Competing Through Information Provision*

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Abstract

This paper studies the competition between sellers who choose how much information to provide to potential buyers. We analyse the symmetric equilibria in information provision of a game in which two sellers with unit supplies compete to attract two buyers with unit demands. Sellers compete ex ante; they commit to a level of information provision and to a sale mechanism (e.g. a second-price auction). More informed buyers have better differentiated private valuations and trade yields them higher informational rents. Our focus is on this critical trade-off faced by sellers: promising information attracts buyers (traffic effect) but lowers profits-per-buyer (rents effect). When the sale mechanisms are common and exogenously fixed, we find that sellers' equilibrium profits can be higher under mechanisms that yield more rents to buyers. High-rent mechanisms inhibit market-stealing and soften the competition between sellers, which lowers equilibrium levels of information provision. High rent levels may also intensify the competition for goods between the buyers, which compresses the traffic-rents trade-off and further dampens the competition between sellers. When sellers promise both information and sale mechanisms, we show that they can capture the efficiency gains of increased information so that all symmetric equilibria in a large class have full information provision.

1 Introduction

Competing sellers are typically modelled as proposing prices to buyers, or more generally sale mechanisms. However, as the quality of buyers' information about a good affects their gains from trade, sellers may try to attract buyers by offering better information. This paper considers a market in which sellers simultaneously announce levels of information provision to potential buyers, who then choose which seller to visit. When choosing their strategies, sellers trade off market share against the cost of selling goods to buyers with better private information.

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Privately informed buyers gain informational rents through trade. Conceptually, a buyer's information about his value for a good has two elements; the private knowledge of some personal characteristics, along with an understanding of how these personal attributes relate to the good's properties. Sellers cannot influence the first kind of knowledge, but controlling the available information about their good affects the second kind. By providing less information to buyers before trading, sellers give out fewer informational rents in the exchange process. Yet, and this is the focus of this paper, if sellers compete for buyers, the latter may shun low-information selling sites.

This paper is captures some traits of competitive markets where sellers make decisions about the precision of the information received by buyers before actual trades occur. An interesting example is provided by an episode in the competition between the auction houses of Christie's and Sotheby's. In that industry, the services surrounding an auction play a critical role in allowing potential customers to better evaluate an object's worth to them. In the early 1990's, competition between the auction houses stiffened considerably, and expanding the services that provide information to potential buyers became an important competitive tool.

> (In the early 1990's,) the auction houses embarked on cutthroat competition to get goods for sale. (...) A greater problem was created by the perceived need to provide ever more luxurious services. Catalogues became ever fatter, printed in colour, on glossy art paper. Pre-sale viewings of selected works from major collections were held across the world. (...) On the inside page of Sotheby's catalogue of the Old Master paintings sale held in London on Dec. 13 (2001), six "specialists in charge" are listed. (...) They identify the paintings, research them, know which world specialist on this or that painter needs to be contacted, and, more mundanely, which client is most likely to be interested in what painting, etc.[1]

Furthermore, later in the decade Phillips, a minor auction house, tried to break the Christie's-Sotheby's duopoly. It did so by providing high guarantees to sellers who committed their pieces with them, but it also tried to match the big auction houses' superior capacity to inform buyers by luring away some of their teams of experts.[2] However, eventually Phillips became "less willing to provide lavish guarantees and loans. It emerged that Phillips's cash, rather than its expertise, had lured sellers of high-quality art; they returned to Christie's and Sotheby's."[3]

As another example, the website of Multiple Listing Service, mls.com, allows real-estate agents to advertise houses for sale by posting pictures and descriptions. Rival agents can adopt very different strategies and the quality of the information revealed in the advertisements varies widely. Some agents post a bare-bones description of the house along with a

[1]International Herald Tribune, 12/01/2002.
[2]The Economist, 01/03/2001.
[3]The Economist, 21/02/2002.

picture of the house's exterior. Others provide pictures of some of the rooms, some even post a full slide show. Bergemann and Pesendorfer (2003) provide other examples of both monopolistic and competitive markets where information provision decisions are important.

This paper presents a simple model in which two sellers with unit supplies compete for the unit demands of two buyers by promising information. A simple game in extensive form is studied that is analogous to that presented in the literature on competing mechanism designers.[4] In this game, sellers first simultaneously communicate the informational conditions to prevail at their sites to the buyers. We assume that sellers can credibly commit to information provision. Buyers then choose which seller to visit, and sales take place. Sorting takes place ex ante; buyers obtain their private information only once they choose a seller and this information is mediated by the information structure chosen by the seller. The presence of a single good at each site provides incentives for buyers to avoid meeting at the same site, allowing sellers to exploit the competition between buyers.

As is typical in the directed search literature, in the subgame following the sellers' announcements, we assume that buyers sort into sale sites according to that subgame's unique symmetric mixed strategy equilibrium. This rules out any implicit coordination among buyers and ensures smooth responses in sellers' profits to changes in their announcements. In equilibrium, sellers face a random demand, whose distribution they affect through their choice of strategy. The equilibria of the game between the sellers with demand generated by the symmetric equilibrium of the buyers' subgame are subgame perfect equilibria of the full game. We consider two variants for the sellers' strategy sets. In the first, sellers only commit to information provision, while in the second they commit both to information and to sale mechanisms. In both cases, we restrict attention to symmetric equilibria of the game between the sellers.

In the first variant of the model, presented in Section 3, sale mechanisms are exogenously fixed, common to both sites and known to buyers before they decide which seller to visit. Sellers can attract buyers only by promising information. Information provision increases buyer rents across demand states (i.e., when one buyer or two buyers are present) and generates a trade-off for sellers; higher information attracts more traffic yet decreases profits-per-head. Fixing the mechanism determines the shape of this trade-off, which, through competition between the sellers, determines equilibrium information provision. For a class of mechanisms broad enough to include common mechanisms such as auctions and prices, we present comparative statics results for interior symmetric equilibria in information provision. First, the equilibrium level of information provision is decreasing in the rents offered to buyers in either demand state. Under mechanisms that are more generous to buyers, providing more information is costly for sellers. This reduces the gains to traffic-stealing and lowers the intensity of competition between the sellers. Reduced competition leads to lower equilibrium

[4] Following McAfee (1993).

information provision. Second, sellers' equilibrium profits are not monotone in the rents offered to buyers by the sale mechanisms. Increased rents generate two opposing effects; equilibrium information provision decreases and so profits are higher, but payouts to buyers are higher for any level of information provision. We show that the first effect dominates if a mechanism offers higher rents in the one-buyer demand state, while the second effect may dominate if a mechanism offers higher rents in the two-buyer demand state. When both buyers visit the same site, they compete to obtain a single good. If the difference in rent levels across demand states is large, buyers are strongly averse to meeting at a site and offers of information will draw little traffic. That is, mechanisms that induce strong competition between buyers flatten the traffic-rents trade-off and dampen the competition between sellers. Increasing the rents offered by a sale mechanism in the one-buyer state makes the two-buyer state relatively unattractive and stiffens the competition between the buyers, and so always increases sellers' profits. On the other hand, increasing rents in the two-buyer demand state reduces buyers' aversion to meeting at a site. The decrease in the intensity of competition between the sellers is then less important than in the case of increases in rents in the one-buyer state, and this limits the drop in the equilibrium level of information provision.

We illustrate the results of Section 3 with two examples. In the first, we consider mechanisms that specify fixed state-dependent prices. In that case, the sellers' preferred mechanism gives the good away for free in the one-buyer demand state and charges a high price in the two-buyer demand state. A low price in the one-buyer state increases buyers' expected rents and allows equilibrium information provision to drop, while a high price in the two-buyer state props up equilibrium information provision but increases the competition between buyers. In the second example we consider the game between sellers who commit to information provision but cannot commit to sale mechanisms. In that case, mechanisms are ex post optimal. These offer low rents to buyers, hence the competition between sellers is intense and in the unique symmetric equilibrium sellers promise full information.

Section 4 presents the most general version of the model that allows sellers to commit to both sale mechanisms and information provision. In this case, buyers are attracted to a seller's site not only by promises of information, but also by promises about terms of trade. Sellers can disentangle their rent-provision and information-provision decisions. Furthermore, in the presence of more than one buyer, higher information provision increases the social surplus as better information helps to identify the buyer who most values the good. Under a no-exclusion assumption and exploiting the ex ante nature of rent and information promises, we characterize a class of symmetric equilbria in which sellers capture the efficiency benefits of increased information by suitably compensating buyers through transfers. In all these symmetric equilibria, sellers promise full information and effectively compete over the rents accruing to buyers in each demand state. There exists a continuum of symmetric equilibria differentiated by the sharing of a fixed level of surplus between buyers and sellers. In all equilibria, competition drives rents in the one-buyer state to the marginal buyer's contribution to available surplus. This leaves additional surplus to be divided and the equilibria differ in the extent to which buyers are penalized when they meet at the same site. Any rent level in the two-buyer state between those offered in the equilibrium most favourable to sellers and those offered in the equilibrium most favourable to buyers can be supported as the rent level of some equilibrium choice of mechanisms. Fixing a given equilibrium, although a marginally lower rent level in the two-buyer state (i.e., a marginally higher 'penalty' imposed on a buyer when both are present) can itself by supported in some other equilibrium, deviating to a mechanism offering that level of rents is not profitable for sellers. The trade-off between traffic and rents is sharp at symmetric profiles and any such deviation results in a loss of traffic large enough to offset the gains in profits-per-head.

1.1 Related Literature

Recent work in mechanism design, auctions and optimal pricing has found that monopolists will often choose to substantially alter the informational attributes of their customers, when given some means of doing so. However, the question of how these incentives extend to a competitive market has received little attention in the literature to date. At this point it should be noted that the recent formulation of seller-controlled information structures, due to Bergemann and Pesendorfer (2003), is fundamentally different from the well-known *linkage principal* of Milgrom and Weber's (1982). The latter show that an auctioneer facing bidders with affiliated valuations gains by making any additional information about the good public. Recent work has focused on problems with private values and, more importantly, on buyers receiving private signals correlated with their true valuations.

To clarify how recent contributions have modelled seller-controlled information, we give a simplified presentation of *information structures*, as originally defined in Bergemann and Pesendorfer (2003). Buyers at a sale site receive private signals that contain information about their valuation for the good. The purpose of an information structure is to control how a buyer obtains an estimate of his valuation through these signals. Let V be the random variable that represents the possible valuations of buyers, and S be the random variable representing the signals they can receive. The distributions of both these variables are commonly known by buyers and sellers. An *information structure* determines how a buyer translates a signal realisation s into a belief about V. More precisely, an information structure consists of a joint distribution $F_{V,S}$ for V and S.[5] Since sellers do not observe signals received by buyers, a buyer in a market where the information structure is given by the distribution $F_{V,S}$ will, from the point of view of the seller, have a valuation distributed

[5]Whose marginals must be consistent with the distributions of V and S.

according to the conditional distribution $F_{V|S}$.

Bergemann and Pesendorfer (2003) study optimal information structures when mechanisms are designed after the distribution of information. This ex post modelling of the mechanism design problem of the seller contrasts with the approach of Esö and Szentez (2006), who show that when an auctioneer is allowed to 'sell' information by designing a mechanism where buyers report both private and seller-supplied signals, he can capture all rents accruing from the information he controls. This leads the auctioneer to release any signals he controls. Our result that all symmetric equilibria have full information provision when sellers simultaneously offer information and sale mechanisms bears some resemblance to theirs.[6] In other results in this literature, it is generally the case that monopoly sellers find it optimal to provide substantially less than full information. In particular, the results of Bergemann and Pesendorfer (2003) imply that significant distortions of information are optimal. In their model a seller optimally chooses monotone finite partitional information structures, according to which the bidders' space of possible values is partitioned into a finite number of subintervals, and bidders receive signals that tell them only the subinterval which contains their value. Ganuza and Penalva (2006) specialize this framework to the case where information structures are ordered by 'precision'[7] and study the (in)efficiency of the information structures offered by a second-price auctioneer to a fixed pool of buyers, and their relationship to the number of buyers. As the number of buyers increases, rents are compressed for any level of information provision and sellers can increase information to capture some of the efficiency gains. In an optimal monopoly pricing framework, Johnson and Myatt (2006) model a seller who chooses the dispersion of consumers' demand through various activities, notably advertising. Similarly to Ganuza and Penalva (2006), they let the monopolist's strategy choice (other than its pricing decision) be an index of the 'informativeness' of the advertising for the product.[8] In a result recalling that of Lewis and Sappington (1994), they find conditions under which a seller's optimal choice of information provision is to release either all or none of the available signals. What all these papers have in common is a monopolistic market structure and a fixed buyer pool. Bergemann and Valimäki's (2006) survey provides more references to related literature.

Damiano and Li (2007) present a model, related to that of Moscarini and Ottaviani (2001), in which two sellers compete for a single buyer who has independent binary valuations for the sellers' goods. Sellers choose the informativeness of a binary signal received by buyers about his value for their good. Once the buyer is informed, the sellers compete through prices. With ex post competition, their model differs starkly from ours. While

[6] There are nevertheless significant differences between their framework and ours. This is discussed further in Section 4.1.

[7] They assume the set of distributions $F_{V|S}$ from which a seller can choose be ordered by *dispersion*. See Müller and Stoyan (2002).

[8] They assume the various demand functions are ordered by what they term a sequence of rotations, which is related to more usual concepts of stochastic orders for random variables.

in our model, information promises attract buyers and affect the performance of ex post mechanisms, in theirs information provision determines the characteristics of the subsequent price competition. Furthermore, in their model, the presence of a single buyer implies that information provision has no efficiency-enhancing effect. Their equilibrium results suggest that in their model the primary role of information provision is to differentiate goods ex post to soften competition.

The other literature close to this paper studies competing auctioneers. The central trade-off is between rents offered to bidders through, for example, lower reserve prices, and the compression of rents through competition between bidders at crowded sites. In our model, this trade-off is critical, except that rents offered to bidders also come in the form of more precise information, not only through price reductions. This is not a very large literature; among its contributions are McAfee (1993), Peters and Severinov (1997), Hernando-Veciana (2005) and Burguet and Sàkovics (1999). Of these, only the last paper considers competition among a small number of sellers, while the first three consider the limiting case in which the numbers of buyers and sellers is large and individual sellers' decisions have negligible effects on the aggregate economy. Burguet and Sàkovics (1999) consider two second-price auctioneers sharing a potential demand of n buyers and competing through reserve prices. In their model, buyers sort ex post, that is, after knowing their values. The complexity of buyers' sorting decisions makes it difficult to determine the global characteristics of sellers' profits. The authors can show that the sellers' symmetric equilibrium strategies will not involve zero reserve prices, but they cannot fully characterize the equilibrium. Having only two buyers greatly improves the tractability of our model.

2 Model

2.1 The Game

We present here the extensive-form game studied in our paper. We focus on its general structure. A complete description of payoffs follows in Section 2.2. The essential details pertaining to sale mechanisms are also described there. A more complete, and standard, presentation of the sale mechanisms is reported to Appendix XXX.

Sellers: Two sellers, a and b, have a single good for sale.

Buyers: Two buyers have unit demands. An informed buyer's valuation for either seller's good is either θ_H or θ_L, with $\theta_H > \theta_L$. The sellers' goods are *ex ante similar* to buyers; the prior distribution of buyer valuations for either good is (p_H, p_L).

Information Provision: In the first stage of the game, sellers commit to information provision. The information structures we consider are as follows: seller k posts a probability π_k with which information about the good is revealed at site k to buyers who choose to attend it. The information structure affects all buyers at site k. That is, ex post, either all buyers at site k are informed or all are uninformed. The goods' ex ante characteristics can be interpreted as that pool of public knowledge about their value which all potential buyers can access. By promising more information, sellers commit to the provision of private signals that allow buyers to differentiate their private values from the public expectations. Strictly speaking, in our model sellers promise more information by promising a higher likelihood that buyers will get access to their true private valuations.

