formally and to identify the part of the information that is "new" to buyer i, we orthogonally decompose the posterior, V_i. This decomposition also enables us to state our remaining assumptions on the environment, which pertain to the effect of buyer i's original signal on his ex-post valuation and its substitutability with the part of the seller-controlled signal that is "new" to him.

2.2 Decomposition of V_i and Decreasing-Returns Conditions

Our goal is to find a function $u_i : \mathbb{R}^2 \to \mathbb{R}$ and a random variable s_i such that $V_i \equiv u_i(v_i, s_i)$. Furthermore, we want u_i to be strictly increasing in both arguments, and s_i, called buyer i's shock, to be *independent* of v_i. Since s_i is orthogonal to the buyer's initial value-estimate, it can be considered as the part of the seller-controlled information that is "new" for buyer i. Using this notation, the seller disclosing all the information she has is equivalent to disclosing, without observing, s_i. In the following lemma we show a simple way to construct s_i and u_i satisfying these requirements.

Lemma 1 *(i) There exists a random variable s_i independent of v_i such that $V_i \equiv u_i(v_i, s_i)$ where u_i is strictly increasing both of its arguments.*

(ii) Any two random variables satisfying (i) are positive monotonic transformations of each other.

Proof. Let H_{iv_i} denote the conditional distribution of $V_i - v_i$ given v_i, and assume that H_{iv_i} is strictly increasing on the entire real line.[11]

(i) Define $s_i \equiv H_{iv_i}(V_i - v_i)$. The random variable s_i is independent of v_i as it is uniformly distributed on $[0,1]$ irrespective of the value of v_i. To see this, take any $y \in [0,1]$, and note that $\Pr(H_{iv_i}(V_i - v_i) \leq y) = \Pr(V_i - v_i \leq H_{iv_i}^{-1}(y)) = H_{iv_i}(H_{iv_i}^{-1}(y)) = y$. Therefore, $s_i \equiv H_{iv_i}(V_i - v_i)$ is uniform on [0,1]. Let $u_i(v_i, s_i) = v_i + H_{iv_i}^{-1}(s_i)$, which is by definition identical to buyer i's ex-post valuation, V_i. Note that u_i is strictly increasing in v_i and s_i.

(ii) Suppose that s_i and $\tilde{s}_i$ are independent of v_i, and there exist functions u_i and $\tilde{u}_i$, strictly increasing in their second argument, such that $u_i(v_i, s_i) \equiv \tilde{u}_i(v_i, \tilde{s}_i) \equiv V_i$.[12] We want to show that $\tilde{s}_i$ is a monotonic transformation of s_i, that is, there exists an increasing function λ such that for all v_i and σ,

$$\Pr\left(u_i(v_i, s_i) \leq u_i(v_i, \sigma)\right) = \Pr\left(\tilde{u}_i(v_i, \tilde{s}_i) \leq \tilde{u}_i(v_i, \lambda(\sigma))\right). \tag{1}$$

The issue is that λ must be the same function for all realizations of v_i.

Define λ so that for all σ, $\Pr(s_i \leq \sigma) = \Pr(\tilde{s}_i \leq \lambda(\sigma))$. (The function λ associates the same percentiles of s_i and $\tilde{s}_i$ with each other.) Note that this function is increasing and does not depend on the realization of v_i as both s_i and $\tilde{s}_i$ are independent of v_i. By the monotonicity of u_i in its second argument, the left-hand side of (1) equals $\Pr(s_i \leq \sigma)$, which in turn equals $\Pr(\tilde{s}_i \leq \lambda(\sigma))$ by the definition of λ. By the monotonicity of $\tilde{u}_i$ in its second argument, $\Pr(\tilde{s}_i \leq \lambda(\sigma))$ equals $\Pr(\tilde{u}_i(v_i, \tilde{s}_i) \leq \tilde{u}_i(v_i, \lambda(\sigma)))$, which is the right-hand side of (1), therefore the equality indeed holds. ■

We make two assumptions regarding the shape of u_i that are independent of the choice of s_i satisfying the conditions of Lemma 1. (In other words, the following conditions on marginal returns are invariant to monotonic transformations of s_i.) Assume $u_i(v_i, s_i)$ is twice differentiable and denote the partial derivatives by $u_{i1} = \partial u_i / \partial v_i$, $u_{i2} = \partial u_i / \partial s_i$, $u_{i11} = \partial^2 u_i / \partial v_i^2$, and $u_{i12} = \partial^2 u_i / \partial v_i \partial s_i$.

Assumption 1: $u_{i12} \leq 0$.

[11] As a result, any realization of the ex-post valuation, V_i, is possible given any type v_i of buyer i. This assumption is purely for convenience.

[12] The latter condition means that for all $\xi \in \mathbb{R}$, $\Pr(u_i(v_i, s_i) \leq \xi) = H_{iv_i}(\nu) = \Pr(\tilde{u}_i(v_i, \tilde{s}_i) \leq \xi)$.

Assumption 2: $u_{i11}/u_{i1} \leq u_{i12}/u_{i2}$.

Assumption 1 says that the marginal impact of s_i on buyer i's valuation is non-increasing in i's type, v_i. Assumption 2 means that an increase in i's type, holding the ex-post valuation constant, weakly decreases the v_i's marginal value.[13] The two assumptions imply a certain monotonicity condition that is *necessary* for our results. For completeness, in a second appendix (not for publication) we derive conditions equivalent to Assumptions 1 and 2 in terms of the joint distribution of v_i and V_i.[14]

In order to see that Assumptions 1 and 2 are invariant to the choice of s_i (as long as s_i is independent of v_i with $u_i(v_i, s_i) \equiv V_i$), consider a positive monotonic transformation of s_i, $\tilde{s}_i \equiv \lambda(s_i)$, and a corresponding $\tilde{u}_i$ with $\tilde{u}_i(v_i, \tilde{s}_i) \equiv u_i(v_i, \lambda(s_i))$. For simplicity assume λ is differentiable, hence $\lambda' > 0$. Since $\tilde{u}_{i1} = u_{i1}$, $\tilde{u}_{i2} = u_{i2}\lambda'$, $\tilde{u}_{i11} = u_{i11}$, and $\tilde{u}_{i12} = u_{i12}\lambda'$, Assumptions 1 and 2 hold for $\tilde{u}_i$ whenever they hold for u_i.

The pairs (v_i, s_i) are assumed to be independent across i; the independence of information across buyers is a standard assumption that rules out Crémer–McLean (1988) full rent extracting mechanisms. We will use v to denote the vector of types and s to denote the vector of shocks. We will also use the usual shorthand notation for the vector of types of buyers other than i, v_{-i}, and let s_{-i} denote $(s_j)_{j \neq i}$.

Let us comment about the interpretation of the seller's information disclosure in the model. We have assumed that the seller has the ability to disclose without observing additional signals that make buyer i learn his posterior valuation, V_i. Alternatively, we could have assumed that the seller can in fact observe the signals that she releases, but she does not know how they affect the valuations of the buyers. For example, the seller of a car can see and disclose the color of her vehicle without knowing how that information changes the buyers' willingness to pay. This fits into the model where buyer i has an initial valuation for the car, v_i, and the seller can allow him to learn V_i by specifying the color.

The seller can design any (indirect) mechanism, which can consist of several rounds of communication between the parties (i.e., sending of messages according to rules

[13]To see this interpretation of Assumption 2 note that the total differential of u_{i1} (the change in the marginal value of i's type) is $u_{i11}dv_i + u_{i12}ds_i$. Keeping u_i constant (moving along an "iso-value" curve) means $ds_i = -u_{i1}/u_{i2}dv_i$. Substituting this into the total differential of u_{i1} yields $(u_{i11} - u_{i12}u_{i1}/u_{i2})dv_i$. This expression is non-positive for $dv_i > 0$ if and only if $u_{i11}/u_{i1} \leq u_{i12}/u_{i2}$.

[14]As we will see in the next subsection, the substitutability conditions sometimes allow but do not imply affiliation or other frequently used assumptions on the joint distribution of v_i and $V_i - v_i$. We believe that the conditions are best expressed and interpreted in the form given above.

specified by the seller); the seller may also generate and release (without observing) signals to each buyer i, including the realization of his shock, s_i. Transfers of the good and money may also occur as a function of the history. The set of all mechanisms is rather complex, and the standard revelation principle cannot be applied in the quest for the optimal mechanism. (The problem is that a buyer's original type does not represent all information acquired in the mechanism, which in turn is designed by the seller.) However, this issue is avoided by the approach that we take in Sections 3-5.

2.3 Two Special Cases

A simple specification of our general model is the following. Buyer i's original private information, v_i, is his estimated valuation for the good; his ex-post valuation differs from this by an additive and independent noise, s_i. That is, i's ex-post valuation is

$$u_i(v_i, s_i) = v_i + s_i,$$

where s_i is independent of v_i, and all signals are independent across i's. The seller is able to resolve the uncertainty in buyer i's valuation by disclosing without observing the realization of s_i.[15]

Since s_i is independent of v_i, the buyer's original private information pertains only to the *expected value* of the good for him. For example, the knowledge of v_i conveys no information about how precise his estimate is. This problem has motivated our research in the first place: the seller can release, without observing, signals that "refine" the buyers' original value-estimates.

Another special case of the general model is a familiar "sampling" problem. Suppose that buyer i's ex-post valuation is the realization of a random variable, $\tilde{V}_i$, distributed according to $\mathcal{N}(\bar{u}_i, 1/\tau_{u_i})$. Let v_i and η_i be conditionally independent draws from normal distributions with mean $\tilde{V}_i$ and precisions τ_{v_i} and τ_{η_i}, respectively. (Here the support of v_i is the real line instead of a compact interval, but this does not cause a problem in the analysis.) Suppose that buyer i originally observes v_i, and the seller can allow him to observe the second signal (η_i) as well.

The buyer's initial estimate of his valuation given v_i is normal with mean $E[\tilde{V}_i|v_i] =$

[15] For example, the object for sale is a car and the seller-controlled information is the color of the car. It seems reasonable to assume that a buyer's initial willingness to pay for the car (v_i) and his color preference (s_i) are statistically independent.

$(\tau_{u_i}\bar{u}_i + \tau_{v_i}v_i)/(\tau_{u_i} + \tau_{v_i})$ and precision $\tau_{u_i} + \tau_{v_i}$. This implies that η_i conditional v_i is also normally distributed, with mean $E[\tilde{V}_i|v_i]$ and variance $1/(\tau_{u_i} + \tau_{v_i}) + 1/\tau_{\eta_i}$. Clearly, η_i and v_i are strictly affiliated. Denote the cdf corresponding to η_i's conditional distribution by H_{iv_i}.

In this application, buyer i's expected ex-post valuation given v_i and η_i is

$$V_i = E[\tilde{V}_i|v_i, \eta_i] = \frac{\tau_{u_i}\bar{u}_i + \tau_{v_i}v_i + \tau_{s_i}\eta_i}{\tau_{u_i} + \tau_{v_i} + \tau_{\eta_i}}.$$

Define $s_i \equiv H_{iv_i}(\eta_i)$, and $u_i(v_i, s_i) = E\left[\tilde{V}_i|v_i, \eta_i = H_{iv_i}^{-1}(s_i)\right]$. Note that s_i is independent of v_i and that u_i is strictly increasing in both arguments. Moreover,

$$\begin{aligned}\frac{\partial}{\partial v_i}u_i(v_i, s_i) &= -\frac{\partial H_{iv_i}(\eta_i)/\partial v_i}{h_{iv_i}(\eta_i)}\frac{\partial}{\partial \eta_i}E[\tilde{V}_i|v_i, \eta_i]\bigg]_{\eta_i = H_{iv_i}^{-1}(s_i)} \\ &= \frac{\tau_{v_i}}{\tau_{u_i} + \tau_{v_i}}\frac{\tau_{s_i}}{\tau_{u_i} + \tau_{v_i} + \tau_{s_i}},\end{aligned}$$

which is a constant between 0 and 1. Therefore u_i is separable in v_i and s_i and linear in v_i; all of our assumptions on u_i are satisfied.

