Stochastic Mechanisms in Settings without Monetary Transfers: The Regular Case*

Eugen Kováč[†] Tymofiy Mylovanov[‡]

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Abstract

We analyze relative performance of stochastic and deterministic mechanisms in an environment that has been extensively studied in the literature on communication (e.g., Crawford and Sobel, 1982) and optimal delegation (e.g., Holmström, 1984): a principal-agent model with hidden information, no monetary transfers, and single-peaked preferences. We demonstrate that under the common assumption of quadratic payoffs and a certain regularity condition on the distribution of private information and the agent's bias, the optimal mechanism is deterministic. We also provide an explicit characterization of this mechanism.

JEL codes: D78, D82, L22, M54.

Keywords: optimal delegation, cheap talk, principal-agent relationship, no monetary transfers, stochastic mechanisms.

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[†]Department of Economics, University of Bonn and CERGE-EI, Charles University, Prague; e-mail: `eugen.kovac@uni-bonn.de`.

[‡]Department of Economics, University of Bonn and Kyiv School of Economics; e-mail: `mylovanov@uni-bonn.de`.

1 Introduction

The literature on optimal mechanisms in the principal-agent model with hidden information, no monetary transfers, and single-peaked preferences has restricted its attention to deterministic mechanisms (Alonso and Matouschek [3], Holmström [17], [18], Martimort and Semenov [30], and Melumad and Shibano [31]). This restriction may contain some loss of generality since stochastic mechanisms can outperform deterministic ones.[1] The purpose of our paper is to address this very question.

Our main results are obtained for the setting in which both parties have quadratic preferences, the most frequently studied setting in the literature.[2] In Proposition 1 we show that, under a certain regularity condition, the optimal *stochastic* mechanism is deterministic; we then explicitly characterize this mechanism. The regularity condition in this proposition is satisfied in most applications and is similar to the regularity condition in the optimal auction in Myerson [35], which requires virtual valuation function to be monotone.

In addition, we also consider a setting in which the principal's preferences are given by an absolute value loss function and the agent's preferences are quadratic. We show that stochastic mechanisms perform strictly better than deterministic ones (Proposition 3) and can implement outcomes *arbitrarily close* to the first-best (Proposition 4). In this setting, the parties' payoffs have different degrees of curvature: the agent has a quadratic loss function, whereas the principal has an absolute value loss function. This allows the principal to use variance to improve an agent's incentives without imposing any cost on itself; this is not possible if the principal's preferences are also quadratic.

The characterization of the optimal mechanism in Proposition 1 for the case of quadratic preferences is closely related to the existing results for deterministic mechanisms: In the settings in which the regularity condition is satisfied, this proposition implies Proposition 3 in Alonso and Matouschek [3] (henceforth, AM); Propositions 2–3 in Martimort and Semenov [30] (henceforth, MS); and Proposition 3 in Melumad and Shibano [31].[3] Hence, our results complement the existing literature by showing that optimal deterministic mechanisms are also optimal in the *entire* set of incentive-compatible mechanisms, including the ones that result in stochastic allocations.

[1] For example, this is the case in the standard principal-agent model with monetary transfers (Stiglitz [37], Arnott and Stiglitz [6], and Strausz [38]).

[2] The setting with quadratic preferences is the leading example in Crawford and Sobel [10], having been applied in models in political science, finance, monetary policy, design of organizations, etc. Quadratic preferences have recently been used (i) as the main framework in Alonso [1], Alonso, Dessein and Matouschek [2], Alonso and Matouschek [4], Ambrus and Takahashi [5], Dessein and Santos [12], Ganguly and Ray [14], Goltsman, Hörner, Pavlov, and Squintani [15], Kraehmer [23], Krishna and Morgan [24], [26], Li [29], Li and Madarasz [28], Morgan and Stocken [33], Morris [34], Ottaviani and Squintani [36], and Vidal [40], and (ii) to obtain more specific results in Blume, Board, and Kawamura [8], Chakraborty and Harbaugh [9], Dessein [11], Gordon [16], Ivanov [19], Kartik, Ottaviani and Squintani [21], Kawamura [22], Krishna and Morgan [25], [27], and Szalay [39]. For a survey of the earlier literature, see Ganguly and Ray [13].

[3] AM and Melumad and Shibano [31] provide results for the case in which our regularity condition does not hold. Moreover, the environment in AM is more general than in this paper because their results do not require quadratic preferences of the agent.

In a related paper, Goltsman, Hörner, Pavlov, and Squintani [15] (henceforth GHPS) study optimal communication rules that transform messages from the agent into recommendations for the principal; these rules can be interpreted as an outcome of communication through an arbitrator. In the benchmark case, they allow the principal to commit to follow the recommendations of the rule, which is equivalent to commitment to a stochastic mechanism, and demonstrate a result similar to Proposition 1 in this paper. Nevertheless, our results have been obtained independently and our methods of proof differ. Furthermore, the results in GHPS are obtained for a setting with a uniform distribution of private information and a constant bias of the agent. Proposition 1 in this paper allows for a broader set of distributions and conflict of preferences.