Terms of Trade: How goods are delivered to the buyer(s) attending site k may be exogenously fixed or committed to by the seller in tandem with π_k. Terms of trade are incentive compatible sale mechanisms that depend on demand and information states. To focus on competition in information provision, in Section 3 sale mechanism are fixed and a strategy for seller k is a probability π_k. In Section 4, sellers compete by promising both information and mechanisms, and a strategy for seller k is a probability π_k along with a demand and information state-contingent incentive-compatible direct mechanism. Throughout the paper, we consider only symmetric equilibria in the sellers' strategies.

Buyers' Subgame: Given sale mechanisms and information offers (π_a, π_b), buyers simultaneously choose which site to visit, picking at most one. Once sorted into selling sites, buyers then either receive information about the good or not, learn the realisation of the demand state, and take part in the sale mechanism. Let $\eta \in \{1, 2\}$ denote the *demand state* of a sale site and $\tau \in \{i, u\}$ its *information state*, where i stands for informed and u for uninformed. The *state* of a sale site is given by $(\eta, \tau) \in \{1, 2\} \times \{i, u\}$.

2.2 Payoffs and the Buyers' Subgame

Sale Mechanisms: Terms of trade at site k are given by incentive compatible mechanisms that depend on demand and information states. These mechanisms specify outcomes, probabilities of obtaining the good and monetary transfers, as functions of reported types for all information and demand states of the market. The mechanisms are also constrained to be anonymous; they cannot depend on a buyer's identity. Our use of sale mechanisms is standard, and their details are presented in Appendix XXX. Let Γ be the set of incentive compatible mechanisms for our model. Importantly, any mechanism $\gamma_k \in \Gamma$ at site k induces *ex ante rents in state* (η, τ) for buyers in each state (η, τ). Denote these rents by $\left\{R_k^{\eta,\tau}\right\}_{\substack{\eta\in\{1,2\}\\ \tau\in\{i,u\}}}$. These rents are computed before buyers learn their types. Thus, in informed states, ex ante rents are the average of θ_H and θ_L-type rents. We denote the *ex ante surplus at site k in state* (η, τ) *under mechanism* γ_k by $\mathcal{S}_k^{\eta,\tau}$. The ex ante surplus is obtained by averaging total gains from trade in state (η, τ) over buyer types, and it depends on mechanisms' allocation rules.

Sorting Equilibrium: In the buyers' subgame, we consider the symmetric mixed strategy equilibrium where each buyer visits site a with probability q. It has been argued, notably by

Levin and Smith (1994) in the context of a single auction with entry and by Burdett, Shi and Wright (2001) in a directed search model, that the equilibria with symmetric mixed strategies by buyers and random demand are more intuitively appealing than asymmetric pure strategy equilibria which generate fixed demand. The latter type of equilibria impose a form of implicit coordination among the buyers. Other than the equilibria in which buyers respond to sellers' offers with pure strategies, Burdett, Shi and Wright (2001) also show that there exists a large number of equilibria with pure actions by buyers on the equilibrium path but in which they support sellers' offers by threatening to revert to the mixed strategy equilibrium in the buyers' subgame. Such sophistication and coordination improve buyers' payoffs relative to the mixed strategy equilibrium, yet the behaviour displayed in these equilibrium is not truly relevant to the questions studied here. Even the pure strategy equilibria in which buyers simply react to sellers' first round offers are not satisfactory. The trade-off between attracting demand and extracting rents, central to this paper, is discontinuous with pure strategies, whereas it is smooth when buyers use mixed strategies. In this way, the probability with which buyers visit a seller admits a market-share interpretation that takes the ideas of the model beyond the simple two-buyer case.

Buyers' Subgame Equilibrium Rents: As mentioned above, we focus on symmetric mixed strategies by buyers where each buyer visits seller a with probability q. Given strategy (π_a, γ_a) and q, a buyer attending site a expects rents

$$\begin{aligned}\mathcal{R}_a(\pi_a, \gamma_a, q) &= \mathbf{E}_\eta \mathbf{E}_\tau R_a^{\eta,\tau} \\ &= q\Big[\pi_a R_a^{2,i} + (1-\pi_a) R_a^{2,u}\Big] \\ &\quad + (1-q)\Big[\pi_a R_a^{1,i} + (1-\pi_a) R_a^{1,u}\Big]. \end{aligned} \tag{1}$$

The first expectation on the top line is taken with respect to the binomial distribution with parameter q of the number of opponents faced by a buyer present at site a, and the second with respect to the binomial distribution with parameter π_a over information states at site a. Similarly, given (π_b, γ_b) and q, a bidder attending auction site b expects rents

$$\begin{aligned}\mathcal{R}_b(\pi_b, \gamma_b, q) &= \mathbf{E}_\eta \mathbf{E}_\tau R_b^{\eta,\tau} \\ &= (1-q)\Big[\pi_b R_b^{2,i} + (1-\pi_b) R_b^{2,u}\Big] \\ &\quad + q\Big[\pi_b R_b^{1,i} + (1-\pi_b) R_b^{1,u}\Big]. \end{aligned}$$

In the unique symmetric mixed strategy equilibrium of the buyers' subgame the level of traffic must satisfy

$$q \begin{cases} = 0 & \text{if } \mathcal{R}_a(\pi_a, \gamma_a, 1) \geq \mathcal{R}_b(\pi_b, \gamma_b, 0), \\ \in (0,1) & \text{if } \mathcal{R}_a(\pi_a, \gamma_a, q) = \mathcal{R}_b(\pi_b, \gamma_b, q), \\ = 1 & \text{if } \mathcal{R}_a(\pi_a, \gamma_a, 1) \leq \mathcal{R}_b(\pi_b, \gamma_b, 0). \end{cases} \tag{2}$$

In the second case, the equilibrium q is found by solving the equation $\mathcal{R}_a(\pi_a, \gamma_a, q) = \mathcal{R}_b(\pi_b, \gamma_b, q)$. To lessen notation, the equilibrium level of q generated by $(\pi_a, \gamma_a, \pi_b, \gamma_b)$ will simply be denoted by q, with its dependence on information provision and mechanisms understood. With the equilibrium in the buyers' subgame fixed, buyer behaviour is characterized by q, whose responses to information provision and mechanisms is given by (2). In the rest of the paper, an equilibrium refers to a subgame perfect equilibrium of the full game with buyer strategies given by q. Only sellers' equilibrium strategies are left to be determined.

Sellers' Profits: Profits of seller k, given $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$, can be expressed as surplus less rents as

$$\mathcal{P}_k(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k}) = \mathbf{E}_\eta \mathbf{E}_\tau \left[\mathcal{S}_k^{\eta,\tau} - \eta R_k^{\eta,\tau} \right]. \tag{3}$$

The first expectation is taken with respect to the binomial distribution with parameter q (if $k = a$) or $1 - q$ (if $k = b$) of demand at site k, and the second with respect to the binomial distribution with parameter π_k over information states at site k.

A Characterization of Incentive Compatible Mechanisms: Note that buyer sorting decisions, as expressed by (2), depend only on information provision and expected rents $R_k^{\eta,\tau}$. In particular, buyer decisions are not affected by how rents are shared between types conditional on being informed. This ex ante feature of rent promises allows a useful characterization of incentive-compatible mechanisms. Crucially, as Lemma 6 illustrates, we can restrict θ_H-type incentive-compatibility constraints to be binding in states $(1, i)$ and $(2, i)$. This is without loss of generality since any incentive-compatible mechanism at site k that achieves rents $\left\{R_k^{\eta,\tau}\right\}_{\substack{\eta \in \{1,2\} \\ \tau \in \{i,u\}}}$ with non-binding θ_H-type incentive constraints can be replaced by an incentive compatible mechanism that achieves the same levels of expected rents with the same allocations, but in which these constraints bind. Under this new mechanism, profits are unchanged and all traffic and information provision incentives are preserved. The intuition of the proof is simple: given an incentive compatible mechanism where $\text{IC}_k^{1,i}(\theta_H)$ does not bind, we can simply increase θ_L-type rents and decrease θ_H-type rents through transfers until the constraint binds, while ensuring that the expected rents in demand state $(1, i)$ are unchanged. The ratio between both shifts which ensures this, a function of the ex ante distribution of types, also keeps profits unchanged.

This result is useful in that binding θ_H-type incentive-compatibility constraints pin down θ_H-type rents as a function of the rents of θ_L-types and the information rents accruing to θ_H-types. The latter depend on the allocations to θ_L-types. Denote *low-type rents under mechanism* γ_k *in state* (η, τ) by $r_k^{\eta,\tau}$. These are the rents offered to θ_L-types in informed states and to the uninformed otherwise. Let $I_k^{\eta,i}$ be the informational rents to θ_H-types in state (η, i). Then we can rewrite the expected rents promised at site k as

$$R_k^{\eta,u} = r_k^{\eta,u} \text{ for } \eta \in \{1,2\},$$
$$R_k^{1,i} = r_k^{1,i} + I_k^{1,i},$$
$$R_k^{2,i} = r_k^{2,i} + I_k^{2,i}.$$

3 Fixed Mechanisms

In this section, sale mechanisms are exogenously fixed and common to both sale sites. Seller k's strategy consists solely of a choice of $\pi_k \in [0,1]$, the probability with which buyers are informed at site k. That sellers are exogenously held to specific terms of trade constrains the rent offers they can extend to buyers through their choice of information provision. Understanding how the characteristics of the sale procedure affect equilibrium information provision is the goal of this section. The focus is the sellers' trade-off between traffic and profit-per-buyer, which varies with sale mechanisms. In particular, we examine how symmetric equilibria in information provision respond to shifts in the sale mechanisms.

3.1 An Example: Second-Price Auctions

We start with an example in which sellers hold second-price auctions without reserve prices irrespective of the buyers visits. A model in which a monopolist chooses information provision while holding a second-price auction with two buyers has been studied in Board (2007) and Ganuza and Penalva (2006). Our example then constitutes a useful benchmark to gauge the effects of allowing sellers to compete through offers of information.

With second-price auctions, buyers obtain the good for free in the one-buyer state, and capture the full surplus $\bar{\theta}$. In the two-buyer state, to bid their best estimate of their true value is a weakly dominant strategy for buyers. When uninformed, this best estimate is $\bar{\theta}$.

A buyer that attends site a, given π_a and q, expects rents

$$\mathcal{R}_a(\pi_a, q) = q\pi_a p_H p_L(\theta_H - \theta_L) + (1-q)\bar{\theta},$$

while a bidder attending site b, given π_b and q, expects rents

$$\mathcal{R}_b(\pi_b, q) = (1-q)\pi_b p_H p_L(\theta_H - \theta_L) + q\bar{\theta}.$$

In the mixed strategy equilibrium of the buyers' subgame, the probability with which buyers visit site a, q_{sp}, is given by

$$q_{sp} = \frac{\bar{\theta} - \pi_b p_H p_L(\theta_H - \theta_L)}{\bar{\theta} - \pi_a p_H p_L(\theta_H - \theta_L) + \bar{\theta} - \pi_b p_H p_L(\theta_H - \theta_L)}. \tag{4}$$

The profits of seller a, given (π_a, π_b) and the resulting q_{sp}, are given by

$$\begin{aligned}\mathcal{P}_a(\pi_a, \pi_b) &= q_{sp}^2 \left[\pi_a \left(p_H^2 \theta_H + (1 - p_H^2)\theta_L\right) + (1 - \pi_a)\bar{\theta}\right] \\ &= q_{sp}^2 \left[\bar{\theta} - \pi_a p_H p_L (\theta_H - \theta_L)\right]. \end{aligned} \tag{5}$$

The term in the brackets of (5) is the expected price paid by the buyer who obtains the good in the two-buyer state. This price decreases in π_a, since the seller then gives away a higher share of the surplus as informational rents. Denote this price by $w_a(\pi_a)$. Suppose a single second-price auctioneer faced a fixed set of two buyers, and can propose information structures as defined in our model and indexed by π. His profits given information provision π would be $w(\pi)$. In this case, we immediately obtain the following result.

Proposition 1. (No Information Under Monopoly With Second-Price Auction) *A second-price auctioneer with no reserve price facing two buyers maximises profits by setting* $\pi = 0$.

This result is known from Board (2006) and Ganuza and Penalva (2006). With this benchmark in hand, we return to our model. Note that we can rewrite (4) as

$$q_{sp} = \frac{w_b(\pi_b)}{w_a(\pi_a) + w_b(\pi_b)}. \tag{6}$$

Since buyers get all the surplus if alone, q_{sp} depends only on how much profits sellers get from demand states with two buyers. Thus (5) becomes

$$\begin{aligned}\mathcal{P}_a(\pi_a, \pi_b) &= \left[\frac{w_b(\pi_b)}{w_a(\pi_a) + w_b(\pi_b)}\right]^2 w_a(\pi_a) \\ &= w_b(\pi_b) \left[\frac{w_b(\pi_b)}{w_a(\pi_a) + w_b(\pi_b)} \cdot \frac{w_a(\pi_a)}{w_a(\pi_a) + w_b(\pi_b)}\right] \\ &= w_b(\pi_b) q_{sp}(1 - q_{sp}). \end{aligned} \tag{7}$$

Clearly, seller a's choice of information influences profits in (7) only through its effect on $q_{sp}(1 - q_{sp})$. This term reaches a maximum when $q = \frac{1}{2}$. Seller a can attain this maximum by setting $\pi_a = \pi_b$. This leads to the following result.

Proposition 2. (Continuum of Symmetric Equilibria with Competition and SPA) *When the sale mechanism is a second-price auction with no reserve price,* (π_a, π_b) *is an equilibrium if and only if* $\pi_a = \pi_b$.

This surprising result states that a seller's best-response to any information promise by an opponent is to match that promise. Any seller that induces asymmetry in market conditions will lose profits. This result differs sharply from that for a monopolist with fixed demand.

If we rewrite the rents of a buyer attending site a, we get

$$\mathcal{R}_a(\pi_a, q) = \bar{\theta} - q w_a(\pi_a). \tag{8}$$

That is, it is as though seller a gives an entering buyer a 'fixed fee' $\bar{\theta}$, but imposes a 'congestion charge' of $w_a(\pi_a)$ when the other buyer is also present. Rents can be rewritten in this particular form only when the sale mechanism is a second-price auction with no reserve price. Say buyers' true valuations were instead given by some continuous random variable Y with mean $\bar{\theta}$. Denote by $Y_{1:2}$ and $Y_{2:2}$ the expected values of the first and second best draws out of two from the distribution of Y. Then we have that $Y_{1:2} + Y_{2:2} = 2\bar{\theta}$. Rewriting rents as in (8) uses the discrete version of this identity. This in turn allows the representation of profits in (7). This shows that the result of Proposition 2 is not due to our model's linear information framework, but that it does depend critically on there being only two buyers and two sellers.[9] Indeed, the continuum of symmetric equilibria identified by Proposition 2 arise in the two buyer, two-seller case with continuous distributions of types and information structures defined as in Ganuza and Penalva (2006). In that case, if we assume that information choice π_k orders the ex post type distributions of buyers by dispersion but leaves the mean unaffected, the expected price paid by the winning buyer is the expected value of the second of two draws from the distribution indexed by π_k, which is decreasing in π_k. Buyer decisions as expressed by (6) and seller profits in (7) are unaffected and the suitable version of Lemma 2 follows.