3 Preview of the Results for a Single Buyer

In this section we preview the solution of the problem for the case of a single buyer. This special case allows us to outline the main results of the paper and the arguments of the proofs. We will state the results precisely, but only provide a heuristic derivation. In Section 4, we provide the results and the proofs in full generality. The case of a single buyer also showcases the practical aspects of the optimal mechanism and its relation to mechanisms observed in reality, in particular, option contracts.[16]

In the rest of this section, we assume that there is only one buyer and drop subscripts referring to the buyer's identity. Furthermore, for the sake of simplicity, we also assume that the buyer's ex-post valuation, $u(v, s)$, is $v + s$, where v is his private information and s is an independent shock that the seller can release without observing.

Recall that the Revelation Principle in its standard form cannot be applied in our setup, and the set of all mechanisms is difficult to characterize. Therefore, instead

[16]We thank Juuso Välimäki for suggesting that we include this section and emphasize the realistic features of the optimal contract with a single buyer.

of trying to find the optimal mechanism in a "canonical" set, we proceed as follows. We first characterize the optimal contract in the case where the seller can actually observe s after having designed the mechanism. In this benchmark case, the seller's maximal payoff is an upper bound on her revenue in our original problem where she cannot observe s. Then, we show that the optimal allocation and payment rules found in the benchmark can in fact be implemented by the seller without knowing s. This mechanism is therefore optimal because the largest revenue attainable in the benchmark is obviously an upper bound on the seller's revenue in the original problem.

Let us consider the benchmark problem where the seller can observe s after having committed to a mechanism. Here, the Revelation Principle applies, so without loss of generality, any mechanism can be represented by an incentive compatible direct revelation mechanism (the buyer reports his type, v, only). Such a mechanism consists of the functions $X^*(v,s)$ and $T^*(v,s)$, where v is the buyer's *reported* type, s is the actual realization of the shock (observed by the seller); X^* is the probability of the buyer getting the good and T^* is the transfer he pays no matter whether or not trade takes place.[17]

We can determine the optimal mechanism in the benchmark case using well-known techniques of mechanism design (see, e.g., Myerson (1981)). If the buyer reports type $\hat{v}$ while his actual type is v, his payoff in the mechanism is

$$\pi^*(v,\hat{v}) = \int [(v+s)\ X^*(\hat{v},s) - T^*(\hat{v},s)]\, dG(s). \tag{2}$$

The mechanism is incentive compatible if $\pi^*(v,\hat{v})$ is maximized in $\hat{v}$ at $\hat{v}=v$. The necessary first- and second-order conditions of that are $\partial\pi^*(v,\hat{v})/\partial\hat{v} = 0$ and $\partial^2\pi^*(v,\hat{v})/\partial\hat{v}^2 \leq 0$ at $\hat{v}=v$. By totally differentiating the former condition and substituting it into the latter one, the second order condition can be rewritten as $\partial^2\pi^*(v,\hat{v})/\partial\hat{v}\partial v \geq 0$ at $\hat{v}=v$. Assuming (for the sake of the heuristic derivation only) that X^* and T^* are differentiable, the two conditions become

$$\int [(v+s)\ X_v^*(v,s) - T_v^*(v,s)]\, dG(s) = 0,$$

$$\int X_v^*(v,s) dG(s) \geq 0,$$

[17] Throughout the paper, starred functions refer to the benchmark case.

where a subscript denotes a partial derivative. By the first-order condition, $d\pi^*(v,v)/dv = \partial\pi^*(v,\hat{v})/\partial v$ at $\hat{v}=v$. From (2), $\partial\pi^*(v,\hat{v})/\partial v$ at $\hat{v}=v$ equals $\int X^*(v,s)dG(s)$. Integrating this from $\underline{v}$ to v yields the following formula for the buyer's equilibrium payoff:

$$\pi^*(v,v) = \pi^*(\underline{v},\underline{v}) + \int_{\underline{v}}^{v}\int X^*(\xi,s)dG(s)d\xi. \tag{3}$$

From this expression, the ex-ante expectation of the buyer's surplus is,

$$\begin{aligned}\int_{\underline{v}}^{\bar{v}} \pi^*(v,v)dF(v) &= \pi^*(\underline{v},\underline{v}) + \int_{\underline{v}}^{\bar{v}}\int_{\underline{v}}^{v}\int X^*(\xi,s)dG(s)d\xi \; dF(v) \\ &= \pi^*(\underline{v},\underline{v}) + \int_{\underline{v}}^{\bar{v}}\int X^*(v,s)dG(s)\frac{1-F(v)}{f(v)}dF(v),\end{aligned}$$

where the we used integration by parts (or Fubini's theorem) on the second line.

The seller's expected payoff is the difference between social surplus and the buyer's expected surplus, which by the previous expression can be written as

$$W = \int_{\underline{v}}^{\bar{v}}\int \left(v+s-\frac{1-F(v)}{f(v)}\right) X^*(v,s)dG(s)dF(v) - \pi^*(\underline{v},\underline{v}). \tag{4}$$

We want to maximize W by choosing X^* and $\pi^*(\underline{v},\underline{v})$ while making sure that X^* is weakly increasing in v (which is the second-order condition), and that $\pi^*(v,v) \geq 0$ (which is the participation constraint). By (3), the latter constraint is satisfied as long as $\pi^*(\underline{v},\underline{v}) \geq 0$. Hence, in order to maximize (4), let $\pi^*(\underline{v},\underline{v}) = 0$ and in order to maximize the integrand in (4) pointwise set

$$X^*(v,s) = \begin{cases} 1 & \text{if } v+s-(1-F(v))/f(v) \geq 0, \\ 0 & \text{otherwise.}\end{cases} \tag{5}$$

The resulting X is strictly increasing in v because of the monotone likelihood condition. This is the allocation rule of the optimal mechanism in the benchmark case. The buyer's expected payoff, $\pi^*(v,v)$, is given by (3), while the corresponding payment scheme can be calculated from (2) at $\hat{v}=v$. Therefore, T^* has to satisfy

$$\int T^*(v,s)dG(s) = \int \left[(v+s)\; X^*(v,s) - \int_{\underline{v}}^{v} X^*(\xi,s)d\xi\right] dG(s). \tag{6}$$

Notice that (5) and (6) characterize the optimal mechanism in the benchmark case.

Now we return to our original problem where s, when released, cannot be observed by the seller. We claim that the optimal mechanism of the benchmark case can still be implemented by the seller. Consider mechanisms where the buyer first reports his type, v, then obtains the additional information, i.e., the realization of s from the seller. Then the seller charges him a fee $c(v)$ in case he does not buy, and an *additional* premium $p(v)$ in case he does. (The functions c and p are announced by the seller before v is reported.)

The buyer with type v who initially announces $\hat{v}$ buys the good in the end if and only if $v + s - p(\hat{v}) \geq 0$ as $c(\hat{v})$, which is paid no matter whether he buys, is sunk. If we set $p(v) = (1 - F(v))/f(v)$ for all v then the buyer who reports v truthfully gets the good if and only if $v + s - (1 - F(v))/f(v) \geq 0$, which is exatly what the benchmark optimal mechanism prescribes (see (5)). By reporting $\hat{v}$, the buyer obtains a payoff of

$$\pi(v, \hat{v}) = \int_{p(\hat{v})-v}^{\infty} (v + s - p(\hat{v}))\, dG(s) - c(\hat{v}). \tag{7}$$

In order to make the buyer announce v truthfully (and hence implement the optimal allocation rule of the benchmark) we set c such that

$$c(\underline{v}) = \int_{p(\underline{v})-\underline{v}}^{\infty} (\underline{v} + s - p(\underline{v}))\, dG(s), \quad \text{and} \tag{8}$$

$$c'(v) = -p'(v)\left[1 - G(p(v) - v)\right]. \tag{9}$$

where $p = (1 - F)/f$. The second line is the first-order condition for maximizing (7) in $\hat{v}$ at $\hat{v} = v$, which is the incentive compatibility constraint. The first line corresponds to $\pi(\underline{v}, \underline{v}) = 0$, the participation constraint. The local second-order condition of this maximization problem is $p' < 0$, which holds by the monotone hazard rate condition.[18] (In the treatment of the general case in Section 4 we will show that the corresponding local condition is in fact globally sufficient.) The buyer's expected profit in this mechanism is

$$\pi(v, v) = \int_{p(v)-v}^{\infty} (v + s - p(v))\, dG(s) - c(v). \tag{10}$$

The mechanism where the buyer announces v, learns s, and pays $c(v) + p(v)$ if he

[18] It is interesting to note that this second-order condition is stronger than the second-order condition of incentive compatibility in the benchmark, which, as we have seen, is that $v - (1 - F(v))/f(v)$ is increasing in v.

buys and $c(v)$ if he does not, where $p = (1 - F)/f$ and c is given by (8)–(9), implements the allocation rule of the benchmark optimal mechanism, the only remaining question is whether the seller gets the same expected payoff as well. In order to show this, it is enough to prove that $\pi(v, v) = \pi^*(v, v)$ for all v, that is, the buyer's payoff is the same in this mechanism as in the benchmark. We know that $\pi(\underline{v}, \underline{v}) = \pi^*(\underline{v}, \underline{v}) = 0$; moreover, by differentiating (10),

$$\begin{aligned}\frac{d}{dv}\pi(v, v) &= \int_{p(v)-v}^{\infty} (1 - p'(v))\, dG(s) - c'(v) \\ &= 1 - G(p(v) - v),\end{aligned}$$

where on the second line we used (9). From (3) and (5),

$$\frac{d}{dv}\pi^*(v, v) = \int X^*(v, s) dG(s) = 1 - G((1 - F(v))/f(v) - v).$$

Since $p = (1 - F)/f$, we have $d\pi(v, v)/dv = d\pi^*(v, v)/dv$ for all v. We conclude that indeed, $\pi(v, v) = \pi^*(v, v)$ for all v. The seller's revenue is the difference between the social surplus and the buyer's payoff. In this mechanism, the buyer's and the social surpluses are the same as their respective counterparts in the benchmark case. Therefore, the seller can indeed attain the same expected revenue as in the benchmark.

The mechanism that we just described—consisting of $(c(v), p(v))$ pairs for all v—implements the same allocation with the same expected revenue for the seller as if she could observe the realization of s, i.e., the *difference* between the buyer's posterior valuation and original estimate, after having committed to a selling procedure.

This mechanism is remarkably simple, and can be thought of as a *menu of European call options* offered by the seller. These types of options are widely used in financial forward markets. In our model, it is the monopolist seller, not "time", that reveals new information to the buyer of the asset. However, this distinction makes no difference because the seller reveals all her information anyway. In the optimal mechanism, the buyer picks a fee, $c(v)$, depending on his type, from a list provided by the seller. After having learned his posterior from the seller's disclosure, the buyer has to pay a corresponding additional $p(v)$ "strike-price" in case he decides to buy the good. Since $p = (1 - F)/f$ is decreasing in v, higher buyer-types pick options with lower strike prices but larger upfront fees. (In financial markets, a European call option on the same future asset also costs more if the strike price is lower.) In our model, the reason why different buyers may choose options with different strike prices is that they have heterogeneous initial estimates regarding the asset's future value.

4 The Optimal Mechanism in the General Case

We now turn to the characterization of the expected revenue maximizing mechanism in the model introduced in Section 2. We show that this mechanism achieves the same expected revenue *as if* the seller could observe the realizations of the shocks that affect the buyers' valuations. In other words, while the buyers still enjoy information rents from their types, all their rents from observing the shocks can be appropriated by the seller. This is what we consider the main result of the paper.

Our strategy to find the optimal selling mechanism incorporating information disclosure is the following. We first characterize the optimal mechanism under the assumption that the seller, after having committed to a mechanism, is able to observe the realizations of the shocks. The seller's revenue in this "benchmark" case is an upper bound on her revenue when she cannot observe the shocks.[19] Then, we show that the same allocation rule and transfers can be implemented even if the seller cannot directly observe the shocks, but can control their release to the buyers. The constructive proof yields the optimal mechanism in our original model.

4.1 Benchmark: The Seller Can Observe the Shocks

Suppose first, for benchmarking purposes only, that the seller can observe the s_i's after having committed to a selling mechanism.[20] The Revelation Principle applies, and hence we can restrict attention to mechanisms where the buyers report their types, and the seller determines the allocation and the transfers as a function of the reported types and the realization of the shocks. We will analyze truthful equilibria of direct mechanisms that consist of an allocation rule, $x_i^*(v_i, v_{-i}, s_i, s_{-i})$ for all i, and an (expected) transfer scheme, $t_i^*(v_i, v_{-i}, s_i, s_{-i})$ for all i. Here, $x_i^*(v_i, v_{-i}, s_i, s_{-i})$ is the probability that buyer

[19] This is so because in the benchmark, the seller can commit to ignore the shocks.

[20] Since the seller observes the shocks only after having committed to a mechanism, "informed principal" type problems do not arise in the benchmark. On the other hand, it does not matter whether the buyers can observe the shocks as long as the mechanism is verifiable (i.e., the seller cannot lie about s_i).

i receives the good, and $t_i^*(v_i, v_{-i}, s_i, s_{-i})$ is the transfer that he expects to pay, given the reported types and the realization of the shocks.

Fix a direct mechanism in the benchmark case, and define

$$X_i^*(v_i, s_i) = \iint x_i^*(v_i, v_{-i}, s_i, s_{-i})\, dF_{-i}(v_{-i}) dG_{-i}(s_{-i}), \tag{11}$$

which is buyer i's expected probability of winning in the mechanism when his type is v_i and the realization of his shock is s_i, and all buyers report their types truthfully. If buyer i with type v_i reports type $\hat{v}_i$ then his profit is,

$$\pi_i^*(v_i, \hat{v}_i) = \iint \left[x_i^*(\hat{v}_i, v_{-i}, s_i, s_{-i}) u_i(v_i, s_i) - t_i^*(\hat{v}_i, v_{-i}, s_i, s_{-i})\right] dF_{-i}(v_{-i}) dG(s). \tag{12}$$

The mechanism is incentive compatible, if and only if,

$$\pi_i^*(v_i, \hat{v}_i) \leq \pi_i^*(v_i, v_i), \quad \text{for all } i \text{ and all } v_i, \hat{v}_i \in [\underline{v}, \bar{v}]. \tag{13}$$

Define $\Pi_i^*(v_i) \equiv \pi_i^*(v_i, v_i)$ as the equilibrium (or indirect) profit function of buyer i. The seller's expected revenue can be written as the difference of the social surplus and the buyers' expected profits,

$$\sum_{i=1}^{n} \iint \left[u_i(v_i, s_i) x_i^*(v_i, v_{-i}, s_i, s_{-i}) - \Pi_i^*(v_i)\right]\, dF(v) dG(s). \tag{14}$$

The benchmark problem is to maximize (14) subject to the buyers' incentive compatibility constraints, (13), their participation constraints, $\Pi_i^*(v_i) \geq 0$ for all i and $v_i \in [\underline{v}, \bar{v}]$, and $\sum_{i=1}^n x_i(v_i, v_{-i}, s_i, s_{-i}) \leq 1$ for all v, s.

Using the tools of Bayesian mechanism design, we obtain the following solution to the benchmark problem.

Proposition 1 *In the revenue-maximizing mechanism of the benchmark case (when the seller can observe the s_i's after having committed to a selling mechanism), the allocation rule sets $x_i^*(v_i, v_{-i}, s_i, s_{-i}) = 1$ for the buyer with the highest non-negative "shock-adjusted virtual valuation," $W_i(v_i, s_i)$, where*

$$W_i(v_i, s_i) = u_i(v_i, s_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} u_{i1}(v_i, s_i). \tag{15}$$

(Ties are broken randomly.) The profit of buyer i with type v_i is

$$\Pi_i^*(v_i) = \int_{\underline{v}_i}^{v_i} \int u_{i1}(y, s_i) X_i^*(y, s_i) dG_i(s_i) dy, \tag{16}$$

where X_i^* is defined by the optimal allocation rule and (11). The seller's revenue in the benchmark is

$$R^* = \iint \max_i \left\{ u_i(v_i, s_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} u_{i1}(v_i, s_i) \vee 0 \right\} dF(v) dG(s). \tag{17}$$

Proof. See the Appendix. ■

From now on, we will refer to the revenue-maximizing mechanism of the benchmark case (characterized in Proposition 1) as $\{x_i^*, t_i^*\}_{i=1}^n$, and let X_i^* and Π_i^* denote i's expected probability of winning and profit functions, respectively. It is useful, for use in subsequent steps of the analysis, to further describe some properties of the allocation rule.

Corollary 1 *X_i^* induced by the optimal allocation rule and (11) is*
(i) continuous in both of its arguments,
(ii) weakly increasing in both of its arguments, and
(iii) if $v_i > \hat{v}_i$, $s_i < \hat{s}_i$ and $u_i(v_i, s_i) = u_i(\hat{v}_i, \hat{s}_i)$ then $X_i^*(v_i, s_i) \geq X_i^*(\hat{v}_i, \hat{s}_i)$.

Proof. Continuity follows because the distributions are atomless. X_i^* is weakly increasing in v_i and s_i because (15) is strictly increasing in both variables, which in turn follows from the monotone hazard rate condition, $u_{i1} > 0$, $u_{i11} \leq 0$, $u_{i2} > 0$, and $u_{i12} \leq 0$ (see Assumption 1 above). Finally, (iii) is another monotonicity property stating that X_i^* is weakly increasing in v_i even if s_i is adjusted to keep $u_i(v_i, s_i)$ constant. To see this, consider $v_i > \hat{v}_i$ and $s_i < \hat{s}_i$ with $u_i(v_i, s_i) = u_i(\hat{v}_i, \hat{s}_i)$, and compare (15) for (v_i, s_i) and $(\hat{v}_i, \hat{s}_i)$. The first term of (15) is the same by assumption; in the second term $(1 - F_i(v_i))/f_i(v_i) \leq (1 - F_i(\hat{v}_i))/f_i(\hat{v}_i)$ by the monotone hazard rate condition and $u_{i1}(v_i, s_i) \leq u_{i1}(\hat{v}_i, \hat{s}_i)$ because $u_{i11}/u_{i1} \leq u_{i12}/u_{i2}$ (see Assumption 2 and Footnote 9). ■

In the proof of Proposition 1 we show that an allocation rule, $X_i(v_i, s_i)$ for all i, is incentive compatible in the benchmark if it is weakly increasing in v_i. For the optimal rule, $X_i^*(v_i, s_i)$ for all i, this property follows from the monotone hazard rate condition and the concavity of u_i in v_i (for all i). Assumptions 1 and 2 regarding the shape of the u_i's, which imply the stronger monotonicity properties (ii)–(iii) of Corollary 1, are clearly not necessary for incentive compatibility in the benchmark case. However, such monotonicity conditions on the allocation rule turn out to be essentially necessary for incentive compatibility in the original problem. That is the reason why we make Assumptions 1 and 2 and prove (ii)–(iii) in Corollary 1.

4.2 The Solution to the Seller's Problem

We now show that the (benchmark) allocation rule and the seller's revenue characterized in Proposition 1 can be implemented even if the seller cannot observe the shocks, as long as she can allow the buyers to observe them. For this implementation we will use a rather restricted class of mechanisms: two-stage, incentive-compatible direct mechanisms. In the first stage of such a mechanism buyers report their v_i's; in the second stage, each buyer observes his own s_i and reports it back. For all reporting profiles (v, s) and for all i, the seller allocates the good to buyer i with probability $x_i(v_i, v_{-i}, s_i, s_{-i})$, and buyer i pays $t_i(v_i, v_{-i}, s_i, s_{-i})$; these functions are set so that truth-telling is incentive compatible in both reporting stages. Clearly, the class of these mechanisms does not exhaust or represent all possible mechanisms. However, since we will be able to replicate the outcome of the benchmark even in this restricted class, we need not look any further.[21]

Given a two-stage mechanism $\{x_i, t_i\}_{i=1}^n$, define

$$X_i(v_i, s_i) = \iint x_i(v_i, v_{-i}, s_i, s_{-i})\, dF_{-i}(v_{-i}) dG_{-i}(s_{-i}),$$
$$T_i(v_i, s_i) \equiv \iint t_i(v_i, v_{-i}, s_i, s_{-i})\, dF_{-i}(v_{-i}) dG_{-i}(s_{-i}).$$

These quantities correspond to buyer i's expected probability of winning and expected transfers, respectively, when he has type v_i and shock s_i, and everybody (including i) reports truthfully in both rounds of the two-stage mechanism.

We will now analyze the consequences of incentive compatibility going backwards, starting in the second stage of the mechanism. In Lemma 1 we show how the allocation

[21] In contrast, two-stage revelation mechanisms can be (and are indeed) used to represent *all* possible indirect mechanisms in dynamic principal-agent models where contracts are signed in the first of two periods (e.g., Baron and Besanko (1984), Courty and Li (2000)) because there the shocks are inevitably observed by the agent and the Revelation Principle holds. This is not so in our model because the seller does not have to disclose (or may only partially disclose) the shocks to the buyers.

rule pins down the buyers' second round profit functions given truthful revelation of types in the first round. Then, in Lemma 2 we describe what happens off the equilibrium path—after buyer i reports his type untruthfully and observes his shock. It turns out that the deviator will report an untruthful value for his shock in such a way that the two lies "cancel" each other, and his actual valuation, $u_i(v_i, s_i)$, is correctly inferred. Using these results, in Lemmas 3 and 4 (the final steps before proving Theorem 1), we derive the indirect profit function of buyer i.

In the second reporting stage, after truthful first round, buyer i with type v_i who observes s_i and reports $\hat{s}_i$ gets

$$\tilde{\pi}_i(s_i, \hat{s}_i; v_i) = u_i(v_i, s_i)X_i(v_i, \hat{s}_i) - T_i(v_i, \hat{s}_i). \tag{18}$$

Incentive compatibility in the second reporting stage is equivalent to

$$\tilde{\pi}_i(s_i, \hat{s}_i; v_i) \leq \tilde{\pi}_i(s_i, s_i; v_i) \quad \text{for all } i,\ v_i,\ s_i, \text{ and } \hat{s}_i. \tag{19}$$

The following lemma provides the conditions for a mechanism to be incentive compatible in the second reporting stage after a truthful first round.