3.2 Equilibrium Comparative Statics

The case of second price auctions, while special, demonstrates that the set of equilibria in information provision for any exogenous incentive compatible mechanism will be difficult do deal with in general. To study how changes in the constraints on rent offers represented by exogenous sale mechanisms affect information provision, we restrict attention to a particular class of mechanisms, which we call *regular*. Regularity restricts the allocations of mechanisms. To this end the following definition is needed.

Definition 1. (No Waste) *A mechanism $\gamma_k \in \Gamma$ has no waste if and only if the good is always delivered to some buyer.*

A mechanism has no waste if no type is excluded from trade in the one-buyer or uninformed states and if it is always the case that the good is allocated to some buyer in the two-buyer state. In a mechanism with no waste, the full surplus ($\bar{\theta}$) is realized in the

[9] The result of Proposition 2 also depends critically on the other assumptions of the model, for example that information provision is costless. Say providing information required a cost of c. Then seller a's profits would be given by (7), less some cost term that depends on q and c. Thus at symmetric profiles marginal profits are negative, so that the only symmetric equilibrium has no information provision.

one-buyer and uninformed states, while in state $(2, i)$ the full surplus is realized only when a θ_L-type never obtains the good when a θ_H-type is present. We can now define regular mechanisms.

Definition 2. (Regular Mechanisms)
An incentive compatible mechanism γ is regular if and only if

i. (Exploiting the uninformed) $R^{1,u} = R^{2,u} = 0$.

ii. (Congestion effects) $R^{1,i} > R^{2,i}$.

iii. γ has no waste.

Property *i* states that in uninformed states a regular mechanism fully exploits the buyers' lack of information. As noted above, sellers benefit from restrictions in buyers' information through an easing of incentive constraints. The assumption that buyers get no rents when uninformed carries this idea to the limit. Property *ii* states that regular mechanisms have *congestion effects*. That is, a buyer strictly prefers being alone at a selling site to being accompanied by the other buyer. Common mechanisms such as auctions and posted prices have this property, which is intuitive given the presence of a single good at each site. Finally, that regular mechanisms have *no waste* is a sufficient condition for expected (over information states) surplus in the two-buyer state to be increasing in information provision. That is, no waste implies that $\mathcal{S}^{2.i} \geq \bar{\theta}$. Total available surplus in the two-buyer state always increases in information provision, yet the sale mechanism's allocation rules may sufficiently restrict delivery of the good in informed states that realized surplus decreases in information provision. In the one-buyer state, no waste implies that the surplus is $\bar{\theta}$, independently of information provision.

Regular mechanisms combine the properties that make the study of ex ante competition through information provision interesting: sellers extract more rents from poorly informed buyers; buyers, who compete for goods, dislike the presence of other buyers; and information provision does not solely redistribute rents, but enhances total surplus. In informed states, standard mechanisms that always deliver the good to some buyer, such as auctions with reserve prices lower than θ_L, or a uniform price (independent of demand state) less than θ_L, can be components of regular mechanisms when combined with take-it-or-leave-it offers of $\bar{\theta}$ in uninformed states.

Under a regular mechanism γ, seller a's profits are

$$\mathcal{P}_a(\pi_a, \gamma, \pi_b, \gamma) = q^2 \left[\pi_a \mathcal{S}^{2,i} + (1-\pi_a)\bar{\theta} - 2\pi_a R^{2,i}\right] + 2q(1-q)\left[\bar{\theta} - \pi_a R^{1,i}\right]. \tag{9}$$

Seller a's profits in the one-buyer state, $\bar{\theta} - \pi_a R^{1,i}$, are clearly decreasing in π_a. That is, under regular mechanisms, sellers that increase information provision give out more rents to buyers in the one-buyer state. Furthermore, as shown in Appendix XXX, that γ has no waste implies that seller a's profits in the two-buyer state, $\pi_a \mathcal{S}^{2,i} + (1-\pi_a)\bar{\theta} - 2\pi_a R^{2,i}$, are also decreasing in π_a.

At symmetric profiles, the market is shared equally between the two sellers. In particular, an equally split market maximises the probability that a seller is visited by a single buyer ($2q(1-q)$), which means that marginal shifts in information provision at symmetric profiles have no effect on this probability.[10] This simplifies the expression for marginal profits at symmetric profiles under regular mechanism γ, which is given by

$$\begin{aligned}\left.\frac{\partial \mathcal{P}_a(\pi_a, \gamma, \pi_b, \gamma)}{\partial \pi_a}\right|_{\pi_a=\pi_b=\pi} = \left.\frac{\partial q}{\partial \pi_a}\right|_{\pi_a=\pi_b=\pi} &\left[\pi \mathcal{S}^{2,i} + (1-\pi)\bar{\theta} - 2\pi R^{2,i}\right] \\ &+ \frac{1}{4}\left[\mathcal{S}^{2,i} - \bar{\theta} - 2R^{2,i}\right] - \frac{1}{2}R^{1,i}. \end{aligned} \tag{10}$$

The first term of (10) is the *increased traffic effect* of an increase in information provision. Traffic to site a increases slightly and seller a gains two-buyer state profits more often. The two last terms are the *decreased profit-per-head effect.* Seller a now hands over more rents to all visiting buyers in each state. The lefthand side of (10) can cross 0 at most once, which implies that for regular mechanisms a unique candidate profile for symmetric equilibrium can be identified.

Lemma 1. (Unique Candidate for Symmetric Equilibrium)
In games with regular mechanisms, there is a unique candidate profile for symmetric equilibrium in information provision, given by

$$\pi^* \equiv \begin{cases} \frac{-(R^{1,i}+R^{2,i})\bar{\theta}}{2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))} & \textit{if } \bar{\theta} < 2R^{1,i} \textit{ and } R^{1,i}+R^{2,i} < -\frac{2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta})}{\bar{\theta}-2R^{1,i}}, \\ 1 & \textit{otherwise.} \end{cases} \tag{11}$$

(DO I DEAL WITH WHY PI*=0 NEVER HAPPENS) Lemma 1 depends on the fact that the decreased profit-per-head effect is negative and does not depend on π_a by the linearity of the information structures. Also, we show in Appendix XXX that $\frac{\partial q}{\partial \pi_a}\big|_{\pi_a=\pi_b=\pi}$ is decreasing in π,[11] the symmetric level of information provision. This, along with (17), implies that the increased traffic effect, though positive, is decreasing in π. That is, buyers are less sensitive to information provision when in a high-information environment, and also in such environments the profits generated by more frequent buyer visits are lower.

[10] This observation will often be useful in the the rest of the paper. Its interpretation is not entirely bound to the two-buyer, two-seller setup, but is rather due to the binomial distribution of demand at sale sites. That is, if $X \sim B(n,q)$ then $\frac{\partial \Pr(X=k)}{\partial q} > 0$ whenever $k > qn$, where qn is the mean state of X. If qn is an integer, then $\frac{\partial \Pr(X=qn)}{\partial q} = 0$. That is, if q is marginally increased, states above the mean state become more likely and states below the mean less likely, while the probability of the mean state is unchanged.

[11] The analysis here differs from the second-price auction example of Section 3.1 since in that case the conclusion that $\frac{\partial q}{\partial \pi_a}\big|_{\pi_a=\pi_b=\pi}$ decreases in π fails. This condition depends on information provision affecting rents in the one-buyer state, which is not true in a second-price auction with no reserve price.

Since the profit function in (9) is not concave in π_a, (10) alone is not sufficient to establish the existence of symmetric equilibria. In fact, even in such a simple model, the behaviour of (9) in π_a is complex. In Appendix XXX, we present conditions on mechanisms' rents that are sufficient for seller a's profit function to be single-peaked around $\pi_a = \pi^*$ when $\pi_b = \pi^*$ and $\pi^* < 1$.[12] In the same way, it is possible to derive sufficient conditions for the existence of full-information equilibria when $\pi^* = 1$. We do not present these the conditions, which by themselves are not intuitive. Furthermore, we focus on interior symmetric equilibria in order to derive comparative statics results that describe how marginal movements within the space of regular mechanisms affect equilibrium levels of information provision and seller profits.

We want to analyse the comparative statics of symmetric equilibria for those games with mechanisms that we can guarantee lead to symmetric equilibria with $\pi^* < 1$, and for which derivatives like $\frac{\partial \pi^*}{\partial R^{\eta,i}}$ for $\eta \in \{1, 2\}$ are defined. We consider shifts in rents that leave expected surplus unchanged. Such shifts could be implemented through changes in transfers, without affecting allocations. The next result shows that increases in rents in either state always have a more important impact on the decreased profit-per-head effect than on the increased traffic effect, thus leading to lower equilibrium information provision.

Proposition 3. (Equilibrium Information Provision is Decreasing in Rents)
For a regular mechanism γ, suppose

i. The information provision game between sellers with mechanism γ has a symmetric equilibrium (π^*, π^*) with $\pi^* < 1$.

ii. There exists a neighbourhood N of γ such that any $\hat{\gamma} \in N$ is a regular mechanism that induces a symmetric equilibrium in information provision $(\hat{\pi}^*, \hat{\pi}^*)$ with $\hat{\pi}^* < 1$.

Then the symmetric equilibrium information provision π^* has

$$\frac{\partial \pi^*}{\partial R^{1,i}} < \frac{\partial \pi^*}{\partial R^{2,i}} \leq 0,$$

with $\frac{\partial \pi^*}{\partial R^{2,i}} = 0$ if and only if $\mathcal{S}^{2,i} = \bar{\theta}$.

Mechanisms that are more generous to buyers lead to lower equilibrium information provision, since higher rents dampen the competition between sellers by increasing the cost of attracting more buyers. However, the drop in equilibrium information provision is more pronounced when rents in the one-buyer state, rather than in the two-buyer state, are increased. To explain this asymmetry, rewrite buyer rents from attending site a in (1) as

$$\mathcal{R}_a(\pi_a, \gamma, q) = \pi_a \left[R^{1,i} - q(R^{2,i} - R^{1,i}) \right]. \tag{12}$$

[12] See the proof of Proposition 3.

That is, when attending site a and conditional on being informed, it is as though a buyer is paid a 'fixed amount' $R^{1,i}$, while he suffers a 'congestion charge' of $(R^{2,i} - R^{1,i})$ whenever the other buyer is also present. An increase in $R^{1,i}$ affects buyer rents to attending site a in two ways; both the fixed payment and the congestion charge increase. The second effect reduces buyers' incentives to visit a deviating seller with higher probability, as this increases their chance of meeting at the same site. This buyer inertia softens the competition between sellers. On the other hand, an increase in $R^{2,i}$ reduces the congestion charge suffered by a buyer at site a. By making buyers less averse to meeting their opponents at a site, this increases sellers' incentives to deviate from symmetric profiles and hence intensifies competition between them.

As surplus in the two-buyer state approaches surplus in the one-buyer state (i.e., $\mathcal{S}^{2,i} \to \bar{\theta}$), $\frac{\partial \pi^*}{\partial R^{2,i}}$ approaches 0. As seen from (10) a decrease in $\mathcal{S}^{2,i} - \bar{\theta}$ both softens the increased traffic effect and intensifies the decreased profit-per-head effect, this may seem paradoxical. But because of these two effects, a reduction in $\mathcal{S}^{2,i} - \bar{\theta}$ leads to a drop in π^*, which in turn favours the increased profit-per-head effect. The result that $\frac{\partial \pi^*}{\partial R^{2,i}}$ increases to 0 as $\mathcal{S}^{2,i} - \bar{\theta}$ decreases to 0 states that the indirect encouragement of a drop in $\mathcal{S}^{2,i} - \bar{\theta}$ to the increased profit-per-head outweighs its direct weakening.

We now examine the effect on equilibrium information provision of increasing the efficiency of sellers' sale mechanisms. These changes can be implemented by changing mechanisms' allocations and compensating buyers through transfers to keep rents unaffected.

Proposition 4. (Equilibrium Information Provision is Increasing in Surplus) *For a regular mechanism γ, suppose conditions i and ii of Proposition 3 hold. Then*

$$\frac{\partial \pi^*}{\partial \mathcal{S}^{2,i}} > 0.$$

Higher surplus in the two-buyer state increases the gains to information provision. This leads to inreased competition between sellers, and higher equilibrium information provision. This reveals a complimentarity between mechanism efficiency and equilibrium information provision; more efficient mechanisms generate more efficient informational environments.

Turning to the behaviour of equilibrium profits, we can identify the two equilibrium effects of an increase in the rents offered to buyers. First, by Proposition 3, the equilibrium level of information provision decreases, and hence sellers extract more profits from buyers given a fixed level of rents. However, buyer rents increase for every level of information provision. Furthermore, the magnitude of these effects depends on whether the rents in the one-buyer or two-buyer demand states are shifted. The next result partially determines the total effect of shifts in rents on seller profits.

Proposition 5. (Responses of Equilibrium Profits to Rents)

For a regular mechanism γ, suppose conditions i and ii of Proposition 3 hold. Then

$$\frac{\partial \mathcal{P}_a(\pi^*,\gamma,\pi^*,\gamma)}{\partial R^{1,i}} > 0$$
$$\text{and} \quad \frac{\partial \mathcal{P}_a(\pi^*,\gamma,\pi^*,\gamma)}{\partial R^{2,i}} \geq (<)\ 0 \ \text{if} \ \ R^{1,i}+R^{2,i} \leq (>) \frac{\mathcal{S}^{2,i}-\bar{\theta}}{\sqrt{2}}.$$

According to Proposition 5, when rents in the one-buyer state increase, the drop in the equilibrium level of information provision raises profits enough to compensate for the rent increase, while this is not always the case for increases in rents in the two-buyer state. For example, note that under any mechanism in which $\mathcal{S}^{2,i}=\bar{\theta}$, we have that $\frac{\partial \mathcal{P}_a(\pi^*,\gamma,\pi^*,\gamma)}{\partial R^{2,i}} < 0$.

It should be noted that since changes in rents are achieved through transfers, the different mechanisms considered in these results are equally efficient with respect to informed buyers' allocations. However, according to Proposition 5, the mechanisms preferred by sellers may foster highly inefficient outcomes since they often lead to low levels of information provision.

As illustrated by Proposition 4, an increase in mechanism efficiency in state $(2,i)$ intensifies the competition between sellers and increases equilibrium information provision, hence decreasing profits. However, the increased surplus in state $(2,i)$ is captured by sellers. The following result shows that the overall effect of the increased allocative efficiency on seller's equilibrium profits is negative.

Proposition 6. (Equilibrium Profits are Decreasing in Surplus)
For a regular mechanism γ, suppose conditions i and ii of Proposition 3 hold. Then

$$\frac{\partial \mathcal{P}_a(\pi^*,\gamma,\pi^*,\gamma)}{\partial \mathcal{S}^{2,i}} < 0.$$

The reason for this is that increased profits affect only state $(2,i)$, while increased competition reduces profits in all states.

DISCUSS RELATIONSHIP WITH DIRECTED SEARCH COMPARATIVE STATIC; INFO AS A PRICE

3.3 Examples

3.3.1 Pricing Mechanisms

MUST CHECK PRICE POSTING WITH UNIFORM PRICE, AND JUSTIFY LOOKING AT DEMAND CONTINGENT PRICES.

We close this section with some illustrations of the preceding results. We first consider *pricing mechanisms*, where $t^{\eta,\tau}$ is the price charged by the sellers in state (η,τ). When two buyers are present at the same site and both their values exceed the relevant price, each obtains the good with equal probability. Such a pricing mechanism is regular if *i. (Exploiting the uninformed)* $t^{\eta,u} = \bar{\theta}$ *for* $\eta \in \{1,2\}$.

ii. (Congestion effects) $\bar{\theta} - t^{1,i} > \frac{1}{2}\left(\bar{\theta} - t^{2,i}\right)$.

iii. (No waste) $t^{1,i}, t^{2,i} \leq \theta_L$.