Lemma 2 *If a two-stage mechanism is incentive compatible and $X_i(v_i, s_i)$ induced by the allocation rule is continuous in s_i then for all $s_i > \hat{s}_i$,*

$$\tilde{\pi}_i(s_i, s_i; v_i) - \tilde{\pi}_i(\hat{s}_i, \hat{s}_i; v_i) = \int_{\hat{s}_i}^{s_i} u_{i2}(v_i, z)X_i(v_i, z)dz. \tag{20}$$

Moreover, if (20) holds and X_i is weakly increasing in s_i then the two-stage mechanism is incentive compatible in the second round after truthful revelation in the first round.

Proof. See the Appendix. ■

In order to complete the analysis of the second round of the mechanism, we also need to know what buyer i will do if he misreports his type in the first round. The following lemma claims that he will "correct" his lie by reporting a shock such that his ex-post valuation at the reported type and shock coincides with his true ex-post valuation.[22]

[22]Such a correction is possible because we assumed that for all v_i, $\hat{v}_i$, and s_i there exists $\hat{s}_i$ such that $u_i(v_i, s_i) = u_i(\hat{v}_i, \hat{s}_i)$. If this assumption did not hold then the seller could detect deviations more easily.

Lemma 3 *In the second round of an incentive compatible two-stage mechanism, buyer i with type v_i who reported $\hat{v}_i$ in the first round and has observed s_i will report $\hat{s}_i = \sigma_i(v_i, \hat{v}_i, s_i)$ such that*

$$u_i(v_i, s_i) \equiv u_i(\hat{v}_i, \sigma_i(v_i, \hat{v}_i, s_i)). \tag{21}$$

Proof. Had buyer i indeed have type $\hat{v}_i$ (as reported) and observed $\hat{s}_i$, incentive compatibility in the second round would require

$$u_i(\hat{v}_i, \hat{s}_i)X_i(\hat{v}_i, \hat{s}_i) - T_i(\hat{v}_i, \hat{s}_i) \geq u_i(\hat{v}_i, \hat{s}_i)X_i(\hat{v}_i, s_i') - T_i(\hat{v}_i, s_i'),$$

for all s_i'. By (21), that is $u_i(v_i, s_i) = u_i(\hat{v}_i, \hat{s}_i)$, this is equivalent to

$$u_i(v_i, s_i)X_i(\hat{v}_i, \hat{s}_i) - T_i(\hat{v}_i, \hat{s}_i) \geq u_i(v_i, s_i)X_i(\hat{v}_i, s_i') - T_i(\hat{v}_i, s_i'),$$

which means that type v_i who reported $\hat{v}_i$ in the first round and then observed s_i is indeed better off reporting $\hat{s}_i$ in the second round rather than any other s_i'. ■

Now we move back to the first round and examine the consequences of incentive compatibility there. Our goal is to derive the buyers' equilibrium profit functions, and we proceed as follows. Using the result of Lemma 2 regarding continuation play after a first-round deviation (buyer i misreporting his type) we first derive the deviating buyer's profit function (see Lemma 3). In Lemma 4 we use these formulas for the deviator's payoff to derive the buyers' equilibrium (or indirect) profit functions.

Lemma 4 *In an incentive compatible two-stage mechanism, if type v_i of buyer i reports $\hat{v}_i < v_i$ in the first round then his payoff is*

$$\pi_i(v_i, \hat{v}_i) = \pi_i(\hat{v}_i, \hat{v}_i) + \int \int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i)X_i(\hat{v}_i, \sigma_i(y, \hat{v}_i, s_i))dy\, dG_i(s_i). \tag{22}$$

If type $\hat{v}_i$ of buyer i reports $v_i > \hat{v}_i$ then he gets

$$\pi_i(\hat{v}_i, v_i) = \pi_i(v_i, v_i) - \int \int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i)X_i(v_i, \sigma_i(y, v_i, s_i))dy\, dG_i(s_i). \tag{23}$$

Proof. Buyer i's expected profit when his type is v_i but reports $\hat{v}_i$ in the first round is,

$$\pi_i(v_i, \hat{v}_i) = \int [u_i(v_i, s_i)X_i(\hat{v}_i, \sigma_i(v_i, \hat{v}_i, s_i)) - T_i(\hat{v}_i, \sigma_i(v_i, \hat{v}_i, s_i))]dG_i(s_i).$$

Using (21) and (18) we have,

$$\pi_i(v_i, \hat{v}_i) = \int \tilde{\pi}_i(\sigma_i(v_i, \hat{v}_i, s_i), \sigma_i(v_i, \hat{v}_i, s_i); \hat{v}_i) dG_i(s_i). \tag{24}$$

Suppose $v_i > \hat{v}_i$. Note that by (21) and the monotonicity of u_i,

$$\sigma_i(v_i, \hat{v}_i, s_i) > s_i > \sigma_i(\hat{v}_i, v_i, s_i).$$

By (20), we can rewrite $\pi_i(v_i, \hat{v}_i)$ in (24) as

$$\pi_i(v_i, \hat{v}_i) = \int \left[\tilde{\pi}_i(s_i, s_i; \hat{v}_i) + \int_{s_i}^{\sigma_i(v_i, \hat{v}_i, s_i)} u_{i2}(\hat{v}_i, z) X_i(\hat{v}_i, z) dz \right] dG_i(s_i).$$

This becomes, by $\pi_i(\hat{v}_i, \hat{v}_i) = \int \tilde{\pi}_i(s_i, s_i, \hat{v}_i) dG_i(s_i)$,

$$\pi_i(v_i, \hat{v}_i) = \pi_i(\hat{v}_i, \hat{v}_i) + \int \int_{s_i}^{\sigma_i(v_i, \hat{v}_i, s_i)} u_{i2}(\hat{v}_i, z) X_i(\hat{v}_i, z) dz \, dG_i(s_i). \tag{25}$$

Note that σ_i defined by (21) is continuous and monotonic. Hence the image of $\sigma_i(y, \hat{v}_i, s_i)$ on $y \in [\hat{v}_i, v_i]$ is $[s_i, \sigma_i(v_i, \hat{v}_i, s_i)]$, and thus we can change the variable of the inside integral in (25) from $z \in [s_i, \sigma_i(v_i, \hat{v}_i, s_i)]$ to $y \in [\hat{v}_i, v_i]$ to get

$$\pi_i(v_i, \hat{v}_i) = \pi_i(\hat{v}_i, \hat{v}_i) + \int \int_{\hat{v}_i}^{v_i} u_{i2}(\hat{v}_i, \sigma_i(y, \hat{v}_i, s_i)) X_i(\hat{v}_i, \sigma_i(y, \hat{v}_i, s_i)) \sigma_{i1}(y, \hat{v}_i, s_i) dy \, dG_i(s_i).$$

By differentiating (21) in v_i (using the Implicit Function Theorem),

$$u_{i1}(v_i, s_i) = u_{i2}(\hat{v}_i, \sigma_i(v_i, \hat{v}_i, s_i)) \, \sigma_{i1}(v_i, \hat{v}_i, s_i). \tag{26}$$

The last two equations imply (22).

Similarly, when type $\hat{v}_i$ reports $v_i > \hat{v}_i$, one can show his payoff is

$$\pi_i(\hat{v}_i, v_i) = \pi_i(v_i, v_i) - \int \int_{\sigma_i(\hat{v}_i, v_i, s_i)}^{s_i} u_{i2}(v_i, z) X_i(v_i, z) dz \, dG_i(s_i).$$

By a change of variable in the inside integral and (26), we get (23). ■

Incentive compatibility in the first round is equivalent to, for all $v_i > \hat{v}_i$,

$$\pi_i(v_i, \hat{v}_i) \leq \pi_i(v_i, v_i) \text{ and } \pi_i(\hat{v}_i, v_i) \leq \pi_i(\hat{v}_i, \hat{v}_i). \tag{27}$$

Equations (22) and (23) are used in the following lemma to characterize the buyers' indirect profit functions in an incentive compatible two-stage mechanism.

Lemma 5 *If a two-stage mechanism is incentive compatible and $X_i(v_i, s_i)$ induced by the allocation rule is continuous then buyer i's indirect profit (as a function of his type) can be written as*

$$\Pi_i(v_i) = \Pi_i(\underline{v}) + \int_{\underline{v}_i}^{v_i} \int u_{i1}(y, s_i) X_i(y, s_i) dG_i(s_i) dy. \tag{28}$$

Proof. Using (22) and (23), incentive compatibility in the first round (the inequality-system (27)) is equivalent to, for all $v_i > \hat{v}_i$,

$$\int \frac{\int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) X_i(\hat{v}_i, \sigma_i(y, \hat{v}_i, s_i)) dy}{v_i - \hat{v}_i} dG_i(s_i)$$
$$\leq \frac{\pi_i(v_i, v_i) - \pi_i(\hat{v}_i, \hat{v}_i)}{v_i - \hat{v}_i}$$
$$\leq \int \frac{\int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) X_i(v_i, \sigma_i(y, v_i, s_i)) dy}{v_i - \hat{v}_i} dG_i(s_i).$$

By $X_i \leq 1$ and the concavity of u_i in v_i,

$$\frac{\int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) X_i(\hat{v}_i, \sigma_i(y, \hat{v}_i, s_i)) dy}{v_i - \hat{v}_i} \leq \frac{\int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) dy}{v_i - \hat{v}_i} \leq u_{i1}(\hat{v}_i, s_i),$$

and by assumption $u_{i1}(\hat{v}_i, s_i)$ has a finite expectation with respect to s_i. Therefore, by the Lebesgue Convergence Theorem,

$$\lim_{v_i \to \hat{v}_i} \int \frac{\int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) X_i(\hat{v}_i, \sigma_i(y, \hat{v}_i, s_i)) dy}{v_i - \hat{v}_i} dG_i(s_i)$$
$$= \int \lim_{v_i \to \hat{v}_i} \frac{\int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) X_i(\hat{v}_i, \sigma_i(y, \hat{v}_i, s_i)) dy}{v_i - \hat{v}_i} dG_i(s_i)$$
$$= \int u_{i1}(\hat{v}_i, s_i) X_i(\hat{v}_i, s_i) dG_i(s_i).$$

By analogous reasoning,

$$\lim_{\hat{v}_i \to v_i} \int \frac{\int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) X_i(v_i, \sigma_i(y, v_i, s_i)) dy}{v_i - \hat{v}_i} dG_i(s_i)$$

$$= \int u_{i1}(v_i, s_i) X_i(v_i, s_i) dG_i(s_i).$$

Therefore, if X_i is continuous in v_i then $\Pi_i(v_i) \equiv \pi_i(v_i, v_i)$ is differentiable everywhere, and

$$\Pi_i'(v_i) = \int u_{i1}(v_i, s_i) X_i(v_i, s_i) dG_i(s_i).$$

Since this derivative is finite for all v_i, Π_i is Lipschitz-continuous and hence it can be recovered from its derivative, and we obtain (28). ■

The significance of equation (28) is that it closely resembles equation (16), the profit function of buyer i under the benchmark case. By comparing the two formulas we see that if the optimal allocation rule (which is continuous) can be implemented with $\Pi_i(\underline{v}) = 0$ then $\Pi_i(v_i) = \Pi_i^*(v_i)$, and the seller's revenue is the same as in the benchmark. The following result—which is the main result of the paper—shows that this is indeed the case.