Thus, by Lemma 1, any information-provision game with pricing mechanisms respecting i, ii and iii has a unique candidate π^* as the level of information provision in a symmetric equilibrium. Furthermore, if $\pi^* < 1$, this candidate profile is indeed a symmetric equilibrium.[13] From (11), we have that $\pi^* < 1$ if

$$t^{1,i} < \frac{\bar{\theta}}{2}. \tag{13}$$

The second condition of (11) is always satisfied for pricing mechanisms since $\mathcal{S}^{2,i} = \bar{\theta}$. Thus, for $t^{1,i}$ and $t^{2,i}$ satisfying ii, iii and (13) the level of information provision in symmetric equilibrium is given by

$$\pi^* = \frac{\bar{\theta}}{2(\bar{\theta} - t^{1,i})} < 1,$$

which does not depend on $t^{2,i}$ and is increasing in $t^{1,i}$ (decreasing in $R^{1,i}$). Applying Proposition 5 to this example, we have that

$$\begin{aligned} &\frac{\partial \mathcal{P}_a(\pi^*, \gamma, \pi^*, \gamma)}{\partial t^{1,i}} > 0 \\ \text{and} \quad &\frac{\partial \mathcal{P}_a(\pi^*, \gamma, \pi^*, \gamma)}{\partial t^{2,i}} < 0. \end{aligned}$$

That is, a seller's preferred mechanism has as high a $t^{2,i}$ and as low a $t^{1,i}$ as possible while still respecting ii, iii and (13). This happens when $t^{1,i} = 0$ and $t^{2,i} = \theta_L$. In this pricing mechanism, sellers give away the good when one buyer is present but charge the highest price that leads to no exclusions in the two-buyer state. Equilibrium information provision is $\pi^* = \frac{1}{2}$. The sellers' favoured pricing mechanism has low information provision and makes buyers very averse to meeting at a site by providing large rents to a buyer who is alone.

Different regular pricing mechanisms are equally efficient from the point of view of informed allocations. Yet the mechanisms most preferred by sellers is informationally the least efficient in the class of regular pricing mechanisms.

3.3.2 Ex Post Optimal Mechanisms

We can use the results of this section to study the situation in which sellers commit to levels of information provision but cannot commit to sale mechanisms. In that case, once buyers

[13] This is by (19) in the Appendix.

have chosen sale sites, sellers deliver their good through the *ex post optimal mechanisms*. When buyers are uninformed, sellers optimally make take-it-or-leave-it offers of $\bar{\theta}$. When buyers are informed, the optimal mechanisms for both the one and two-buyer states depend on whether or not sellers prefer to exclude θ_L-types and sell only to θ_H-types. For both demand states, a seller strictly prefers to sell to θ_L-types whenever $\theta_L > p_H\theta_H$. When θ_L-types are excluded, sellers extract all informational rents from θ_H-types. In that case, buyers expect no rents from any demand state regardless of the level of information provision. The interesting case is when $\theta_L > p_H\theta_H$ and informed θ_H-types obtain rents. This condition for the non-exclusion of θ_L-types will also be useful later, so we state it here as an assumption.

Assumption 1. (No Exclusions Under Ex Post Optimal Mechanisms)
$\theta_L > p_H\theta_H$.

Note also that under Assumption 1, the ex post optimal mechanisms are regular and can be described by rent levels for low types $r^{\eta,\tau} = 0$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$ and informational rents $I_k^{1,i} = p_H(\theta_H - \theta_L)$ and $I_k^{2,i} = \frac{1}{2}p_L p_H(\theta_H - \theta_L)$.

By Lemma 1, there is a unique candidate π^* for symmetric equilibrium and since under ex post optimal mechanisms $R^{1,i} = p_H(\theta_H - \theta_L)$, we have that

$$\begin{aligned}
2R^{1,i} - \bar{\theta} &= p_H\theta_H - p_L\theta_L - 2p_H\theta_L \\
&= p_H\theta_H + p_L\theta_L - 2\theta_L \\
&< \theta_L(p_L - 1) \\
&< 0,
\end{aligned}$$

CHECK CONDITIONS WITH CORRECTIONS **** where the first inequality follows from $\theta_L > p_H\theta_H$. Thus, by (11), under optimal sale mechanisms, the only candidate information component of a symmetric equilibrium is $\pi = 1$. Further, it can be shown that $\pi = 1$ is indeed a symmetric equilibrium.

Proposition 7. (Full Information With No Commitment to Mechanisms)
Under Assumption 1 and ex post optimal mechanisms, the only symmetric equilibrium has full information provision.

To show that full information provision is indeed a symmetric equilibrium, we show that seller a's profits are increasing in π_a when $\pi_b = 1$. When buyers face the optimal mechanisms once sorted, expected rents are low. This increases the sensitivity of their sorting decisions to shifts in information provision and enhances sellers' traffic-stealing incentives. With optimal mechanisms sellers achieve their favoured ex post outcome, yet competition leads them to make their most costly ex ante information commitments.

4 Endogenous Mechanisms

The previous section showed that sale mechanisms interact strongly with the competitive incentives to provide information. However, it may not be desirable to assume that sellers can compete through information structures but cannot competitively alter the terms of sale at their sites. This section allows sellers to commit to any incentive compatible mechanism, and explores a set of equilibria in which sellers provide full information and compete over the direct rents offered to buyers through transfers. In these equilibria, sellers disentangle information and rent provision even in the presence of competition. Sellers' interest in doing so stems from the fact that while providing rents is redistributive, providing information is efficiency-enhancing.

In this section, seller k's strategy is a pair $(\pi_k, \gamma_k) \in [0,1] \times \tilde{\Gamma}$. Given the characterization of incentive-compatible mechanisms in Lemma 6, sellers in effect choose monotone allocation probabilities and θ_L-type and uninformed rents, along with information provision.

We will start by stating the main result of this section. The rest of the section will explain the steps used to establish the result through a series of lemmas. Section XXX then shows how these results establish the existence of the class of equilibria of interest. We first present the following definitions, which relate to the properties of the allocations of sale mechanisms.

Definition 3. (Partial and Full Allocative Efficiency)
A mechanism $\gamma_k \in \Gamma$ has partial allocative efficiency (PAE) if and only if the good is always sold to some buyer in uninformed states, and to a θ_H-type in informed states if such a type is present. A mechanism $\gamma_k \in \Gamma$ has full allocative efficiency (FAE) if and only if it has partial allocative efficiency and the good is always sold to a θ_L-type in informed states if no θ_H-type is present.

To relate this to our earlier definitions, any mechanism with FAE has no waste, but a mechanism with no waste may allocate the good to a θ_L-buyer in the presence of a θ_H-buyer in state $(2,i)$. Under FAE, the surplus in state $(2,i)$ is maximized, and we denote it by $\bar{S}^{2,i}$. A mechanism with PAE always allocates the good to θ_H-types and uninformed buyers, while it may exclude θ_L-types. An example would be a price of θ_H in informed states along with a price of $\bar{\theta}$ in uninformed states.

The following result provides a characterization of the symmetric equilibria with endogenous mechanisms under Assumption 1.

Proposition 8. (Symmetric Equilibrium)
Under Assumption 1, $(\pi, \gamma, \pi, \gamma) \in ([0,1] \times \Gamma)^2$ is a symmetric equilibrium if and only if $\pi = 1$, γ has full allocative efficiency, $R^{2,i} \leq R^{1,i}$ and $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$.

A series of lemmas establishes this result, which can be organized along three themes; information provision, efficiency and rent levels.

4.1 Equilibrium Information Provision

This section derives necessary conditions for full information provision in equilibrium. Intuitively, as information increases the potential size of the 'pie', it allows Pareto-improving deviations for sellers. Information provision also has a distributive effect through rents as it shifts probability among information states within and across demand states. However, since sellers promise both state-contingent rents and information ex ante, they can increase information provision and offset the effect this has on buyer rents through transfers. In this way, buyer behaviour is unaffected and sellers pocket the newly generated surplus. The one proviso to the above argument is that the mechanisms must be such that more information actually increases the surplus expected over demand states at site k, $\mathbf{E}_\eta \mathbf{E}_\tau \mathcal{S}_k^{\eta,\tau}$. If some buyer types are excluded by the mechanism, this need not be the case. However, in this case, reduced information provision will generate efficiency gains that the seller can capture through transfers.

Lemma 2. (Full or No Information In Equilibrium)
Suppose that $(\pi_a, \gamma_a, \pi_b, \gamma_b) \in \left([0,1] \times \tilde{\Gamma}\right)^2$ *is an equilibrium, that* $\mathbf{E}_\eta \mathbf{E}_\tau \mathcal{S}_a^{\eta,\tau}$ *is increasing (decreasing) in* π_a*, and that it is not the case that* γ_a *and* γ_b *are the ex post optimal mechanisms. Then* $\pi_a = 1$ *(*$\pi_a = 0$*).*

The proof shows that given an equilibrium with $\pi_a < 1$, unless it is the case that $r^{2,i} = r^{2,u} = r^{1,i} = r^{1,u} = 0$, seller a can always increase information provision and adjust transfers so as to keep buyer rents constant. Allocations are unchanged and so, by assumption, higher surplus is generated. Since rents and traffic are unaffected, seller profits increase (as seen from (16)), contradicting the fact that the original profile is an equilibrium. Lemma 2 will be useful later, as it ensures that a large class of mechanisms must be paired with full information in any equilibrium. Checking whether a mechanism belongs to that class relies most importantly on its allocation rules.

Full-information symmetric equilibria could arise for some of the exogenous mechanisms of Section 3. However, as sellers could not commit to sale mechanisms, the rationale for their existence was quite different. There, increasing information provision was profitable only if the increase in traffic generated compensated the seller for the higher rents now offered to buyers. It was shown that the shape of this traffic-rents trade-off depends on the sale mechanisms, and that full-information equilibria exist when rents are such that incentives for traffic-stealing are high. With ex ante promises of mechanisms, Lemma 2 shows that a seller can deviate to a full information profile without concerning himself with traffic effects, since he directly controls state-contingent rents. He can use this control to trade off information against rents and capture the efficiency benefits of information. Although this commonality of interest between sellers and buyers with respect to information also exists when mechanisms are fixed, sellers lack the tools to exploit it.

The intuition that sellers can exploit efficiency gains through ex ante offers is very general. In the literature on information provision by monopolists, Esö and Szentez (2006) have proved a result similar to ours in spirit. There are nevertheless significant differences between their paper and ours. They study a general monopolistic auction design problem in which buyers have ex ante private information and the seller controls access to 'new' private information about buyers' values. The seller proposes mechanisms in which buyers first report their ex ante private types, then get access to the new information and finally report this back to the seller. When the seller controls the release of signals but does not observe their realizations, Esö and Szentez show that he can achieve the same allocation and profits as under the optimal mechanism in the case in which he can directly observe the signals. This is implemented by suitably controlling the ex ante type-dependent transfers made by the buyers to the seller before they get access to the new information. Because of the competition between the sellers, our model lacks a benchmark optimal mechanism in the case of observable signals, whose allocation and rent levels sellers would attempt to mimic in the case where they control access to unobservable signals. Given any symmetric profile with $\pi \leq 1$, we need to check whether a seller has a profitable deviation that involves an increase in information provision. For any profile in which expected surplus is increasing in information provision, our result provides such a deviation. In Esö and Szentez's model, efficiency considerations are not mentioned explicitly since the seller is only trying to match the surplus level of the benchmark optimal mechanism in the observable-signals case. More importantly, in our model sellers cannot ask buyers to make any transfers before information is released.

4.2 Euilibrium Allocations

This section presents results that restrict equilibrium allocations in the game with endogenous mechanisms. The first result shows that, irrespective of an opponent's strategy, a seller never leaves a good unsold in the presence of θ_H or uninformed types. No such strong result exists for θ_L-type allocations; as in the monopoly case, sellers may sometimes find it optimal to exclude θ_L-types. The section's second result, however, shows that Assumption 1, which ensures that monopolists never exclude θ_L-types in either demand state, is sufficient to guarantee that θ_L-types are not excluded in all symmetric equilibria of the competitive game.

Lemma 3. (No Exclusions of θ_H or Uninformed Types)
A strategy $(\pi_k, \gamma_k) \in [0,1] \times \Gamma$ for seller k in which γ_k does not have full allocative efficiency is strictly dominated.

More specifically, for any profile in which seller k posts a mechanism that does not have PAE, we can find a alternative mechanism with PAE that leaves traffic unchanged and yields strictly higher profits to seller k. This result states that not only will equilibrium mechanisms have PAE, but that it is without loss when searching for equilibria to consider deviations from candidate profiles that have PAE.

The proof deals with θ_H-type and uninformed allocations separately, and mirrors analogous results in the monopoly framework. Using the characterization of mechanisms in $\tilde{\Gamma}$ of Lemma 7, it shows that profits can be increased and θ_L-types made less willing to mimic θ_H-types if seller k increases θ_H-type allocations and transfers simultaneously, keeping θ_H-types at the same level of rents.[14] Similarly, a profile in which uninformed buyers are excluded with positive probability is vulnerable to a deviation where a seller increases both probabilities and transfers, keeping buyers at the same level of rents.

Unlike the case of θ_H-type and uninformed buyers, the optimality of allocative efficiency with respect to θ_L-types is not guaranteed. In our model, the classic arguments determining θ_L-type allocations are further complicated by their competitive effects on traffic across sale sites. As noted above, in the monopoly case, Assumption 1 determines whether the seller excludes θ_L-types in either demand state. This is shown by noting that when Assumption 1 holds and θ_L-types are excluded with some probability, the seller can increase profits by increasing both θ_L-types' allocation probabilities and transfers, keeping their rent level constant, while still increasing θ_H-type rents (through the binding incentive compatibility constraint for θ_H-types). This increases rents expected over informed types. The problem with this argument in our framework is that in general, an increase in rents in any state increases traffic but may decrease the likelihood of the one-buyer state (when $q > \frac{1}{2}$). So even if a seller has a deviation that increases both profits and rents in some state, the effect on total profits may depend on the relation between profits in the one-buyer and two-buyer states.

The next result, unlike Lemma 3, presents only a necessary condition on θ_L-type allocations for symmetric equilibrium. Furthermore, it relies on Assumption 1.

Lemma 4. (No Exclusion of θ_L-types in Symmetric Equilibrium)
Under Assumption 1, if $(\pi, \gamma, \pi, \gamma) \in ([0, 1] \times \Gamma)^2$ is a symmetric equilibrium, then γ has full allocative efficiency.

We know from Lemma 3 that PAE is necessary for equilibrium, so what needs to be shown is that in a symmetric equilibrium θ_L-types always receive the good in the absence of θ_H-types. The proof applies the argument for the monopoly case outlined above to find a deviation from any symmetric equilibrium that violates FAE. The difficulty mentioned

[14]In the two-buyer state, it may be the case that θ_L-types receive the good even in the presence of θ_H-types, and that the ressource constraint binds, so that the seller cannot allocate the good more often to θ_H-types without allocating it less often to θ_L-types. But then the seller can simply 'free up' allocation probabilities by delivering the good less often to θ_L-types and keep their rents constant by decreasing their transfers. Such modification to θ_L-type contracts make θ_H-type buyers less willing to mimic θ_L-types as they have higher valuations for allocation probabilities.

above is dealt with by the fact that at a symmetric profile small increases in traffic have a negligible effect on the probability of the one-buyer state. A marginal deviation in either demand state of the type described above raises seller profits unambiguously as traffic in the two-buyer state always increases in q.