Theorem 1 *In the model where the seller can disclose but cannot observe the buyers' shocks, the revenue-maximizing auction can be implemented as a two-stage incentive compatible direct mechanism where the good is allocated to the buyer with the highest $W(v_i, s_i)$ defined in (15) and the buyers' profits are Π_i^* as in (16). Moreover, the seller's expected revenue equals R^*, given by (14). That is, the seller's revenue in the optimal mechanism is the same as it would be if she could observe the realizations of the shocks.*

Proof. Set $X_i = X_i^*$ and suppose that all buyers except i report their types truthfully. Consider buyer i with type v_i contemplating to misreport to $\hat{v}_i < v_i$. Note that his deviation payoff is

$$\pi_i(v_i, \hat{v}_i) - \pi_i(v_i, v_i) = [\pi_i(v_i, \hat{v}_i) - \pi_i(\hat{v}_i, \hat{v}_i)] - [\pi_i(v_i, v_i) - \pi_i(\hat{v}_i, \hat{v}_i)].$$

By (22), and (28), the difference of the two bracketed expression can be written as

$$\int \int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) X_i^*(\hat{v}_i, \sigma_i(y, \hat{v}_i, s_i)) dy dG_i(s_i) - \int \int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) X_i^*(y, s_i) dy dG_i(s_i) \quad (29)$$

But since for all $y \in [\hat{v}_i, v_i]$, by property (iii) of X_i^* (Corollary 1),

$$X_i^*(\hat{v}_i, \sigma_i(y, \hat{v}_i, s_i)) \leq X_i^*(y, s_i),$$

the difference in (29), and hence $\pi_i(v_i, \hat{v}_i) - \pi_i(v_i, v_i)$, is non-positive. A similar argument can be used to rule out deviation to $\hat{v}_i > v_i$. ■

A key feature of the optimal allocation rule that makes the proof work is that for all $v_i, \hat{v}_i \in [\underline{v}, \overline{v}]$ and $s_i, \hat{s}_i \in \mathbb{R}$ such that $v_i > \hat{v}_i$ and $u_i(v_i, s_i) = u_i(\hat{v}_i, \hat{s}_i)$, the allocation rule "favors" the pair (v_i, s_i), that is, $X_i^*(\hat{v}_i, \hat{s}_i) \leq X_i^*(v_i, s_i)$, as seen in property (iii) in Corollary 1. In words, buyer i with type v_i and a given ex-post valuation wins the object more often than he does with a lower type $\hat{v}_i$ but the same ex-post valuation. This monotonicity property of X_i^* is a consequence of Assumption 2 concerning u_i (requiring that $u_{i1}(v_i, s_i)$ decreases when v_i increases but s_i decreases to keep $u_i(v_i, s_i)$ constant), together with the monotone hazard rate condition imposed on the F_i's. Note that the above condition on X_i^* is not necessary for the incentive compatibility of the same allocation rule in the benchmark case (this can be seen from the proof of Proposition 1). However, the condition is crucial in Theorem 1: if it fails then one can easily construct an example where the two-stage mechanism is not incentive compatible.

4.3 Discussion of the Optimal Mechanism

It is interesting to observe that in the optimal mechanism, two buyers with the same ex-post valuation do not have the same probability of winning. According to Corollary 1, increasing v_i and decreasing s_i so that $u_i(v_i, s_i)$ remains constant ("compensated") *increases* the probability that i wins the object. This is so because $W(v_i)$ in equation 15 is increasing in v_i even if $u_i(v_i, s_i)$ is compensated. The economic reason for this property is that the seller uses discrimination—giving preference to a buyer with higher original type (estimated valuation) among those with the same ex-post valuation—in order to screen the buyers.

This property, however, implies that the auction does not achieve full efficiency, even under ex-ante symmetry of the bidders and conditional on the object being sold. In contrast, in the classical setup with deterministic valuations, the optimal auction of Myerson (1981) and Riley and Samuelson (1981) is efficient conditional on sale, provided that the buyers are ex ante symmetric.

In order to better explain Theorem 1, consider a setup where the buyers are ex ante symmetric (the v_i's are identically distributed), and the shocks are additive, mean zero random variables (such as in one of the applications of the next section). Let us compare the optimal allocation rule in the case when the seller can observe the shocks with that of the revenue maximizing auction when *nobody* (neither the seller nor the buyers) can observe them. In the latter case, the seller should allocate the good to the buyer with the largest non-negative virtual value-estimate, $v_i - (1 - F(v_i))/f(v_i)$. If the seller can observe the shocks, then, in the optimal mechanism, the good will be allocated more efficiently, as the winner will now be the buyer with the highest non-negative shock-adjusted virtual valuation, $v_i + s_i - (1 - F(v_i))/f(v_i)$.[23] According to Theorem 1, the seller, by controlling the release of the shocks and without actually observing them, can implement the same allocation while entirely appropriating the increase in efficiency.[24]

5 The Handicap Auction

In this section we show how to implement the optimal mechanism derived in Section 3 when u_{i1} is constant (as it is in the two applications discussed in Section 2.2).

5.1 The Rules of the Auction

A handicap auction consists of two rounds. In the first round, each buyer i, knowing his type, chooses a price premium p_i for a fee $C_i(p_i)$, where C_i is a fee-schedule published by the seller. The buyers do not observe the premia chosen by others. The second round is a traditional auction, and the winner is required to pay his premium over the price. Between the two rounds, the seller may send messages to the buyers. In the optimal handicap auction the seller allows every buyer to learn the realization of his shock between the two rounds, and the second round is a second price (or English) auction with a zero reservation price.

[23] It is easy to see that if $v_i - (1 - F(v_i))/f(v_i) < v_j - (1 - F(v_j))/f(v_j)$, but, by adding the shocks to both sides the inequality is reversed, then $v_i + s_i > v_j + s_j$. Therefore, an allocation based on the shock-adjusted virtual valuations "pointwise" improves efficiency. (This may not be true if the F_i's are not identical.)

[24] If the buyers' ex ante type-distributions are not identical then, as the seller gets to observe the signals, the efficiency of the optimal mechanism may only improve in ex ante expectation. Still, there will be some efficiency gain, which will be extracted by the seller even if she cannot observe the additional signals.

We call this mechanism a handicap auction because in the second round, the buyers compete under unequal conditions; a bidder with a smaller premium has a clear advantage. An interesting feature of our auction is that the bidders *buy* their premium in the initial round, which allows for some form of price discrimination. We will come back to the issue of price discrimination later.

An alternative way of formulating the rules of the handicap auction would be by using price discounts (or rebates) instead of price premia. In this version, each bidder first has to buy a discount from a schedule published by the seller. Then the buyers are allowed to learn the realizations of the shocks, and are invited to bid in a second price auction with a reservation price r, where the winner's discount is applied towards his payment. The reader can check that a handicap auction can be easily transformed into a mechanism like this by setting r sufficiently high (larger than the highest p_i in the original fee-schedules), and specifying that a discount $d_i = r - p_i$ is sold for a price $C(p_i)$ in the first round. In what follows, however, we will use the original form of the handicap auction.

If there is only a single buyer, then, as we have seen it in Section 3, the handicap auction simplifies to a *menu of buy options*: p_i can be thought of as the strike price, and the upfront fee, $C_i(p_i)$, is the cost of the option. In the second round, the buyer can exercise his option to buy at price p_i (there is no other bidder, so the second-highest bid is zero), for which he initially paid a fee of $C_i(p_i)$. We will revisit the single-buyer case one more time in the context of a numerical example below.

Observe that it is a dominant strategy for buyer i who chose premium p_i in the first round, has type v_i and observes s_i before the second round to bid $u_i(v_i, s_i) - p_i$ in the second stage of the handicap auction. Assuming that the buyers follow this weakly dominant strategy in the second round, the handicap auction can be represented by pairs of functions, $p_i : [\underline{v}, \bar{v}] \to \mathbb{R}$ and $c_i : [\underline{v}, \bar{v}] \to \mathbb{R}$, for $i = 1, ..., n$, where $p_i(v_i)$ is the price premium that type v_i chooses (in equilibrium) for the fee of $c_i(v_i) \equiv C_i(p_i(v_i))$.

Proposition 2 *If u_{i1} is constant then the optimal mechanism of Theorem 1 can be implemented via a handicap auction $\{c_i, p_i\}_{i=1}^n$, where*

$$p_i(v_i) = \frac{1 - F_i(v_i)}{f_i(v_i)} u_{i1}, \tag{30}$$

and $c_i(v_i)$ *is defined by*

$$c_i(v_i) = \iint (W_i(v_i, s_i) - W^0_{-i}(v_{-i}, s_{-i}))\mathbf{1}_{\{W_i(v_i,s_i) \geq W^0_{-i}(v_{-i},s_{-i})\}} dF_{-i}(v_{-i})dG(s) - \iint \left[\int_{\underline{v}}^{v_i} u_{i1}\mathbf{1}_{\{W_i(y,s_i) \geq W^0_{-i}(v_{-i},s_{-i})\}} dy \right] dF_{-i}(v_{-i})dG(s), \quad (31)$$

where $W_i(v_i, s_i) = u_i(v_i, s_i) - p_i(v_i)$ *as defined by (15) and (30), and* $W^0_{-i}(v_{-i}, s_{-i}) = \max_{j \neq i}\{W_j(v_j, s_j), 0\}$.

From the seller's perspective, the premium-fee schedule offered in the first round of the handicap auction works as a device to discriminate among buyers with different value estimates. When a buyer decides to participate in the handicap auction, he knows his type (expected valuation), which tells him whether he is more or less likely to win. Therefore, in the first round, a buyer with a high type chooses a small price premium for a large fee in order not to pay much when he wins. Using analogous reasoning, low types choose large price premia, which are cheaper, but make winning more expensive. In the optimal handicap auction, just like in the general optimal mechanism, two buyers with the same ex-post valuation do not have the same probability of winning. The buyer with the larger v_i will choose a smaller price premium, bid higher in the second round, and will be more likely to win.

One may suggest that the way the seller can appropriate all rents from the additional information is that in the handicap auction, she essentially charges the buyers a type-dependent up-front fee equal to the "value" of the information they are about to receive. This intuition may be appealing, but it overly simplifies the workings of the mechanism. The value of the additional information to the participants is not well-defined because it depends on the rules of the selling mechanism. This value could be different if the seller chose a mechanism different from the handicap auction.

5.2 Determining the Optimal Handicap Auction: A Numerical Example

It may be useful to compute a numerical example not only for illustrative purposes, but also, to see how a seller may be able to compute the parameters of the optimal handicap auction (the price premium–fee schedule) in a practical application.

We will consider the following setup. Types and shocks are additive; the types are distributed independently and uniformly on $[0,1]$, the shocks are distributed independently according to a standard logistic distribution.[25]

First, assume that there is a single buyer, that is, $n = 1$. Then, the handicap auction can be thought of as a menu of buy options, represented by $C_1(p_1)$, where p_1 is the strike price and $C_1(p_1)$ is the fee of the option. In the first round, the buyer chooses a price p_1 and pays $C_1(p_1)$; in the second round (after having observed s_1), he has the option to buy the good at price p_1. Again, we represent this menu as a pair of functions, $c_1(v_1)$ and $p_1(v_1)$, $v_1 \in [0,1]$.

In the uniform-logistic example with $n = 1$, the expected revenue maximizing strike price-schedule is given by (30),

$$p_1(v_1) = 1 - v_1.$$

The fee-schedule in (31) becomes,

$$c_1(v_1) = \frac{1}{2}\ln(1+e) - 1 + v_1 + \frac{1}{2}\ln\left(1 + e^{1-2v_1}\right).$$

We can also express the cost of the option as a function of the strike price,

$$c_1 = C_1(p_1) = \frac{1}{2}\ln\left[(1+e)\left(1+e^{2p_1-1}\right)\right] - p_1.$$

This (downward-sloping) schedule is depicted as the top curve in Figure 1.