Without Assumption 1, a seller wants to exclude θ_L-types to depress θ_H-type rents. Marginally, whether this is profitable depends on whether the increased profits from θ_H-types compensate the drop in traffic in the two-buyer state. This traffic-rents trade-off will also involve the level of information provision. Without Assumption 1, it is then not possible to derive a simple necessary condition on θ_L-type allocations which, as above, does not depend on information provision.

4.3 Equilibrium Rents

Lemma 5. (Equilibrium Rents)
Any symmetric equilibrium with ????????? has $R^{2,i} \leq R^{1,i}$ and $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$.

Under the regular mechanisms of Section 3, buyers faced congestion effects and preferred being alone at an auction site when informed. When sellers commit to mechanisms, they may or may not induce congestion effects. Proposition 8 confirms that a seller will always impose congestion effects in a symmetric equilibrium. The intuition for this is as follows. As in (12), we can rewrite a buyer's expected rents at site a from a symmetric profile with $\pi = 1$ as

$$R^{1,i} + q(R^{2,i} - R^{1,i}), \tag{14}$$

that is, as a 'fixed fee' of $R^{1,i}$ along with a 'bonus' ('congestion charge') of $R^{2,i} - R^{1,i}$ when another buyer attends and $R^{2,i} > R^{1,i}$ ($R^{2,i} \leq R^{1,i}$). If $R^{2,i} > R^{1,i}$, decreasing $R^{2,i}$ lowers the bonus, but buyers remain indifferent between attending sites a and b only if this bonus is handed out more often, i.e., if q increases. As sellers can decrease rents while increasing traffic, profiles with $R^{2,i} > R^{1,i}$ admit a profitable deviation.

The condition $\frac{\bar{S}^{2,i}}{2} = R^{1,i}$ states that the marginal buyer[15] attending a site is awarded his marginal contribution to site surplus. To see this, note that seller a's profits at symmetric profiles with FAE are marginally increasing in $R^{1,i}$ (or $R^{2,i}$) whenever $\frac{\bar{S}^{2,i}}{2} > R^{1,i}$.[16] A marginal buyer drawn to site a by a marginal change in rents receives $R^{1,i}$, his 'fixed fee', from seller a. On the other hand, this marginal buyer brings his share of the surplus when another buyer is also present, $\frac{\bar{S}^{2,i}}{2}$, to site a. Since the probability of the one-buyer state is unaffected by small changes in q at a symmetric profile, a marginal buyer brings nothing to that state. A seller will want to attract a marginal buyer whenever his contribution exceeds

[15]This is how we interpret the mass involved in a marginal increase in q.
[16]This can be seen from (26) in the Appendix.

the cost of luring him. Similarly, if $\frac{\bar{S}^{2,i}}{2} < R^{1,i}$, a seller can gain by shedding a marginal buyer through a decrease in rents.

4.4 Symmetric Equilibria

The proof of Proposition XXX follows from the results of the previous sections. The necessity of FAE for symmetric equilibrium has been established in Lemma 4. Under FAE, Lemma 2 states that $\pi = 1$ is necessary for symmetric equilibrium unless both sellers commit to the ex post optimal mechanisms. That is, under FAE, information provision increases the surplus available at a selling site since two buyers generate more surplus when informed than when uninformed, as $\mathcal{S}^{2,i} = \bar{S}^{2,i} > \bar{\theta}$ and $\mathcal{S}^{1,i} = \bar{\theta}$. The necessity of full information under Assumption 1 for ex post optimal mechanisms follows from Proposition 7. Lemma 5 provides the conditions for equilibrium rents. The sufficiency argument is direct; taking a profile satisfying the conditions of the proposition, we show that no deviation can be profitable.

A seller cannot commit to low information provision for essentially the same reason that he cannot commit to excluding θ_H-types from trade; since he can maintain incentive compatibility while increasing available surplus.

Proposition 8 identifies a continuum of symmetric equilibria under Assumption 1. These can be ranked from the least favourable to buyers (with rents $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$ and $r^{2,i} = 0$) to the most favourable to buyers (with rents $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$ and $R^{2,i} = R^{1,i}$). Competition drives sellers to offer marginal buyers their marginal contribution to site surplus, yet because of the traffic structure this only determines rents in the one-buyer state. Competition does not impose a particular division of surplus in the two-buyer state, since a marginal buyer pays out a negligible 'congestion charge', yet receives the full 'fixed fee' $R^{1,i}$. In line with the interpretation of rents in (14), it is as though the 'fee' paid to buyers to attend either site is fixed by competition and sellers coordinate on the 'congestion charge' imposed on buyers.

The equilibria differ in how the surplus is shared between buyers and sellers. Because of full information, FAE and symmetric seller strategies, the full available surplus is realized. In the presence of coordination among buyers, the efficient distribution of buyers across sale sites has one of them with each seller. In the absence of coordination, efficiency requires maximizing the likelihood of having one buyer at each site, which happens when $q = \frac{1}{2}$.

The competition between sellers does not drive profits to zero; sellers make positive profits in all equilibria. In the one-buyer state, profits are positive since they are given by $\bar{\theta} - \frac{\bar{S}^{2,i}}{2}$ and it is the case that $2\bar{\theta} > \bar{S}^{2,i}$. In the two-buyer state, profits are $\bar{S}^{2,i} - 2R^{2,i}$, which is positive except in the equilibrium most favourable to buyers. Sellers cannot undercut each other in equilibrium as the gain in traffic does not compensate the drop in rents-per-buyer. At symmetric profiles where $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$, whatever the level of $R^{2,i} \leq R^{1,i}$, deviations are not profitable. It does not matter whether a seller contemplates deviating from a high-rent or low-rent symmetric profile, the behaviour of profits is qualitatively the same. That sellers do not compete away all profits in the presence of traffic effects has been noted in the literature on competing auctioneers.[17] With congestion effects and mixed strategies by buyers, changes in demand induced by changes in rent offers are continuous, and competition between sellers is much less fierce than in the typical Bertrand model. The indeterminancy of rents, once the necessary condition $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$ is satisfied, however, is novel. As noted above, this stems from the fact that even though competition between the sellers forces them to offer the marginal buyer his contribution to site surplus, a marginal buyer only receives $R^{1,i}$. Rents in the two-buyer state are constrained not by competition, but by the congestion effects condition $R^{2,i} \leq R^{1,i}$.

5 Discussion

In this section, we discuss the main assumptions of our model. In particular, the assumption of two buyers and two sellers, though restrictive, is important. When the number of buyers goes beyond two, it rapidly becomes difficult to deal with the binomial demand generated by buyers' mixed strategies, and hence sellers' profit functions are cumbersome. This is one reason why the literature on competing auctioneers focuses on large economies where a seller's effect on market conditions vanishes. The addition of more sellers does not lead to any pronounced tractability problems or alter qualitative results, but increasing the competition faced by a given seller will definitely affect the level of equilibrium variables.

Having sellers choose the probability of providing information, and not directly choosing some ex post distribution of types, simplifies the model by reducing the ex post information states to two; informed and uninformed. However, choices of (π_a, π_b) differentiate the sites with respect to information ex ante, which is the essential element in having ex ante uninformed buyers choose selling sites on the basis of informativeness. This feature of the model could potentially be replaced with information structures ordered by precision. Qualitatively, results for exogenous mechanisms would not change given some assumptions on how ex post distributions vary in the index variable π. When mechanisms are endogenous, our results would depend on some version of Assumption 1.

The assumption that each seller has a single good for sale is sufficient to generate competition among buyers for goods. Alternatively, sellers may face decreasing returns to scale. When no rarity effects are present at sale sites, all buyers attend the site offering the highest expected rents-per-buyer, and congestion has no influence on rents. The presence of congestion effects among buyers is critical in our model, whose main focus is the trade-off sellers

[17] In the asymptotic model of Peters and Severinov (1997) where, as is the case here, buyers sort into sites before observing their values, the (unique) symmetric equilibrium in reserve prices of second-price auctions is bounded away from cost. This is true also in the duopolistic model of Burguet and Sákovics (1999) where, however, buyers know their values before sorting.

face between providing rents to buyers to attract them and extracting maximal profits per buyer present. Our paper pertains to markets where, once buyers are present at a selling site, it is not the case that the seller can serve all of them at the same cost.

A buyer who has sorted into a site is assumed to be committed to it. He receives (or not) signals from that site only, and cannot renege on his choice and attend the other site if his signal or demand realizations are not favourable. This trait of the model is implicitly an assumption about the presence of transportation or time costs from switching sale locations once one is 'on site' somewhere. Sales may have short time frames, so that a buyer leaving his present site would 'miss' the sales at other sites. This event may arise if sellers either will not or cannot commit to wait for 'late' buyers.

6 Conclusion

In this paper we have analysed the strategic interactions of sellers who compete for buyers by committing to information provision. When mechanisms are exogenously fixed and sellers compete solely through offers of information, we have shown that they may prefer to compete in high-rent environments, as these lessen the intensity of competition and lead to lower information provision. However, such environments lead to different outcomes depending on whether they foster or attenuate the competition between buyers for goods. When sellers commit to both information provision and mechanisms, we have shown that under a no-exclusion assumption, all symmetric equilibria have full information provision. However, a variety of rent levels are supported in equilibrium as a result of different equilibrium offers of mechanisms. This shows that, in a range of cases, sellers prefer to compete through mechanisms rather than through information provision. By doing so they maximize the available surplus, and competition determines the equilibrium share of this surplus going to buyers.

Appendix

6.1 Sale Mechanisms

The next two sections resume the description of the model, first defining the sale mechanisms trough which goods are delivered to buyers and then deriving sellers' payoff functions by determining behaviour in the buyers' subgame. Once buyers have sorted into selling sites, sellers deliver their goods according to state-contingent incentive-compatible direct mechanisms. These mechanisms specify outcomes, probabilities of obtaining the good and monetary transfers, as functions of reported types for all information and demand states of the market. The mechanisms are also constrained to be anonymous; they cannot depend on a buyer's identity.

More formally, let $\eta \in \{1,2\}$ denote the *demand state* of a sale site and $\tau \in \{i,u\}$ its *information state*, where i stands for informed and u for uninformed. The *state* of a sale site is given by $(\eta,\tau) \in \{1,2\} \times \{i,u\}$. Let $\Psi(\eta,\tau)$ denote the set of *report profiles* that can be received by the seller in state (η,τ). That is,

$$\Psi(\eta,\tau) = \begin{cases} \{(\theta_m,\theta_n)\}_{(m,n)\in\{L,H\}^2} & \text{if } \eta = 2 \text{ and } \tau = i, \\ \{\theta_m\}_{m\in\{L,H\}} & \text{if } \eta = 1 \text{ and } \tau = i, \\ \emptyset & \text{if } \tau = u. \end{cases}$$

A *direct mechanism* for seller k is a collection of functions

$$\Big\{\big\{x_k^{\eta,\tau} : \Psi(\eta,\tau) \to [0,1], y_k^{\eta,\tau} : \Psi(\eta,\tau) \to \Re\big\}\Big\}_{\substack{\eta\in\{1,2\},\\ \tau\in\{i,u\}}}$$

where $x_k^{\eta,\tau}(\psi)$ and $y_k^{\eta,\tau}(\psi)$ are, respectively, the probability a buyer obtains the good and the transfer he must pay to seller k when the report profile is $\psi \in \Psi(\eta,\tau)$ in state (η,τ). Since no report is necessary when buyers are uninformed, we write probabilities and transfers as $x_k^{\eta,u}$ and $y_k^{\eta,u}$, respectively, for $\eta \in \{1,2\}$. Also, since mechanisms are anonymous, we define $x_k^{2,i}(\theta_m,\theta_n)$ as the probability that a buyer reporting θ_m obtains the good when the other buyer reports θ_n. A similar remark holds for the transfer function $y_k^{2,i}(\theta_m,\theta_n)$. The probabilities must satisfy

$$\begin{aligned} & x_k^{1,\tau}(\psi) \le 1 \quad \text{for } \psi \in \Psi(1,\tau) \text{ and } \tau \in \{i,u\}, \\ & x_k^{2,u} \le \frac{1}{2}, \\ & x_k^{2,i}(\theta_m,\theta_n) + x_k^{2,i}(\theta_n,\theta_m) \le 1 \quad \text{for } (m,n) \in \{L,H\}^2. \end{aligned} \tag{15}$$

In state $(2,i)$ at site k, each buyer only knows his own valuation. We define the *reduced form* transfers and winning probabilities as

$$X_k^{2,i}(\theta_j) = \mathbf{E}_{\theta_{-j}} x_k^{2,i}(\theta_j,\theta_{-j}),$$

and

$$Y_k^{2,i}(\theta_j) = \mathbf{E}_{\theta_{-j}} y_k^{2,i}(\theta_j, \theta_{-j}) \quad \text{for } j \in \{H, L\}.$$

The direct mechanisms at sale sites must ensure that buyers truthfully report their types once demand and information states are known. *Incentive-compatible direct mechanisms* are defined as respecting a set of state-contingent incentive and participation constraints. When no information is released at site k, no incentive constraints apply. The relevant participation constraints are

$$x_k^{1,u}\bar{\theta} - y_k^{1,u} \geq 0, \tag{PC$_k^{1,u}$}$$
$$x_k^{2,u}\bar{\theta} - y_k^{2,u} \geq 0. \tag{PC$_k^{2,u}$}$$

In state $(1, i)$ at site k, the set of constraints is given by

$$x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) \geq x_k^{1,i}(\theta_L)\theta_H - y_k^{1,i}(\theta_L), \tag{IC$_k^{1,i}(\theta_H)$}$$
$$x_k^{1,i}(\theta_L)\theta_L - y_k^{1,i}(\theta_L) \geq x_k^{1,i}(\theta_H)\theta_L - y_k^{1,i}(\theta_H), \tag{IC$_k^{1,i}(\theta_L)$}$$
$$x_k^{1,i}(\theta_L)\theta_L - y_k^{1,i}(\theta_L) \geq 0. \tag{PC$_k^{1,i}(\theta_L)$}$$

As is well known, the participation constraint of the θ_H-type, (PC$_k^{1,i}(\theta_H)$), is satisfied whenever (IC$_k^{1,i}(\theta_H)$) and (PC$_k^{1,i}(\theta_L)$) hold.

The constraints that need to be satisfied in state $(2, i)$ at site k are given by

$$X_k^{2,i}(\theta_H)\theta_H - Y_k^{2,i}(\theta_H) \geq X_k^{2,i}(\theta_L)\theta_H - Y_k^{2,i}(\theta_L), \tag{IC$_k^{2,i}(\theta_H)$}$$
$$X_k^{2,i}(\theta_L)\theta_L - Y_k^{2,i}(\theta_L) \geq X_k^{2,i}(\theta_H)\theta_L - Y_k^{2,i}(\theta_H), \tag{IC$_k^{2,i}(\theta_L)$}$$
$$X_k^{2,i}(\theta_L)\theta_L - Y_k^{2,i}(\theta_L) \geq 0. \tag{PC$_k^{2,i}(\theta_L)$}$$

As in the single buyer case, the participation constraint of the θ_H-type, PC$_k^{2,i}(\theta_H)$, is satisfied whenever IC$_k^{2,i}(\theta_H)$ and PC$_k^{2,i}(\theta_L)$ hold. The class of incentive compatible direct mechanisms for this problem is denoted by Γ, and a particular mechanism at site k by γ_k.[18] Any mechanism at site k, whether exogenously fixed or proposed by seller k, must be an element of Γ.

Suppose that instead of denoting the probability with which buyers become informed, π_k orders ex post distributions of types according to precision, as in Ganuza and Penalva (2006). In this case, a seller promising more information ex ante faces a more 'spread out' distribution of buyer values ex post. That is, buyers possess more detailed private information and hence the seller faces costlier incentive constraints. In our simplified modelling of information, sellers compete through information provision ex ante but face either informed or uninformed

[18] In what follows, to lighten notation, we will omit the site subscript k for exogenous mechanisms that are common across sale sites and for symmetric profiles when mechanisms are chosen by sellers.

buyers ex post. In this case, a seller promising more information ex ante is more likely to face incentive constraints ex post.

, and they are given by

$$
\begin{aligned}
R_k^{\eta,u} &= x_k^{\eta,u}\bar{\theta} - y_k^{\eta,u} \quad \text{ for } \eta \in \{1,2\}, \\
R_k^{1,i} &= \mathbf{E}_\theta \left[x_k^{1,i}(\theta)\theta - y_k^{1,i}(\theta) \right], \\
R_k^{2,i} &= \mathbf{E}_\theta \left[X_k^{2,i}(\theta)\theta - Y_k^{2,i}(\theta) \right].
\end{aligned}
$$

Note that in informed states these rents are computed before buyers learn their types.

We denote the *surplus at site k in state (η,τ) under mechanism γ_k* by $\mathcal{S}_k^{\eta,\tau}$. That is

$$
\begin{aligned}
\mathcal{S}_k^{1,u} &= x_k^{1,u}\bar{\theta} \\
\mathcal{S}_k^{2,u} &= 2x_k^{2,u}\bar{\theta} \\
\mathcal{S}_k^{1,i} &= \left[p_H x_k^{1,i}(\theta_H)\theta_H + p_L x_k^{1,i}(\theta_L)\theta_L \right], \\
\mathcal{S}_k^{2,i} &= 2\left[p_H X_k^{2,i}(\theta_H)\theta_H + p_L X_k^{2,i}(\theta_L)\theta_L \right].
\end{aligned}
$$

Profits of seller k, given $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$, are

$$
\mathcal{P}_k(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k}) = \mathbf{E}_\eta \mathbf{E}_\tau \mathbf{E}_v \left[\eta y_k^{\eta,\tau}(v) \right].
$$

The first expectation is taken with respect to the binomial distribution with parameter q (if $k = a$) or $1 - q$ (if $k = b$) of demand at site k, and the second with respect to the binomial distribution with parameter π_k over information states at site k. The final expectation is taken with respect to the distribution of truthful reports in state (η,τ), which can be derived from the ex ante distribution of types. We can rewrite profits in the more useful form of site surplus less buyer rents. We denote the *surplus at site k in state (η,τ) under mechanism γ_k* by $\mathcal{S}_k^{\eta,\tau}$. That is

$$
\begin{aligned}
\mathcal{S}_k^{1,u} &= x_k^{1,u}\bar{\theta} \\
\mathcal{S}_k^{2,u} &= 2x_k^{2,u}\bar{\theta} \\
\mathcal{S}_k^{1,i} &= \left[p_H x_k^{1,i}(\theta_H)\theta_H + p_L x_k^{1,i}(\theta_L)\theta_L \right], \\
\mathcal{S}_k^{2,i} &= 2\left[p_H X_k^{2,i}(\theta_H)\theta_H + p_L X_k^{2,i}(\theta_L)\theta_L \right].
\end{aligned}
$$

Profits of seller k, given $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$, can then be expressed in the useful form of surplus less rents as

$$
\mathcal{P}_k(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k}) = \mathbf{E}_\eta \mathbf{E}_\tau \left[\mathcal{S}_k^{\eta,\tau} - \eta R_k^{\eta,\tau} \right]. \tag{16}
$$

7 A Characterization of Incentive-Compatible Mechanisms

Note that buyer sorting decisions, as expressed by (2), depend only on information provision and expected rents $R_k^{\eta,\tau}$. In particular, buyer decisions are not affected by how rents are shared between types conditional on being informed. This ex ante feature of rent promises allows a useful characterization of incentive-compatible mechanisms. Crucially, as Lemma 6 illustrates, we can restrict θ_H-type incentive-compatibility constraints to be binding in states $(1, i)$ and $(2, i)$. This is without loss of generality since any incentive-compatible mechanism at site k that achieves rents $\left\{R_k^{\eta,\tau}\right\}_{\substack{\eta\in\{1,2\}\\ \tau\in\{i,u\}}}$ with non-binding θ_H-type incentive constraints can be replaced by an incentive compatible mechanism that achieves the same levels of expected rents with the same allocations, but in which these constraints bind. Under this new mechanism, profits are unchanged and all traffic and information provision incentives are preserved.

Lemma 6. (θ_H-Type Incentive-Compatibility Constraints Bind)
Given any profile $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$, there exists a mechanism $\tilde{\gamma}_k \in \Gamma$ where $\widetilde{IC}_k^{1,i}(\theta_H)$ and $\widetilde{IC}_k^{2,i}(\theta_H)$ are binding and such that under profile $(\pi_k, \tilde{\gamma}_k, \pi_{-k}, \gamma_{-k})$ all buyers' rents and allocation probabilities and all sellers' profits are the same as under profile $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$.

The result is proved[19] for an original profile where only $(\mathrm{IC}_k^{1,i}(\theta_H))$ is slack. The same proof applies to the case where only $(\mathrm{IC}_k^{2,i}(\theta_H))$ is slack, except that the reduced-form mechanisms replace the mechanisms. But as the proof manipulates mechanisms in different demand states independently, given an original profile where the incentive compatibility constraints of θ_H-types in both demand states are slack, one could find a rent and profit-equivalent mechanism with incentive constraints binding in both states by the same procedure. The intuition of the proof is simple: given an incentive compatible mechanism where $\mathrm{IC}_k^{1,i}(\theta_H)$ does not bind, we can simply increase θ_L-type rents and decrease θ_H-type rents through transfers until the constraint binds, while ensuring that the expected rents in demand state $(1, i)$ are unchanged. The ratio between both shifts which ensures this, a function of the ex ante distribution of types, also keeps profits unchanged.

Lemma 6 identifies, for any incentive compatible mechanism $\gamma_k \in \Gamma$, a profit and rent-equivalent mechanism $\tilde{\gamma}_k \in \Gamma$ in which θ_H-type incentive compatibility constraints are binding and the allocation rules are as in γ_k. We say then that $\tilde{\gamma}_k$ is the *$IC(\theta_H)$-equivalent* of γ_k. Similarly, denote by $\tilde{\Gamma}$ the set of IC(θ_H)-equivalent mechanisms. Given information provision (π_a, π_b), a game with mechanisms $(\gamma_a, \gamma_b) \in (\Gamma \setminus \tilde{\Gamma})^2$ generates the same distribution over outcomes as a game with mechanisms $(\tilde{\gamma}_a, \tilde{\gamma}_b)$, where $\tilde{\gamma}_k$ is the $IC(\theta_H)$-equivalent mechanism of γ_k. That is, excluding mechanisms in $\Gamma \setminus \tilde{\Gamma}$ does not reduce the set of equilibria in terms

[19] All proofs are in the Appendix.

of information provision. On the other hand, when sellers also choose mechanisms, it is not the case that equilibrium mechanisms must belong to $\tilde{\Gamma}$. However, Lemma 6 states that excluding mechanisms in $\Gamma \setminus \tilde{\Gamma}$ does not reduce the set of equilibrium allocations, traffic levels and payoffs. Furthermore, any equilibrium property satisfied by allocations or expected rents $R_k^{\eta,\tau}$ of mechanisms in $\tilde{\Gamma}$ must also hold for equilibria with mechanisms in $\Gamma \setminus \tilde{\Gamma}$. In what follows, by incentive compatible mechanisms we refer to mechanisms in $\tilde{\Gamma}$.

Lemma 6 is useful in that binding θ_H-type incentive-compatibility constraints pin down θ_H-type rents as a function of the rents of θ_L-types. Denote *low-type rents under mechanism γ_k in state (η, τ)* by $r_k^{\eta,\tau}$. These are the rents offered to θ_L-types in informed states and to the uninformed otherwise. Then we can rewrite the expected rents promised at site k as

$$\begin{aligned} R_k^{\eta,u} &= r_k^{\eta,u} \text{ for } \eta \in \{1,2\}, \\ R_k^{1,i} &= r_k^{1,i} + x_k^{1,i}(\theta_L) p_H (\theta_H - \theta_L), \\ R_k^{2,i} &= r_k^{2,i} + X_k^{2,i}(\theta_L) p_H (\theta_H - \theta_L). \end{aligned}$$

Lemma 6 justifies the use of the following well-known result.

Lemma 7. (Characterization of IC(θ_H)-Equivalent Mechanisms)
$\gamma_k \in \tilde{\Gamma}$ if and only if $x_k^{1,i}(\theta_H) \geq x_k^{1,i}(\theta_L)$, $X_k^{1,i}(\theta_H) \geq X_k^{1,i}(\theta_L)$ and $r_k^{\eta,\tau} \geq 0$ for all $\eta \in \{1,2\}$ and $\tau \in \{i, u\}$.

The proof of this result is entirely standard and is omitted. Thus, $IC(\theta_H)$-equivalent mechanisms are characterized by the non-negative rent levels of θ_L and uninformed types and monotone allocation rules in informed states.

Definition 4. (No Waste)
A mechanism $\gamma_k \in \Gamma$ has no waste if and only if

$$\begin{aligned} & x_k^{1,i}(\theta_H) = x_k^{1,i}(\theta_L) = x_k^{1,u} = 1, \\ & x_k^{2,u} = \frac{1}{2}, \\ & x_k^{2,i}(\theta_H, \theta_H) = x_k^{2,i}(\theta_L, \theta_L) = \frac{1}{2} \\ & x_k^{2,i}(\theta_H, \theta_L) + x_k^{2,i}(\theta_L, \theta_H) = 1. \end{aligned}$$

In a mechanism with no waste, the full surplus ($\bar{\theta}$) is realized in the one-buyer and uninformed states, while in state $(2, i)$ the full surplus is realized only when $X_k^{2,i}(\theta_H) = \frac{p_H}{2} + p_L$ and $X_k^{2,i}(\theta_L) = \frac{p_L}{2}$.[20]

[20] In state $(2, i)$, the seller cannot set $X_k^{2,i}(\theta_H) = 1$, since when both θ_H-types are present, only one may obtain the good. Anonymity of the mechanism and feasibility imply that $X_k^{2,i}(\theta_H) \leq p_L + \frac{p_H}{2}$. That is, if with two buyers the seller allocates the good efficiently with respect to θ_H-type buyers, such a buyer obtains the good whenever his opponent is a θ_L-type, while he obtains the good with probability $\frac{1}{2}$ whenever he faces a θ_H-type.

Furthermore, that γ has no waste also implies that seller a's profits in the two-buyer state also decrease in π_a since the term in the first brackets of (9) is linear in π_a and

$$\begin{aligned}
\mathcal{S}^{2,i} - \bar{\theta} - 2R^{2,i} &= \mathcal{S}^{2,i} - \bar{\theta} - 2\Big(r^{2,i} + X^{2,i}(\theta_L)p_H(\theta_H - \theta_L)\Big) \\
&\leq \mathcal{S}^{2,i} - \bar{\theta} - 2X^{2,i}(\theta_L)p_H(\theta_H - \theta_L) \\
&= \theta_H\left[2p_H X^{2,i}(\theta_H) - p_H\right] + \theta_L\left[2p_L X^{2,i}(\theta_L) - p_L\right] - 2X^{2,i}(\theta_L)p_H(\theta_H - \theta_L) \\
&= (\theta_H - \theta_L)\left[p_L - 2X^{2,i}(\theta_L)\right] \\
&\leq 0. \qquad (17)
\end{aligned}$$

The second line follows since $r^{2,i} \geq 0$, the fourth since $p_H X_k^{2,i}(\theta_H) + p_L X_k^{2,i}(\theta_L) = \frac{1}{2}$ under no waste, and the last since $X^{2,i}(\theta_L) \geq \frac{p_L}{2}$ under no waste.

Note also that under Assumption 1, the ex post optimal mechanisms are regular, and they can be described by allocation probabilities

$$\begin{aligned}
x_k^{1,i}(\theta_H) = x_k^{1,i}(\theta_L) = x_k^{1,u} &= 1, \\
x_k^{2,u} &= \frac{1}{2}, \\
X_k^{2,i}(\theta_H) &= \frac{p_H}{2} + p_L, \\
X_k^{2,i}(\theta_L) &= \frac{p_L}{2},
\end{aligned}$$

and rent levels for low types $r^{\eta,\tau} = 0$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$.

Definition 5. (Partial Allocative Efficiency)
A mechanism $\gamma_k \in \Gamma$ has partial allocative efficiency (PAE) if and only if

$$\begin{aligned}
&x_k^{1,i}(\theta_H) = x_k^{1,u} = 1, \\
&x_k^{2,u} = \frac{1}{2}, \\
&x_k^{2,i}(\theta_H, \theta_L) = 1, \text{ and } x_k^{2,i}(\theta_H, \theta_H) = \frac{1}{2}.
\end{aligned}$$

A mechanism has PAE if it always allocates the good to θ_H-types and uninformed buyers, while it may exclude θ_L-types.

Definition 6. (Full Allocative Efficiency)
A mechanism $\gamma_k \in \Gamma$ has full allocative efficiency (FAE) if and only if it has partial allocative efficiency and also

$$\begin{aligned}
&x_k^{1,i}(\theta_L) = 1, \\
&x_k^{2,i}(\theta_L, \theta_L) = \frac{1}{2}.
\end{aligned}$$

A mechanism has FAE if it has PAE and furthermore always allocates the good to a θ_L-type buyer in the absence of a θ_H-type. To relate this to our earlier definitions, any mechanism with FAE has no waste, but a mechanism with no waste may allocate the good to a θ_L-buyer in the presence of a θ_H-buyer in state $(2,i)$. Under FAE, the surplus in state $(2,i)$ is maximized, and we denote it by $\bar{S}^{2,i}$.

Proof of Lemma 6:

Consider an incentive compatible mechanism at site k, $\gamma_k = \left\{ \left\{ x_k^{\eta,\tau}(v), y_k^{\eta,\tau}(v) \right\}_{v \in V(\eta,\tau)} \right\}_{\substack{\eta \in \{1,2\}, \\ \tau \in \{i,u\}}}$ such that $\mathrm{IC}_k^{1,i}(\theta_H)$ is slack. In particular, say

$$x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) = x_k^{1,i}(\theta_L)\theta_H - y_k^{1,i}(\theta_L) + C,$$

with $C > 0$. Consider an alternative mechanism $\hat{\gamma}_k = \left\{ \left\{ \hat{x}_k^{\eta,\tau}(v), \hat{y}_k^{\eta,\tau}(v) \right\}_{v \in V(\eta,\tau)} \right\}_{\substack{\eta \in \{1,2\}, \\ \tau \in \{i,u\}}}$ identical to γ_k except that

$$\begin{aligned} \hat{y}_k^{1,i}(\theta_H) &= y_k^{1,i}(\theta_H) + p_L C \\ \hat{y}_k^{1,i}(\theta_L) &= y_k^{1,i}(\theta_L) - p_H C. \end{aligned}$$

In that case,

$$\begin{aligned} \hat{x}_k^{1,i}(\theta_H)\theta_H - \hat{y}_k^{1,i}(\theta_H) &= x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) - p_L C \\ &= x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) - C + p_H C \\ &= x_k^{1,i}(\theta_L)\theta_H - y_k^{1,i}(\theta_L) + p_H C \\ &= \hat{x}_k^{1,i}(\theta_L)\theta_H - \hat{y}_k^{1,i}(\theta_L). \end{aligned}$$

Thus, $\widehat{IC}_k^{1,i}(\theta_H)$ binds. Since under $\hat{\gamma}_k$ the transfer of type θ_L has been decreased, $\widehat{PC}_k^{1,i}(\theta_L)$ is satisfied. Since both $\widehat{IC}_k^{1,i}(\theta_H)$ and $\widehat{PC}_k^{1,i}(\theta_L)$ hold, then so does $\widehat{PC}_k^{1,i}(\theta_H)$. Finally, we have made the θ_H types worse off and θ_L types better off, so that $\widehat{IC}_k^{1,i}(\theta_L)$ holds. Hence $\hat{\gamma}_k$ is incentive compatible.