If the buyer has a higher estimate then he will choose to buy an option with a lower strike price at a higher cost. For example, if the buyer has the lowest estimate, $v_1 = 0$, then he buys the option of getting the good at $p_1 = 1$, which costs $c_1 = \ln[(1+e)/e] \approx 0.3133$ upfront, and yields zero net surplus. In contrast, the highest type, $v_1 = 1$, buys a call option with zero strike price at a cost of about 0.8133.

Now we turn to the case of many buyers, $n > 1$, in the uniform-logistic example. We will compute the optimal handicap auction represented by $\{c_i, p_i\}_{i=1}^n$. As in the case of $n = 1$, in the revenue-maximizing mechanism, $p_i(v_i) \equiv 1 - v_i$.

Instead of analytically deriving $c_i(v_i)$ for different numbers of buyers, we carry out a more practical Monte Carlo simulation. We take 100,000 random draws from the joint distribution of (s, v_{-i}), and compute $w_j = v_j + s_j - p_j(v_j)$ for all j. Then we determine

[25] The cdf of the standard logistic distribution is $G_i(s_i) = e^{s_i}/(1+e^{s_i})$, $s_i \in (-\infty, +\infty)$.

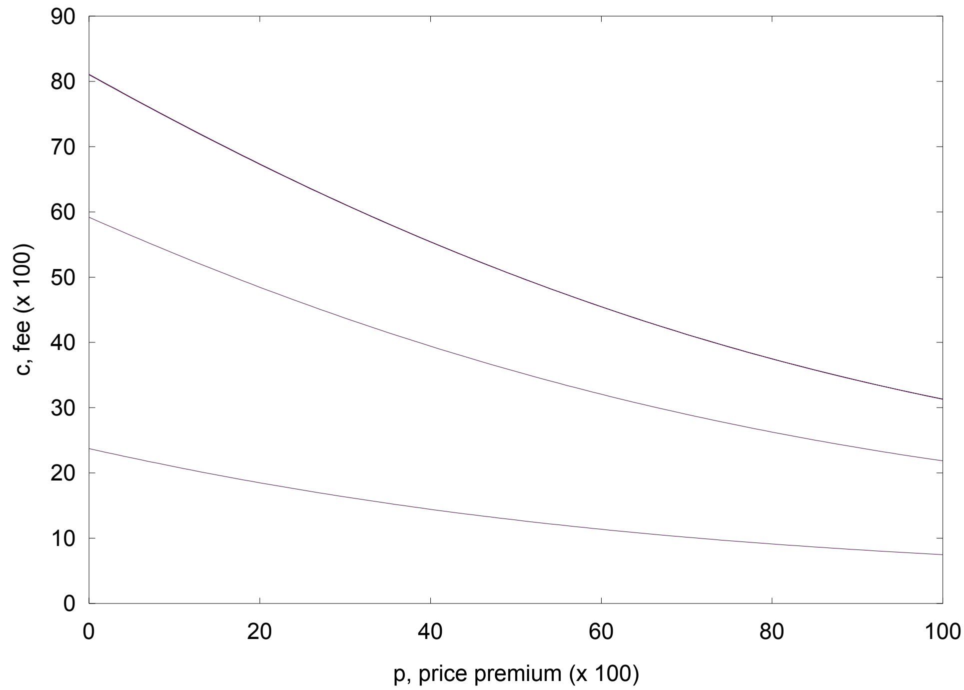

Figure 1: Fee schedules in the revenue maximizing handicap auction (uniform-logistic setup; schedules from top to bottom for n=1, 2, and 5)

$c_i(0)$ from (31), where the expectation is estimated by the sample mean. We compute $c_i(v_i)$ recursively, $c_i(v_i+\text{step}) = c_i(v_i)+\text{step}*c_i'(v_i)$, where step$= 1/100$. The derivative is obtained from (31) as $c_i'(v_i) = \mathbf{E}_{s_i,s_{-i},v_{-i}}\left[\mathbf{1}_{\{v_i+s_i-p_i(v_i)\geq \max_{j\neq i} w_j\}}\right]$. From $c_i(v_i)$ and $p_i(v_i)$ we compute $C_i(p_i) \equiv c_i(p_i^{-1}(p_i))$.

The results of a (typical) simulation are shown in Figure 1. The top curve shows $C_i(p_i)$ for the case of $n = 1$. There are actually two (almost identical) curves superimposed on each other: one graphs the formula that we derived before, the other is the result of the Monte Carlo experiment. The curve in the middle is $C_i(p_i)$ for $n = 2$, and the one in the bottom is $C_i(p_i)$ for $n = 5$. As n increases, $C_i(p_i)$ shifts down and flattens out.

6 Conclusions

In this paper we have analyzed an auction model where the seller controls the release of private signals to the buyers that, together with the buyers' original private information, affect their valuation for the good to be sold. We have derived the expected revenue maximizing mechanism comprising both the rules of information disclosure and those of the transaction.

In the optimal mechanism, the seller discloses all the information that she can and yet obtains the same expected revenue as if she could observe the additional signals (which she can release, but cannot directly observe). The buyers do not enjoy any additional information rents from the signals whose disclosure is controlled by the seller. This result is true under a quite general signal structure: a buyer's type and shock (the part of the information released by the seller that is "new" to the buyer) may affect his ex-post valuation in a non-linear, non-additive way. The only restrictions imposed on the valuation functions (Assumptions 1 and 2) essentially amount to requiring decreasing marginal returns and substitutability of the orthogonally normalized signals.

The optimal mechanism can be implemented via a "handicap auction" in important and interesting applications of the general model. Two such applications that we discussed were (i) a model where the buyer's type is interpreted as a signal solely on the expected value of his valuation, and (ii) a model where buyers obtain signals by "sampling" the distribution of their normally distributed ex-post valuations.

The overall conclusion of our investigation is that under quite general conditions, the seller who controls the "flow of information" in an auction appropriates the rents of that information.

7 Appendix: Omitted Proofs

Proof of Proposition 1. Rewrite (12) as

$$\pi_i^*(v_i, \hat{v}_i) = \pi_i^*(\hat{v}_i, \hat{v}_i) + \int [u_i(v_i, s_i) - u_i(\hat{v}_i, s_i)] X_i(\hat{v}_i, s_i) dG_i(s_i).$$

By switching the roles of v_i and $\hat{v}_i$ we get

$$\pi_i^*(\hat{v}_i, v_i) = \pi_i^*(v_i, v_i) + \int [u_i(\hat{v}_i, s_i) - u_i(v_i, s_i)] X_i(v_i, s_i) dG_i(s_i).$$

Incentive compatibility of the mechanism is equivalent to, for all $v_i > \hat{v}_i$,

$$\pi_i^*(v_i, \hat{v}_i) \leq \pi_i^*(v_i, v_i) \text{ and } \pi_i^*(\hat{v}_i, v_i) \leq \pi_i^*(\hat{v}_i, \hat{v}_i).$$

Using the above expressions for the profit this becomes, for all $v_i > \hat{v}_i$,

$$\begin{aligned} \int \left[u_i(v_i, s_i) - u_i(\hat{v}_i, s_i)\right] X_i(\hat{v}_i, s_i) dG_i(s_i) & \\ &\leq \pi_i^*(v_i, v_i) - \pi_i^*(\hat{v}_i, \hat{v}_i) \\ &\leq \int \left[u_i(v_i, s_i) - u_i(\hat{v}_i, s_i)\right] X_i(v_i, s_i) dG_i(s_i). \end{aligned} \quad (32)$$

Suppose $v_i > \hat{v}_i$. Then, by $u_{i1} > 0$ and $X_i \geq 0$, the first line of the above inequality-system is non-negative, hence $\Pi_i^*(v_i) \equiv \pi_i^*(v_i, v_i)$ is weakly increasing. Cross-divide (32) by $(v_i - \hat{v}_i)$ to get

$$\begin{aligned} \int \frac{u_i(v_i, s_i) - u_i(\hat{v}_i, s_i)}{v_i - \hat{v}_i} X_i(\hat{v}_i, s_i) dG_i(s_i) & \\ &\leq \frac{\Pi_i^*(v_i) - \Pi_i^*(\hat{v}_i)}{v_i - \hat{v}_i} \\ &\leq \int \frac{u_i(v_i, s_i) - u_i(\hat{v}_i, s_i)}{v_i - \hat{v}_i} X_i(v_i, s_i) dG_i(s_i). \end{aligned} \quad (33)$$

Consider the limit of the integral on the first line as v_i approaches $\hat{v}_i$. By concavity of u_i in v_i and $X_i \leq 1$, the integrand (pointwise, for all s_i) is weakly less than $u_{i1}(0, s_i)$, which in turn has a finite expectation with respect to s_i. Hence, by the Lebesgue Convergence Theorem (cf. Royden (1967), p.76), the order of taking the limit and the expectation can be reversed, and

$$\lim_{v_i \to \hat{v}_i} \int \frac{u_i(v_i, s_i) - u_i(\hat{v}_i, s_i)}{v_i - \hat{v}_i} X_i(\hat{v}_i, s_i) dG_i(s_i) = \int u_{i1}(\hat{v}_i, s_i) X_i(\hat{v}_i, s_i) dG_i(s_i).$$

That is, as v_i converges to $\hat{v}_i$, the first inequality of (33) implies

$$\int u_{i1}(\hat{v}_i, s_i) X_i(\hat{v}_i, s_i) dG_i(s_i) \leq \Pi_i^{*\prime}{}_{+}(\hat{v}_i)$$

whenever the right-hand derivative of Π_i^* exists at $\hat{v}_i$. By analogous reasoning, as $\hat{v}_i$

approaches v_i, the second inequality of (33) implies that

$$\int_{s_i} u_{i1}(v_i, s_i)X_i(v_i, s_i)dG_i(s_i) \geq \Pi_{i\ -}^{*\prime}(v_i)$$

whenever the left-side derivative of Π_i^* exists at v_i. Since Π_i^* is weakly increasing, its derivative exists almost everywhere, therefore

$$\Pi_i^{*\prime}(v_i) = \int u_{i1}(v_i, s_i)X_i(v_i, s_i)dG_i(s_i) \tag{34}$$

almost everywhere.