Profits for the seller in state $(1,i)$ under mechanism $\hat{\gamma}_k$ are given by

$$\begin{aligned} p_H \hat{y}_k^{1,i}(\theta_H) + p_L \hat{y}_k^{1,i}(\theta_L) &= p_H y_k^{1,i}(\theta_H) + p_L y_k^{1,i}(\theta_L) + p_H p_L C - p_L p_H C \\ &= p_H y_k^{1,i}(\theta_H) + p_L y_k^{1,i}(\theta_L), \end{aligned}$$

where the last line is profits under γ_k in state $(1,i)$. Profits in other states are also unaffected. As noted in the text, the case where $IC_k^{2,i}(\theta_H)$ is slack is identical, with reduced-form mechanisms replacing the mechanisms. To that end, note that in state $(2,i)$, profits under mechanism γ_k are given by

$$\begin{aligned} &p_H^2 \left[2y_k^{2,i}(\theta_H,\theta_H) \right] + 2p_L p_H \left[y_k^{2,i}(\theta_H,\theta_L) + y_k^{2,i}(\theta_L,\theta_H) \right] + p_L^2 \left[2y_k^{2,i}(\theta_L,\theta_L) \right] \\ &= 2 \left[p_H Y_k^{1,i}(\theta_H) + p_L Y_k^{1,i}(\theta_L) \right]. \end{aligned}$$

$\square$

Proof of Lemma 1: Setting (10) equal to zero and checking the conditions for which $\pi < 1$, we obtain the expression for π^*. By the argument in the text, all that needs to be shown is that $\frac{\partial q}{\partial \pi_a}\big|_{\pi_a=\pi_b=\pi}$ is decreasing in the symmetric probability π. By (2) and using the fact that $R^{1,u} = R^{2,u} = 0$ for regular mechanisms, we have

$$q = \frac{\pi_a R^{1,i} - \pi_b R^{2,i}}{(R^{1,i} - R^{2,i})(\pi_a + \pi_b)},$$

and it can be verified that

$$\left.\frac{\partial q}{\partial \pi_a}\right|_{\pi_a=\pi_b=\pi} = \frac{R^{1,i} + R^{2,i}}{4\pi(R^{1,i} - R^{2,i})},$$

which is decreasing in π.

$\square$

Proof of Proposition 3:

The first part of the proof is the following lemma which provides sufficient conditions for the existence of interior symmetric equilibria.

Lemma 8. *Given a regular mechanism γ that generates rents such that*

i. $\bar{\theta} < 2R^{1,i}$ *and* $R^{1,i} + R^{2,i} < -\frac{2R^{1,i}(\mathcal{S}^{2,i} - \bar{\theta})}{\bar{\theta} - 2R^{1,i}}$.

ii. $2R^{1,i}(R^{2,i})^2 - 6(R^{1,i})^2 R^{2,i} + 8(R^{1,i})^3 - \bar{\theta}(4(R^{1,i})^2 + R^{1,i}R^{2,i} - (R^{2,i})^2) \leq 0$.

the symmetric equilibrium of the game between sellers is $\pi^* = \frac{-(R^{1,i}+R^{2,i})\bar{\theta}}{2R^{1,i}(\mathcal{S}^{2,i} - \bar{\theta} - (R^{1,i}+R^{2,i}))} < 1$.

Proof: Point i of the statement ensures that $\pi^* < 1$. Consider a candidate symmetric profile (π, π) and a deviation by seller a to $\pi + \lambda$, which induces traffic level q^λ. Then we have that

$$\begin{aligned} q^\lambda &= \frac{\pi(R^{1,i} - R^{2,i}) + \lambda R^{1,i}}{(R^{1,i} - R^{2,i})(2\pi + \lambda)} \\ &= \frac{1}{2} + z, \\ \text{with } z &= \frac{\lambda(R^{1,i} + R^{2,i})}{2(R^{1,i} - R^{2,i})(2\pi + \lambda)}. \end{aligned} \tag{18}$$

Also,

$$\begin{aligned}
\mathcal{P}_a(\pi+\lambda,\pi)-\mathcal{P}_a(\pi,\pi) &= z(z+1)\left[\pi\mathcal{S}^{2,i}+(1-\pi)\bar{\theta}-2\pi R^{2,i}\right]-2z^2\left[\bar{\theta}-\pi R^{1,i}\right] \\
&\quad +(\frac{1}{2}+z)^2\left[\mathcal{S}^{2,i}-\bar{\theta}-2R^{2,i}\right]-2\lambda(\frac{1}{2}+z)(\frac{1}{2}-z)R^{1,i} \\
&= \frac{\lambda^2}{D}\Bigg[4R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta})\Bigg[(R^{1,i}+R^{2,i})(R^{1,i}-R^{2,i})\bar{\theta} \\
&\quad -2\lambda(R^{1,i})^2(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))\Bigg] \\
&\quad +(R^{1,i}+R^{2,i})^2\bar{\theta}\Bigg[\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})(5R^{1,i}-R^{2,i}) \\
&\quad +(R^{1,i}+R^{2,i})^2\Bigg]\Bigg] \\
&\leq F\Bigg[(\mathcal{S}^{2,i}-\bar{\theta})(4(R^{1,i})^2+R^{1,i}R^{2,i}-(R^{2,i})^2) \\
&\quad -2R^{2,i}(R^{1,i}+R^{2,i})(2R^{1,i}-R^{2,i})\Bigg] \qquad (19) \\
&< H\Bigg[2R^{1,i}(R^{2,i})^2-6(R^{1,i})^2R^{2,i}+8(R^{1,i})^3 \\
&\quad -\bar{\theta}(4(R^{1,i})^2+R^{1,i}R^{2,i}-(R^{2,i})^2)\Bigg]. \qquad (20)
\end{aligned}$$

Where D, F and $H>0$. The second equality follows from setting $\pi=\pi^*$ and rearranging terms. The first inequality follows from the fact that $q^\lambda\leq 1$ when $\lambda\leq\frac{\bar{\theta}(R^{1,i}+R^{2,i})(R^{1,i}-R^{2,i})}{-2R^{1,i}R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))}$. The last inequality follows since $\pi^*<1$ when $\mathcal{S}^{2,i}-\bar{\theta}<-\frac{(R^{1,i}+R^{2,i})(\bar{\theta}-2R^{1,i})}{2R^{1,i}}$.

□

To show that the set of regular mechanisms that satisfy conditions i and ii of Proposition 3 is nonempty, note that (19) implies that under any mechanism where $\mathcal{S}^{2,i}=\bar{\theta}$ (a pricing mechanism in the two-buyer state), deviations from the symmetric profile (π^*,π^*) are strictly not profitable for seller a. Pick some regular mechanism γ with $X^{2,i}(\theta_L)=X^{2,i}(\theta_H)=\frac{1}{2}$ that satisfies condition i of Lemma 8. Consider an alternative mechanism $\mathring{\gamma}$ identical to γ except that $\mathring{X}^{2,i}(\theta_L)=\frac{1}{2}-\epsilon_L$ and $\mathring{X}^{2,i}(\theta_H)=\frac{1}{2}+\epsilon_H$ and such that the term inside the brackets of (19) is strictly negative. Then $\mathring{\gamma}$ is regular and there exists some neighbourhood N of $(\mathring{X}^{2,i}(\theta_L),\mathring{X}^{2,i}(\theta_H),\mathring{r}^{1,i},\mathring{r}^{2,i})$ such that for all $\hat{\gamma}$ represented by some $n\in N$, $\hat{\gamma}$ is regular and the term inside the brackets of (19) is negative. Thus all such $\hat{\gamma}$ induce a unique symmetric equilibrium (π^*,π^*) with $\pi^*<1$.

Finally, the derivatives mentioned in the proposition can be computed directly to yield

$$\begin{aligned}
\frac{\partial \pi^*}{\partial R^{1,i}} &= -\frac{2\bar{\theta} R^{1,i}(R^{1,i}+R^{2,i}) - 2\bar{\theta} R^{2,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))}{(-2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})))^2} \\
&< 0, \\
\text{and} \quad \frac{\partial \pi^*}{\partial R^{2,i}} &= -\frac{2\bar{\theta} R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta})}{(-2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})))^2} \\
&\leq 0.
\end{aligned}$$

From these it can be checked that

$$\begin{aligned}
\frac{\partial \pi^*}{\partial R^{1,i}} - \frac{\partial \pi^*}{\partial R^{2,i}} &= \frac{-2\bar{\theta}(R^{1,i}+R^{2,i})(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+2R^{2,i}))}{(-2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})))^2} \\
&> 0.
\end{aligned}$$

$\square$

Proof of Proposition 5:

The profits of both sellers at a symmetric equilibrium with a regular mechanism γ_k are given by

$$\frac{1}{4}\left[\pi^*(\mathcal{S}^{2,i}-\bar{\theta}-2R^{2,i})+\bar{\theta}\right]+\frac{1}{2}\left[\bar{\theta}-\pi^* R^{1,i}\right]. \tag{21}$$

Direct computation yields

$$\begin{aligned}
\frac{\partial \mathcal{R}_a(\pi^*,\pi^*)}{\partial R^{1,i}} &= \frac{\bar{\theta}}{8(R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})))^2}\Bigg[R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta})(R^{1,i}+R^{2,i}) \\
&\qquad + R^{2,i}(\mathcal{S}^{2,i}-\bar{\theta}-2(R^{1,i}+R^{2,i}))(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))\Bigg] \\
&> 0 \\
\frac{\partial \mathcal{R}_a(\pi^*,\pi^*)}{\partial R^{2,i}} &= \frac{\bar{\theta} R^{1,i}}{2(R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})))^2}\left[(\mathcal{S}^{2,i}-\bar{\theta})^2-2(R^{1,i}+R^{2,i})^2\right].
\end{aligned}$$

$\square$

Proof of Proposition 7:

We show that $\mathcal{P}_a(\pi_a,1)$ is increasing in π_a.

$$\begin{aligned}
\mathcal{P}_a(\pi_a,1) &= \left(\frac{\pi_a-\frac{p_L}{2}}{(1+\pi_a)(1-\frac{p_L}{2})}\right)^2\bar{\theta} + 2\left(\frac{(\pi_a-\frac{p_L}{2})(1-\frac{\pi_a p_L}{2})}{((1+\pi_a)(1-\frac{p_L}{2}))^2}\right)\big(\bar{\theta}-\pi_a p_H(\theta_H-\theta_L)\big) \\
&= \left(\frac{\pi_a-\frac{p_L}{2}}{((1+\pi_a)(1-\frac{p_L}{2}))^2}\right)\Big(\bar{\theta}(p_H\pi_a+2-\frac{p_L}{2})+2(1-\frac{\pi_a p_L}{2})\pi_a p_H(\theta_H-\theta_L))\Big) \\
&\equiv A(\pi_a)\big(B(\pi_a)+C(\pi_a)\big)
\end{aligned}$$

Where $B(\pi_a)$ is clearly increasing in π_a, while it can be shown that $A(\pi_a)$ and $C(\pi_a)$ are increasing whenever $\pi \leq 1 + p_L$ and $\pi \leq \frac{1}{p_L}$, respectively, which is always true.

□

Proof of Lemma 2:

The reason why Lemma 2 is stated in terms of $IC(\theta_H)$-equivalent mechanisms is that the condition that it not be the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$ is easy to write for these mechanisms. This condition simply states that it is possible, for at least one state, to increase transfers in an incentive compatible way. Any mechanism $\gamma_a \in \Gamma$ that satisfies this last property would have its $IC(\theta_H)$-equivalent mechanism satisfy the property that it not be the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$ (through Lemma 6). The following proof then applies to all incentive compatible mechanisms that are components of some equilibrium, since a best response to a IC(θ_H)-equivalent mechanism is also a best-response to the original mechanism.

Suppose that $(\pi_a, \gamma_a, \pi_b, \gamma_b)$ is an equilibrium, that $\mathbf{E}_\eta \mathbf{E}_\tau \mathcal{S}_a^{\eta,\tau}$ is increasing in π_a, that it is not the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$ and that $\pi_a < 1$. Consider a deviation by seller a to a profile in which

$$\begin{aligned} \hat{\pi}_a &= \pi_a + \lambda \\ \hat{r}_a^{\eta,\tau} &= r_a^{\eta,\tau} - \delta^{\eta,\tau}, \end{aligned}$$

where $\lambda > 0$. For this deviant profile not to affect traffic levels (or buyers' expected rents), we need

$$\begin{aligned} & q\left[(\pi_a + \lambda)\left[r_a^{2,i} - \delta^{2,i} + z_a^{2,i}\right] + (1 - \pi_a - \lambda)\left[r_a^{2,u} - \delta^{2,u}\right]\right] \\ &\quad + (1-q)\left[(\pi_a + \lambda)\left[r_a^{1,i} - \delta^{1,i} + z_a^{1,i}\right] + (1 - \pi_a - \lambda)\left[r_a^{1,u} - \delta^{1,u}\right]\right] \\ &\quad = q\left[\pi_a\left[r_a^{2,i} + z_a^{2,i}\right] + (1-\pi_a)\left[r_a^{2,u}\right]\right] + (1-q)\left[\pi_a\left[r_a^{1,i} + z_a^{1,i}\right] + (1-\pi_a)\left[r_a^{1,u}\right]\right], \end{aligned}$$

or

$$\begin{aligned} & (\pi_a + \lambda)\left[q\delta^{2,i} + (1-q)\delta^{1,i}\right] + (1 - \pi_a - \lambda)\left[q\delta^{2,u} + (1-q)\delta^{1,u}\right] \\ &\quad = \lambda\left[q\left[r_a^{2,i} + z_a^{2,i} - r_a^{2,u}\right] + (1-q)\left[r_a^{1,i} + z_a^{1,i} - r_a^{1,u}\right]\right], \end{aligned} \tag{22}$$

where $z_a^{1,i} = r_a^{1,i} + x_a^{1,i}(\theta_L)p_H(\theta_H - \theta_L) \geq 0$ and $z_a^{2,i} = r_a^{2,i} + X_a^{2,i}(\theta_L)p_H(\theta_H - \theta_L) \geq 0$. The sign of the right-hand side (RHS) of (22) is given by the properties of the mechanism at site a. It is positive if buyers prefer, on average, to be informed at the site, and negative if buyers prefer, on average, to be uninformed.

Independently of the sign of the RHS, the left-hand side (LHS) can be made greater than the RHS by setting $\delta^{\eta,\tau} = r_a^{\eta,\tau}$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$. In that case

$$\begin{aligned} LHS - RHS = &\pi_a \left[q r_a^{2,i} + (1-q) r_a^{1,i} \right] + (1-\pi_a) \left[q r_a^{2,u} + (1-q) r_a^{1,u} \right] \\ &- \lambda \left[q z_a^{2,u} + (1-q) z_a^{1,u} \right], \\ > 0, \end{aligned}$$

since λ can be chosen arbitrarily small and it is not the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$.

Conversely, if $RHS > 0$, setting $\delta^{\eta,\tau} = 0$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$ implies that $LHS - RHS < 0$, so that there exists some choices of $\delta^{\eta,\tau} \in [0, r_a^{\eta,\tau}]$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$ such that $LHS = RHS$ (again, using the fact that it is not the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$).