From the second inequality of (33), concavity of u_i in v_i, and $X_i \leq 1$, it follows that

$$\frac{\Pi_i^*(v_i) - \Pi_i^*(\hat{v}_i)}{v_i - \hat{v}_i} \leq \int u_{i1}(0, s_i)dG_i(s_i) < \infty,$$

hence Π_i^* is Lipschitz continuous. This implies that Π_i^* is also absolutely continuous, therefore it can be recovered from its derivative (Royden (1967), p.91). We conclude that

$$\Pi_i^*(v_i) = \Pi_i^*(0) + \int_0^{v_i} \int_{s_i} u_{i1}(y, s_i)X_i(y, s_i)dG_i(s_i)dy. \tag{35}$$

Given the profit function of buyer type v_i in (35), we can write the seller's expected revenue (after the application of Fubini's Theorem) as

$$\iint \sum_i x_i(v_i, v_{-i}, s_i, s_{-i}) \left[u_i(v_i, s_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} u_{i1}(v_i, s_i)\right] dF(v)dG(s). \tag{36}$$

Our candidate optimal allocation rule will be $x_i^*(v_i, v_{-i}, s_i, s_{-i}) = 1$ for the buyer with the highest non-negative "shock-adjusted virtual valuation,"

$$u_i(v_i, s_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} u_{i1}(v_i, s_i).$$

Clearly, this allocation rule pointwise maximizes the integrand in (36). The only remaining question is whether this rule is incentive compatible. Note that the candidate optimal allocation rule, $x_i^*(v_i, v_{-i}, s_i, s_{-i})$, hence $X_i^*(v_i, s_i)$, is weakly increasing in v_i because, by assumption, u_i is increasing in v_i, $(1 - F_i)/f_i$ is weakly decreasing in v_i (monotone hazard rate), and u_{i1} is weakly decreasing in v_i (u_i is concave in v_i)

We will now show that if (35) holds and $X_i(v_i, s_i)$ is weakly increasing in v_i then a mechanism inducing this allocation rule and Π_i^* profit for buyer i $(i = 1, ..., n)$ is incentive compatible. Suppose $v_i > \hat{v}_i$ and use (35) to rewrite

$$\Pi_i^*(v_i) - \Pi_i^*(\hat{v}_i) = \int \int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) X_i(y, s_i) dy \, dG_i(s_i).$$

Decrease the integrand by replacing $X_i(y, s_i)$ with $X_i(\hat{v}_i, s_i)$,

$$\begin{aligned} \Pi_i^*(v_i) - \Pi_i^*(\hat{v}_i) &\geq \int \int_{\hat{v}_i}^{v_i} u_{i1}(y, s_i) dy \, X_i(\hat{v}_i, s_i) dG_i(s_i) \\ &= \int \left[u_i(y, s_i) - u_i(\hat{v}_i, s_i)\right] X_i(\hat{v}_i, s_i) dG_i(s_i). \end{aligned}$$

But this is exactly the incentive compatibility condition for buyer type v_i not to imitate $\hat{v}_i < v_i$, (32). A similar argument applies when $\hat{v}_i > v_i$. ■

Proof of Lemma 2. Rewrite (19) as

$$\tilde{\pi}_i(s_i, \hat{s}_i; v_i) = \tilde{\pi}_i(\hat{s}_i, \hat{s}_i; v_i) + \left[u_i(v_i, s_i) - u_i(v_i, \hat{s}_i)\right] X_i(v_i, \hat{s}_i) \tag{37}$$

By reversing the roles of s_i and $\hat{s}_i$, we get

$$\tilde{\pi}_i(\hat{s}_i, s_i; v_i) = \tilde{\pi}_i(s_i, s_i; v_i) - \left[u_i(v_i, s_i) - u_i(v_i, \hat{s}_i)\right] X_i(v_i, s_i).$$

Incentive compatibility requires, for all $s_i > \hat{s}_i$,

$$\tilde{\pi}(s_i, \hat{s}_i; v_i) \leq \tilde{\pi}_i(s_i, s_i; v_i) \text{ and } \tilde{\pi}(\hat{s}_i, s_i; v_i) \leq \tilde{\pi}_i(\hat{s}_i, \hat{s}_i; v_i).$$

Using the above expressions for $\tilde{\pi}_i(s_i, \hat{s}_i; v_i)$ and $\tilde{\pi}_i(\hat{s}_i, s_i; v_i)$ incentive compatibility becomes, for all $s_i > \hat{s}_i$,

$$\begin{aligned} \left[u_i(v_i, s_i) - u_i(v_i, \hat{s}_i)\right] X_i(v_i, \hat{s}_i) &\leq \tilde{\pi}_i(s_i, s_i; v_i) - \tilde{\pi}_i(\hat{s}_i, \hat{s}_i; v_i) \\ &\leq \left[u_i(v_i, s_i) - u_i(v_i, \hat{s}_i)\right] X_i(v_i, s_i). \end{aligned} \tag{38}$$

Cross-divide (38) by $(s_i - \hat{s}_i)$. Take $(s_i - \hat{s}_i)$ to zero: the first and third lines of (38) both converge to $u_{i2}(v_i, s_i) X_i(v_i, s_i)$. Therefore $\tilde{\pi}_i(s_i, s_i; v_i)$ is differentiable in s_i everywhere and

$$\frac{d}{ds_i} \tilde{\pi}_i(s_i, s_i; v_i) = u_{i2}(v_i, s_i) X_i(v_i, s_i).$$

This derivative is continuous in s_i (because both u_{i2} and X_i are), hence we can integrate it to get (20).

To see the second part of the claim, suppose buyer i reports $\hat{s}_i < s_i$ after seeing s_i. His deviation gain is

$$\begin{aligned}&\tilde{\pi}_i(s_i, \hat{s}_i; v_i) - \tilde{\pi}_i(s_i, s_i; v_i)\\ &\qquad = [\tilde{\pi}_i(s_i, \hat{s}_i; v_i) - \tilde{\pi}_i(\hat{s}_i, \hat{s}_i; v_i)] - [\tilde{\pi}_i(s_i, s_i; v_i) - \tilde{\pi}_i(\hat{s}_i, \hat{s}_i; v_i)] .\end{aligned}$$

Recall from (37) that the first bracketed difference is

$$[u_i(v_i, s_i) - u_i(v_i, \hat{s}_i)]\, X_i(v_i, \hat{s}_i),$$

and by (20) the second one can be rewritten as

$$\begin{aligned}\int_{\hat{s}_i}^{s_i} u_{i2}(v_i, z) X_i(v_i, z) dz &\geq \int_{\hat{s}_i}^{s_i} u_{i2}(v_i, z) X_i(v_i, \hat{s}_i) dz\\ &= [u_i(v_i, s_i) - u_i(v_i, \hat{s}_i)]\, X_i(v_i, \hat{s}_i).\end{aligned}$$

(The inequality follows from X_i being weakly increasing in s_i.) Therefore, the gain from deviation is non-positive. A similar argument can be used to show that a deviation to $\hat{s}_i > s_i$ is not profitable either. ■

Proof of Proposition 2. If, for all $j = 1, ..., n$ and $v_j \in [\underline{v}, \bar{v}]$, type v_j of buyer j purchases a premium $p_j(v_j) = (1 - F_j(v_j))/f_j(v_j) u_{i1}$ then buyer i will win in the second round if and only if, for all j,

$$u_i(v_i, s_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} u_{i1} \geq \max\left\{ u_j(v_j, s_j) - \frac{1 - F_j(v_j)}{f_j(v_j)} u_{i1}, 0 \right\}.$$

This is so because in the second round, every buyer j bids $u_j(v_j, s_j) - p_j(v_j)$ by the assumption that the buyers follow weakly dominant strategies in the second round. Hence the allocation rule is indeed the same as in the optimal mechanism, provided that all buyers behave "truthfully," i.e., every buyer j with type v_j chooses $p_j(v_j)$ for a fee $c_j(v_j)$ defined in (31).

We show that the handicap auction defined by (30) and (31) is incentive compatible. Let $\pi_i^h(v_i, \hat{v}_i)$ denote the payoff of buyer i with type v_i if he "deviates" to $\hat{v}_i$ (chooses

$c_i(\hat{v}_i)$ and $p_i(\hat{v}_i)$). First, we show that for buyer i there is no incentive to deviate downwards. Suppose $\hat{v}_i < v_i$. Then the deviator bids $u_i(v_i, s_i) - p_i(\hat{v}_i)$ in the second round and his payoff is

$$
\begin{aligned}
&\pi_i^h(v_i, \hat{v}_i) \\
&= -c_i(\hat{v}_i) + \iint \left(u_i(v_i, s_i) - p_i(\hat{v}_i) - W_{-i}^0(v_{-i}, s_{-i})\right) \mathbf{1}_{\{u_i(v_i,s_i) - p_i(\hat{v}_i) \geq W_{-i}^0(v_{-i}, s_{-i})\}}\, dF_{-i} dG.
\end{aligned}
$$

Using the definitions of c_i and W_i,

$$
\begin{aligned}
\pi_i^h(v_i, \hat{v}_i) = &\iint \left[\int_{\underline{v}}^{\hat{v}_i} u_{i1} \mathbf{1}_{\{W_i(y,s_i) \geq W_{-i}^0(v_{-i}, s_{-i})\}} dy\right] dF_{-i} dG \\
&- \iint \left(u_i(\hat{v}_i, s_i) - p_i(\hat{v}_i) - W_{-i}^0(v_{-i}, s_{-i})\right) \mathbf{1}_{\{u_i(\hat{v}_i,s_i) - p_i(\hat{v}_i) \geq W_{-i}^0(v_{-i}, s_{-i})\}} dF_{-i} dG \\
&+ \iint \left(u_i(v_i, s_i) - p_i(\hat{v}_i) - W_{-i}^0(v_{-i}, s_{-i})\right) \mathbf{1}_{\{u_i(v_i,s_i) - p_i(\hat{v}_i) \geq W_{-i}^0(v_{-i}, s_{-i})\}}\, dF_{-i} dG.
\end{aligned}
$$

Notice, if $v_i = \hat{v}_i$, then the terms in the second and the third lines cancel each others, therefore the integral in the first line equals $\pi_i^h(\hat{v}_i, \hat{v}_i)$. This implies that

$$
\pi_i^h(v_i, v_i) = \pi_i^h(\hat{v}_i, \hat{v}_i) + \iint \left[\int_{\hat{v}_i}^{v_i} u_{i1} \mathbf{1}_{\{W_i(y,s_i) \geq W_{-i}^0(v_{-i}, s_{-i})\}} dy\right] dF_{-i} dG \tag{39}
$$

and

$$
\begin{aligned}
\pi_i^h(v_i, \hat{v}_i) = &\ \pi_i^h(\hat{v}_i, \hat{v}_i) \\
&- \iint \left(u_i(\hat{v}_i, s_i) - p_i(\hat{v}_i) - W_{-i}^0(v_{-i}, s_{-i})\right) \mathbf{1}_{\{u_i(\hat{v}_i,s_i) - p_i(\hat{v}_i) \geq W_{-i}^0(v_{-i}, s_{-i})\}} dF_{-i} dG \\
&+ \iint \left(u_i(v_i, s_i) - p_i(\hat{v}_i) - W_{-i}^0(v_{-i}, s_{-i})\right) \mathbf{1}_{\{u_i(v_i,s_i) - p_i(\hat{v}_i) \geq W_{-i}^0(v_{-i}, s_{-i})\}}\, dF_{-i} dG.
\end{aligned} \tag{40}
$$

Since $\partial u_i(v_i, s_i)/\partial v_i = u_{i1}$ is a constant, $u_i(v_i, s_i)$ can be written as $u_{i1} v_i + r_i(s_i)$. We claim that the difference between the integrands in the second and third terms can be rewritten as,

$$
\begin{aligned}
&\left(u_{i1} v_i + r_i(s_i) - p_i(\hat{v}_i) - W_{-i}^0(v_{-i}, s_{-i})\right) \mathbf{1}_{\{u_{i1} v_i + r_i(s_i) - p_i(\hat{v}_i) \geq W_{-i}^0(v_{-i}, s_{-i})\}} \\
&- \left(u_{i1} \hat{v}_i + r_i(s_i) - p_i(\hat{v}_i) - W_{-i}^0(v_{-i}, s_{-i})\right) \mathbf{1}_{\{u_{i1} \hat{v}_i + r_i(s_i) - p_i(\hat{v}_i) \geq W_{-i}^0(v_{-i}, s_{-i})\}} \\
&= \int_{\hat{v}_i}^{v_i} u_{i1} \mathbf{1}_{\{u_{i1} y + r_i(s_i) - p_i(\hat{v}_i) \geq W_{-i}^0(v_{-i}, s_{-i})\}} dy.
\end{aligned} \tag{41}
$$

If $u_{i1}\hat{v}_i + r_i(s_i) - p_i(\hat{v}_i) \geq W^0_{-i}(v_{-i}, s_{-i})$ then the difference is

$$\begin{aligned} u_{i1}\left(v_i - \hat{v}_i\right) &= \int_{\hat{v}_i}^{v_i} u_{i1} dy \\ &= \int_{\hat{v}_i}^{v_i} u_{i1} \mathbf{1}_{\{u_{i1}y + r_i(s_i) - p_i(\hat{v}_i) \geq W^0_{-i}(v_{-i}, s_{-i})\}} dy. \end{aligned}$$

Suppose $u_{i1}v_i + r_i(s_i) - p_i(\hat{v}_i) \geq W^0_{-i}(v_{-i}, s_{-i}) \geq u_{i1}\hat{v}_i + r_i(s_i) - p_i(\hat{v}_i)$. Let $y^* \in \{\hat{v}_i, v_i\}$ be such that $W^0_{-i}(v_{-i}, s_{-i}) = u_{i1}v_i + r_i(s_i) - p_i(\hat{v}_i)$. Then (41) is

$$\begin{aligned} & u_{i1}v_i + r_i(s_i) - p_i(\hat{v}_i) - W^0_{-i}(v_{-i}, s_{-i}) \\ &= u_{i1}\left(v_i - y^*\right) = \int_{y^*}^{v_i} u_{i1} dy \\ &= \int_{\hat{v}_i}^{v_i} u_{i1} \mathbf{1}_{\{u_{i1}y + r_i(s_i) - p_i(\hat{v}_i) \geq W^0_{-i}(v_{-i}, s_{-i})\}} dy. \end{aligned}$$

If $W^0_{-i}(v_{-i}, s_{-i}) \geq u_{i1}y + r_i(s_i) - p_i(\hat{v}_i)$ then the value of both sides of (41) is zero. Therefore (41) indeed holds. From (40) and (41) it follows that

$$\pi_i^h(v_i, \hat{v}_i) = \pi_i^h(\hat{v}_i, \hat{v}_i) + \iint \left[\int_{\hat{v}_i}^{v_i} u_{i1} \mathbf{1}_{\{u_{i1}y + r_i(s_i) - p_i(\hat{v}_i) \geq W^0_{-i}(v_{-i}, s_{-i})\}} dy\right] dF_{-i} dG.$$

Notice that since p_i is decreasing, $u_{i1}y + r_i(s_i) - p_i(\hat{v}_i) \leq u_{i1}y + r_i(s_i) - p_i(y)$ whenever $y \in \{\hat{v}_i, v_i\}$. Hence

$$\begin{aligned} \pi_i^h(v_i, \hat{v}_i) &\leq \pi_i^h(\hat{v}_i, \hat{v}_i) + \iint \left[\int_{\hat{v}_i}^{v_i} u_{i1} \mathbf{1}_{\{u_{i1}y + r_i(s_i) - p_i(y) \geq W^0_{-i}(v_{-i}, s_{-i})\}} dy\right] dF_{-i} dG \\ &= \pi_i^h(v_i, v_i), \end{aligned}$$

where the equality follows from (39). This means that for buyer i there is no incentive to deviate downwards. An almost identical argument shows that there is no incentive to deviate upwards either. ■

8 Appendix (Not for Publication)

In this appendix we translate the conditions imposed on the shape of the u_i functions (Assumptions 1 and 2) into assumptions on the correlation between a buyer's initial estimate, v_i, and his posterior, V_i (the valuation that the buyer learns after the seller releases all the information she has). Recall that the meaning of Assumptions 1-2 are that the orthogonally normalized signals v_i and s_i are substitutes and have decreasing returns in buyer i's valuation function, u_i. The assumptions on the correlation of v_i and V_i given below express exactly the same, and are provided for the sake of completeness.

In order to simplify notation we omit the reference to the identity of buyer i, and define $z \equiv V - v$, which is the "error term" in the buyer's original estimate. Note that z and v may be correlated. Denote the cumulative distribution of z given v by $H_v(z)$ and the corresponding (positive) density by $h_v(z)$. We will call the model where the primitives are v and z as the "model with additive, correlated signals".

Recall that in the paper we transformed the model with additive, correlated signals into one with orthogonally normalized v and s by letting $s \equiv H_v(z)$, and $u(v,s) = v + H_v^{-1}(s)$. Therefore, $u(v,s) \equiv V$.

Differentiating the identity $u(v,s) \equiv v + H_v^{-1}(s)$ yields

$$\begin{aligned} \frac{\partial u(v,s)}{\partial v} &= 1 + \frac{\partial H_v^{-1}(s)}{\partial v} \\ \frac{\partial u(v,s)}{\partial s} &= \frac{\partial H_v^{-1}(s)}{\partial s} = \frac{1}{h_v\left(H_v^{-1}(s)\right)}. \end{aligned} \tag{42}$$

Since $h_v > 0$, the second line is always positive. As far as the first line is concerned, we first show that for $s = H_v(z)$,

$$\frac{\partial H_v^{-1}(s)}{\partial v} = -\frac{\partial H_v(z)/\partial v}{h_v(z)}. \tag{43}$$

To see this, fix s and define $\widetilde{z}(v)$ implicitly by $s = H_v(\widetilde{z}(v))$. Differentiating this identity according to v and rearranging it we get

$$\frac{d\widetilde{z}(v)}{dv} = -\frac{\partial H_v(\widetilde{z}(v))/\partial v}{h_v(\widetilde{z}(v))}.$$

On the other hand, $H_v^{-1}(s) = \widetilde{z}(v)$, hence

$$\frac{d\widetilde{z}(v)}{dv} = \frac{\partial H_v^{-1}(s)}{\partial v}.$$

So, from (42) and (43) we can conclude that the assumption that u is increasing in v in the orthogonally normalized model translates to

$$1 - \frac{\partial H_v(z)/\partial v}{h_v(z)} > 0 \tag{44}$$

in the model with additive, correlated signals.

From the second line of (42), using the rule for derivatives of inverse functions and the chain rule we get,

$$\begin{aligned}\frac{\partial^2 u(v,s)}{\partial s \partial v} &= \frac{\partial^2 H_v^{-1}(s)}{\partial s \partial v} = \frac{\partial\left(1/h_v\left(H_v^{-1}(s)\right)\right)}{\partial v} \\ &= -\frac{1}{h_v^2\left(H_v^{-1}(s)\right)}\left(\left.\frac{\partial h_v(z)}{\partial v}\right|_{z=H_v^{-1}(s)}\right)\frac{\partial H_v^{-1}(s)}{\partial v}.\end{aligned} \tag{45}$$

Using (43) we can further rewrite it as

$$\frac{\partial^2 u(v,s)}{\partial s \partial v} = \left.\frac{1}{h_v^3(z)}\frac{\partial h_v(z)}{\partial v}\frac{\partial H_v(z)}{\partial v}\right|_{z=H_v^{-1}(s)}.$$

Since the density h_v is positive, Assumption 1 requiring $u_{12} \leq 0$ in the orthogonally normalized model translates into

$$\frac{\partial h_v(z)}{\partial v}\frac{\partial H_v(z)}{\partial v} \leq 0 \tag{46}$$

in the model with additive, correlated signals.

From (42) and (43)

$$\begin{aligned}\frac{\partial^2 u(v,s)}{\partial v^2} &= -\left.\frac{\partial\left[\partial H_v(z)/\partial v\right]/h_v(z)}{\partial v}\right|_{z=H_v^{-1}(s)} \\ &= -\left.\frac{h_v(z)\,\partial^2 H_v(z)/\partial v^2 - \left[\partial h_v(z)/\partial v\right]\left[\partial H_v(z)/\partial v\right]}{h_v^2(z)}\right|_{z=H_v^{-1}(s)}.\end{aligned}$$

Therefore, for $z = H_v^{-1}(s)$,

$$\begin{aligned}
&\frac{\partial^2 u(v,s)}{\partial v^2} / \frac{\partial u(v,s)}{\partial v} = \\
&-\frac{h_v(z)\,\partial^2 H_v(z)/\partial v^2 - [\partial h_v(z)/\partial v]\,[\partial H_v(z)/\partial v]}{h_v^2(z)} / \left[1 - \frac{\partial H_v(z)/\partial v}{h_v(z)}\right] \\
&= -\frac{h_v(z)\,\partial^2 H_v(z)/\partial v^2 - [\partial h_v(z)/\partial v]\,[\partial H_v(z)/\partial v]}{h_v^2(z)} / \left[\frac{h_v(z) - \partial H_v(z)/\partial v}{h_v(z)}\right] \\
&= -\frac{h_v(z)\,\partial^2 H_v(z)/\partial v^2 - [\partial h_v(z)/\partial v]\,[\partial H_v(z)/\partial v]}{h_v(z)\,(h_v(z) - \partial H_v(z)/\partial v)}.
\end{aligned}$$

And, by (45) and (42),

$$\frac{\partial^2 u(v,s)}{\partial v \partial s} / \frac{\partial u(v,s)}{\partial s} = \frac{1}{h_v^2(z)} \frac{\partial h_v(z)}{\partial v} \frac{\partial H_v(z)}{\partial v}\bigg]_{z=H_v^{-1}(s)}.$$

Hence the assumption $u_{11}/u_1 \leq u_{12}/u_2$ in the orthogonally normalized model translates to

$$-\frac{h_v(z)\,\partial^2 H_v(z)/\partial v^2 - [\partial h_v(z)/\partial v]\,[\partial H_v(z)/\partial v]}{h_v(z)\,(h_v(z) - \partial H_v(z)/\partial v)} \leq \frac{1}{h_v^2(z)} \frac{\partial h_v(z)}{\partial v} \frac{\partial H_v(z)}{\partial v}.$$

After multiplying both sides by $h_v(z)\,(h_v(z) - \partial H_v(z)/\partial v)$, which is non-negative by (44), we get

$$\begin{aligned}
\frac{\partial h_v(z)}{\partial v} \frac{\partial H_v(z)}{\partial v} - h_v(z) \frac{\partial^2 H_v(z)}{\partial v^2} &\leq \frac{(h_v(z) - \partial H_v(z)/\partial v)}{h_v(z)} \frac{\partial h_v(z)}{\partial v} \frac{\partial H_v(z)}{\partial v} \\
&= \frac{\partial h_v(z)}{\partial v} \frac{\partial H_v(z)}{\partial v} - \frac{(\partial H_v(z)/\partial v)^2}{h_v(z)} \frac{\partial h_v(z)}{\partial v}.
\end{aligned}$$

Notice that the first term is the same on both sides of the inequality. Hence, Assumption 2 is equivalent to

$$\frac{\partial^2 H_v(z)/\partial v^2}{(\partial H_v(z)/\partial v)^2} \geq \frac{\partial h_v(z)/\partial v}{h_v^2(z)}. \tag{47}$$

Summary: The assumption that u is increasing in both arguments is equivalent to

$$1 - \frac{\partial H_v(z)/\partial v}{h_v(z)} > 0.$$

The two numbered assumptions in the paper translate as follows:

$$
\begin{aligned}
&\text{Assumption 1: } u_{12} \leq 0 \Leftrightarrow \frac{\partial h_v(z)}{\partial v}\frac{\partial H_v(z)}{\partial v} \leq 0, \\
&\text{Assumption 2: } \frac{u_{11}}{u_1} \leq \frac{u_{12}}{u_2} \Leftrightarrow \frac{\partial^2 H_v(z)/\partial v^2}{\left(\partial H_v(z)/\partial v\right)^2} \geq \frac{\partial h_v(z)/\partial v}{h_v^2(z)}.
\end{aligned}
$$

The meaning of the latter two conditions, as we discussed above, is the same as that of Assumptions 1 and 2: The buyer's original signal, v, and the component of the error term that is orthogonal to v, have decreasing returns and are substitutes in V. It appears to us that the equivalent formulation where the signals are orthogonally normalized demonstrates this interpretation much more clearly.

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