If $RHS < 0$, buyers prefer being uninformed at site a and must be compensated for increases in information provision if they are to keep their sorting decisions unchanged. This can be done by a suitable choice of $\delta^{\eta,\tau} \leq 0$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$. Finally, if $RHS = 0$, buyers are indifferent between informed and uninformed states at site a and a seller can increase information provision without shifting traffic by setting $\delta^{\eta,\tau} = 0$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$.

In all cases, the argument above yields a deviation for seller a which keeps rent payouts unchanged and strictly increases the surplus available at site a. This implies that $(\pi_a, \gamma_a, \pi_b, \gamma_b)$ is not an equilibrium.

□

Proof of Lemma 3:

Our argument proceeds with mechanisms in $\tilde{\Gamma}$. However, if a mechanism in $\Gamma \setminus \tilde{\Gamma}$ without PAE were a component of an equilibrium, applying the following proof to its IC(θ_H)-equivalent (through Lemma 6) would yield a contradiction, since a best response to a IC(θ_H)-equivalent mechanism is also a best-response to the original mechanism.

Consider an incentive compatible mechanism at site k, $\gamma_k = \left\{ \left\{ x_k^{\eta,\tau}(v), y_k^{\eta,\tau}(v) \right\}_{v \in V(\eta,\tau)} \right\}_{\substack{\eta \in \{1,2\}, \\ \tau \in \{i,u\}}}$ such that $x_k^{1,i}(\theta_H) < 1$. Consider an alternative mechanism $\hat{\gamma}_k = \left\{ \left\{ \hat{x}_k^{\eta,\tau}(v), \hat{y}_k^{\eta,\tau}(v) \right\}_{v \in V(\eta,\tau)} \right\}_{\substack{\eta \in \{1,2\}, \\ \tau \in \{i,u\}}}$, identical to γ_k except that

$$\begin{aligned} \hat{x}_k^{1,i}(\theta_H) &= x_k^{1,i}(\theta_H) + \epsilon \\ \hat{y}_k^{1,i}(\theta_H) &= y_k^{1,i}(\theta_H) + \epsilon \theta_H. \end{aligned}$$

We have $\hat{x}_k^{1,i}(\theta_H) > x_k^{1,i}(\theta_H) \geq \hat{x}_k^{1,i}(\theta_L) > x_k^{1,i}(\theta_L)$ and $\hat{r}^{1,i} = r^{1,i} \geq 0$ since $\gamma_k \in \tilde{\Gamma}$, and so $\hat{\gamma}_k \in \tilde{\Gamma}$. Seller k's profits are higher under $\hat{\gamma}_k$ than under γ_k since θ_H-type transfers in the one-buyer state are higher.

As noted in the text, a complication arises in the proof for the two-buyer state if $X_k^{2,i}(\theta_H) < p_L + \frac{1}{2} p_H$ and if the constraint $x^{2,i}(\theta_H, \theta_L) + x^{2,i}(\theta_L, \theta_H) \leq 1$ from (15) is binding under the original mechanism γ_k. If this is not the case, then the previous proof applies to the reduced-form mechanisms. If the previous proof does not apply it must be that $x^{2,i}(\theta_L, \theta_H) > 0$, that is, a θ_L-type is sometimes allocated the good in the presence of a θ_H-type. Consider an alternative mechanism $\hat{\gamma}_k$ identical to γ_k except that

i. θ_L-types never get preference over θ_H-types, $\hat{x}^{2,i}(\theta_H, \theta_L) = 1$ and $\hat{x}^{2,i}(\theta_L, \theta_H) = 0$, so that

$$\hat{X}^{2,i}(\theta_L) = X^{2,i}(\theta_L) - p_H x_k^{2,i}(\theta_L, \theta_H)$$
$$\hat{X}^{2,i}(\theta_H) = X^{2,i}(\theta_H) + p_L x_k^{2,i}(\theta_L, \theta_H).$$

ii. Transfers are adjusted so that rents to both types are unchanged

$$\hat{Y}^{2,i}(\theta_L) = Y^{2,i}(\theta_L) - \theta_L(X_k^{2,i}(\theta_L) - \hat{X}^{2,i}(\theta_L))$$
$$\hat{Y}^{2,i}(\theta_H) = Y^{2,i}(\theta_H) + \theta_H(\hat{X}_k^{2,i}(\theta_H) - X^{2,i}(\theta_H)).$$

By condition i and since $\gamma_k \in \tilde{\Gamma}$, we have that $\hat{X}^{2,i}(\theta_H) > X^{2,i}(\theta_H) \geq X^{2,i}(\theta_L) > \hat{X}^{2,i}(\theta_L)$. Along with condition ii, this implies that $\hat{\gamma}_k \in \tilde{\Gamma}$.

Profits to seller k in the two-buyer state under $\hat{\gamma}_k$ are given by

$$\begin{aligned} 2\left[p_L \hat{Y}^{2,i}(\theta_L) + p_H \hat{Y}^{2,i}(\theta_H)\right] &= 2\left[p_L Y^{2,i}(\theta_L) + p_H Y^{2,i}(\theta_H) + p_H p_L(\theta_H - \theta_L)x^{2,i}(\theta_L, \theta_H)\right] \\ &> 2\left[p_L Y^{2,i}(\theta_L) + p_H Y^{2,i}(\theta_H)\right], \end{aligned}$$

which is the expression for profits to seller k in the two-buyer state under γ_k. The inequality follows since by hypothesis $x^{2,i}(\theta_L, \theta_H) > 0$. Thus a seller offering mechanism γ_k gains by offering $\hat{\gamma}_k$ since traffic and one-buyer state profits are unchanged and two-buyer state profits are higher. If under $\hat{\gamma}_k$ it is the case that $\hat{X}_k^{2,i}(\theta_H) < p_L + \frac{1}{2}p_H$, we can now apply the proof for the one-buyer state.

Similarly, for uninformed allocations, consider an incentive compatible mechanism at site k, $\gamma_k = \left\{\left\{x_k^{\eta,\tau}(v), y_k^{\eta,\tau}(v)\right\}_{v \in V(\eta,\tau)}\right\}_{\substack{\eta \in \{1,2\} \\ \tau \in \{i,u\}}}$, such that $x_k^{\eta,u} < 1$ for some $\eta \in \{1, 2\}$. Consider an alternative mechanism $\hat{\gamma}_k$, identical to γ_k except that in state (η, u)

$$\hat{x}_k^{\eta,u} = x_k^{\eta,u} + \epsilon$$
$$\hat{y}_k^{\eta,u} = y_k^{\eta,u} + \epsilon m.$$

Thus bidder rents are the same under both mechanisms but seller k's profits are higher in state (r, u) since the good is sold more often at higher prices.

□

Proof of Lemma 4:

Consider an incentive compatible mechanism[21] at site k $\gamma_k = \left\{ \{x_k^{\eta,\tau}(v), y_k^{\eta,\tau}(v)\}_{v \in V(\eta,\tau)} \right\}_{\substack{\eta \in \{1,2\} \\ \tau \in \{i,u\}}}$ such that $x_k^{1,i}(\theta_L) < 1$ and the level of rents provided to type θ_L is given by $r^{1,i} \geq 0$. Then

$$y_k^{1,i}(\theta_L) = x_k^{1,i}(\theta_L) - r^{1,i}, \tag{23}$$

and, by Lemmas 6 and 3

$$y_k^{1,i}(\theta_H) = \theta_H - x_k^{1,i}(\theta_L)(\theta_H - \theta_L) - r^{1,i}. \tag{24}$$

By (23) and (24), we can write seller k's profits conditional on $(\mathrm{IC}_k^{1,i}(\theta_H))$ binding and type θ_L receiving rents $r^{1,i}$ as

$$x_k^{1,i}(\theta_L)(\theta_L - p_H\theta_H) + p_H\theta_H - r^{1,i}. \tag{25}$$

These are increasing in $x_k^{1,i}(\theta_L)$ whenever $\theta_L > p_H\theta_H$. Since $x^{1,i}(\theta_H) = 1$ by Lemma 3, an increase in $x^{1,i}(\theta_L)$ maintains incentives compatibility so seller k can increase profits in state $(1,i)$ by increasing doing so. This increases traffic to site k (since rents to θ_H-types increase). But at a symmetric equilibrium, we have $q = \frac{1}{2}$ and marginal changes in traffic leave unaffected the probability of the one-buyer state $(2q(1-q))$, so that profits of seller k increase with marginal changes in $x^{1,i}(\theta_L)$.

□

Proof of Proposition 8:

Necessity of $R^{2,i} \leq R^{1,i}$: Consider a symmetric equilibrium with $\pi = 1$, FAE and a mechanism[22] γ such that $R^{1,i} < R^{2,i}$. This last fact implies that $r^{2,i} > 0$. Consider a mechanism $\hat{\gamma}_k$ for seller k identical to γ except that $\hat{r}^{2,i} = r^{2,i} - \mathbf{d}r^{2,i}$. By (2) and the argument in the text, $\hat{\gamma}_k$ leads to an infinitesimal increase in the number of buyers visiting site k. Locally, moving away from a symmetric profile does not change the probability of the one-buyer state, while it increases that of the two-buyer state, where rents are now lower. This deviation is thus profitable.

□

Necessity of $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$: We will consider local variations in $R^{1,i}$ and $R^{2,i}$ that leave $\pi = 1$ and allocative efficiency unchanged. Assume for now that $r^{1,i} > 0$ and $r^{2,i} > 0$ to ensure that it is always possible to effect such local changes through transfers. Profits for seller a are given by

$$\mathcal{P}_a(\pi_a, \gamma_a, \pi_b, \gamma_b) = q^2[\bar{S}^{2,i} - 2R_a^{2,i}] + 2q(1-q)[m - R_a^{1,i}].$$

[21] We need only consider mechanisms in $\tilde{\Gamma}$, by the remark in the proof of Lemma 3.
[22] We need only consider mechanisms in $\tilde{\Gamma}$, by the remark in the proof of Lemma 3.

At a symmetric profile, we can ignore the local changes in the term $q(1-q)$, we thus have

$$\frac{\partial \mathcal{P}_a(\pi_a,\gamma_a,\pi_b,\gamma_b)}{\partial R_a^{1,i}} = 2q\left[\frac{\partial q}{\partial R_a^{1,i}}(\bar{S}^{2,i} - 2R_a^{2,i}) - (1-q)\right], \tag{26}$$

where, at a symmetric profile with $\pi = 1$ we have $q = \frac{1}{2}$ and $\frac{\partial q}{\partial R_a^{1,i}} = \frac{1}{4(R_a^{1,i} - R_a^{2,i})}$. Thus

$$\begin{aligned}\frac{\partial \mathcal{P}_a(\pi_a,\gamma_a,\pi_b,\gamma_b)}{\partial R_a^{1,i}} &= \left(\frac{1}{4}\right)\frac{\bar{S}^{2,i} - 2R_a^{2,i}}{R_a^{1,i} - R_a^{2,i}} - \frac{1}{2}\\ &= 0 \quad \text{only when } R^{1,i} = \frac{\bar{S}^{2,i}}{2}.\end{aligned}$$

In the same way, it can be computed that $\frac{\partial \mathcal{P}_a(\pi_a,\gamma_a,\pi_b,\gamma_b)}{\partial R_a^{2,i}} = 0$ only when $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$. That is, the same condition holds for marginal changes in expected rents in both one-buyer and two-buyer states. Since $\frac{\partial \mathcal{P}_a(\pi_a,\gamma_a,\pi_b,\gamma_b)}{\partial R_a^{2,i}} = 0$ and $\frac{\partial \mathcal{P}_a(\pi_a,\gamma_a,\pi_b,\gamma_b)}{\partial R_a^{1,i}} = 0$ yield the same condition, we need to worry about the existence of derivatives only when $r^{1,i} = r^{2,i} = 0$. But then an argument considering deviations $R_a^{1,i} + \mathrm{d}R_a^{1,i}$ or $R_a^{2,i} + \mathrm{d}R_a^{2,i}$ yields the result.

□

Sufficiency: Fixing some profile that satisfies the assumptions of the proposition, we will first show that, fixing $\pi = 1$ and FAE, no deviation consisting of either individual or joint shifts (not necessarily local) in $R^{1,i}$ and $R^{2,i}$ can achieve higher profits. Since the candidate profile has full information and FAE, considering changes in rents where surplus in both states is maximized gives an upper bound on the profitability of deviations that involve the same changes in rents but that include a decrease in information provision and/or allocative efficiency.

Consider some profile with $\pi = 1$ and associated rents $R^{1,i} \geq R^{2,i}$. Consider a deviation profile for seller a where

$$\begin{aligned}\hat{R}_a^{1,\imath} &= R^{1,i} + \Delta^1\\ \hat{R}_a^{2,i} &= R^{2,i} + \Delta^2,\end{aligned}$$

where the Δ^η need not be positive. We have that the new level of traffic q^Δ is given by

$$\begin{aligned}q^\Delta &= \frac{(R^{1,\tau} - R^{2,i}) + \Delta^1}{2((R^{1,i} - R^{2,i})) + \Delta^1 - \Delta^2}\\ &= \frac{1}{2} + z\\ &\quad \text{with } z = \left(\frac{1}{2}\right)\frac{\Delta^1 + \Delta^2}{2((R^{1,i} - R^{2,i})) + \Delta^1 - \Delta^2}.\end{aligned}$$

The difference in profits can be written as

$$\begin{aligned}
\mathcal{P}_a(1,\hat{\gamma_a},1,\gamma_b) - \mathcal{P}_a(1,\gamma_a,1,\gamma_b) &= \left[\bar{S}^{2,i} - 2R^{2,i}\right](x(x+1)) - 2\left[m - R^{1,i}\right]x^2 \\
&\quad - 2\Delta^2\left(\frac{1}{2}+x\right)^2 - 2\Delta^1\left(\frac{1}{2}+x\right)\left(\frac{1}{2}-x\right) \\
&= C\Bigg[\left[\bar{S}^{2,i} - 2R^{2,i}\right]\left(4((R^{1,i}-R^{2,i})) + 3\Delta^1 - \Delta^2\right)\left(\Delta^1+\Delta^2\right) \\
&\quad - 2\left[m - R^{1,i}\right]\left(\Delta^1+\Delta^2\right)^2 \\
&\quad - 8\left((R^{1,i}-R^{2,i})\right)\left((R^{1,i}-R^{2,i}) + \Delta^1\right)\left(\Delta^1+\Delta^2\right)\Bigg],
\end{aligned}$$

where $C = \left(\frac{1}{4}\right)\left[\frac{1}{2(R^{1,i}-R^{2,i}))+\Delta^1-\Delta^2}\right]^2 > 0$. Let us then set the original candidate profile as

$$\begin{aligned}
R^{1,1} &= \frac{\bar{S}^{2,i}}{2} \\
R^{2,i} &= \frac{\bar{S}^{2,i}}{2} - \epsilon, \text{ for } \epsilon \geq 0.
\end{aligned}$$

We can then simplify the profit difference to obtain

$$\begin{aligned}
\mathcal{P}_a(1,\hat{\gamma_a},1,\gamma_b) - \mathcal{P}_a(1,\gamma_a,1,\gamma_b) &= C\left[(\Delta^1+\Delta^2)^2(-2\epsilon - (2m - \bar{S}^{2,i}))\right] \\
&< 0 \text{ for any } (\Delta^1,\Delta^2), \text{ since } \epsilon > 0 \text{ and } 2m > \bar{S}^{2,i}.
\end{aligned}$$

Thus no deviations are profitable.

□

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