A growing body of literature studies multiple extensions of cheap talk communication (Crawford and Sobel [10]): Krishna and Morgan [26], for example, consider two rounds of communication. Ganguly and Ray [14], GHPS, and Krishna and Morgan [26] analyze communication through a mediator. In Blume, Board, and Kawamura [8] and Kawamura [22] there is exogenous noise added to the messages of the agent. This literature often assumes quadratic preferences and identifies equilibria that are Pareto superior to the equilibria in Crawford and Sobel [10]. In these equilibria, the players' behavior induces a lottery over decisions. By contrast, the equilibrium allocation in Crawford and Sobel [10] is deterministic. In these models, the principal has limited commitment power and must make decisions that are optimal given available information. This raises a question as to whether optimal stochastic allocations can also outperform optimal deterministic ones if the principal has full commitment power and could choose any mechanism. Proposition 1 in this paper and Theorem 1 in GHPS answer this question, in the settings considered, negatively.

The technical approach in this paper is distinct from the rest of the literature on optimal mechanisms in settings with single-peaked preferences. In AM, for example, the optimal deterministic mechanism is derived by considering the effects of adding and removing decisions available to an agent within a mechanism. As they observe, this is equivalent to a difficult optimization problem over the power set of available decisions. We do not know how to extend their method to stochastic mechanisms considered in this paper. In a setting with single-peaked preferences and monetary transfers, Krishna and Morgan [27] characterize the optimal deterministic mechanism using optimal control. Their method might be applicable to our setting. It would require, however, a technical assumption that an optimal allocation is piecewise differentiable. The arguments in this paper prove simpler; they do not deal with power sets, do not require piecewise differentiability of an allocation, and avoid the use of optimal control and differential equations. At the same time, we specifically focus our attention on quadratic payoffs.

On the other hand, our approach is similar to the one in the optimal auction literature (e.g., Myerson [35]). For instance, a byproduct of our proof is a characterization of incentive-compatible mechanisms, analogous to the one in the literature on mechanism design with monetary transfers.[4] Nevertheless, there is an additional

[4]The necessary and sufficient conditions for incentive compatibility of deterministic mechanisms in the setting without transfers are given in MS and Melumad and Shibano [31].

difficulty caused by the constraint of non-negative variance. We resolve this difficulty by expressing the principal's payoff in terms of a derivative of a function whose value can be interpreted analogously to virtual valuation in an auction environment. The regularity condition requires this function to be monotone, which ensures that the optimal mechanism is deterministic.[5] Our method relies on the regularity condition and it cannot be directly extended to the settings in which the condition does not hold. A complete characterization of optimal stochastic mechanisms in such cases remains an open problem.

The remainder of the paper is organized as follows: Section 2 introduces the model. The results for quadratic preferences are presented in Section 3, starting with an example in which the private information of the agent is uniformly distributed and the parties have a constant conflict of preferences. Section 4 considers the setting in which the principal has an absolute value loss payoff function. Section 5 provides conclusions. Note that the proofs omitted in the main text are presented in Appendix A.

2 Environment

There is a principal (she) and an agent (he). The agent has one-dimensional private information $\omega \in \mathbb{R}$ called a state of the world. The principal's prior beliefs about ω are represented by a cumulative distribution function $F(\omega)$ with support $\Omega = [0, 1]$ and an atomless density $f(\omega)$ that is positive and absolutely continuous on Ω. The parties must make a decision $p \in \mathbb{R}$. There are no outside options. The agent has a quadratic loss function, $u_a(p, \omega) = -(p - \omega)^2$. We will consider two versions of the principal's payoff: In Section 3 we consider a principal with a quadratic loss function, $u_p(p, \omega) = -[p - z(\omega)]^2$. In Section 4 we assume that the principal has an absolute value loss function, $u_p(p, \omega) = -|p - z(\omega)|$. Moreover, we assume that function $z : \Omega \to \mathbb{R}$ is absolutely continuous function. The value $z(\omega)$ represents principal's ideal decision in state ω. The difference $b(\omega) = \omega - z(\omega)$ then represents the agent's bias.

Let $\mathcal{P}$ be the set of probability distributions on $\mathbb{R}$ with a finite variance. An *allocation* M is a (Borel measurable) function $M : \Omega \to \mathcal{P}$ that maps the agent's information into a lottery over decisions. An allocation M is deterministic if for every $\omega \in \Omega$ the lottery $M(\omega)$ implements one decision with certainty. Let $\mathcal{M}$ denote the set of all allocations.

An allocation has two interpretations. First, it can describe the outcome of interaction of the agent and the principal in some game. Second, it can describe a decision problem for the agent in which he chooses a report $\omega \in \Omega$ and obtains a lottery $M(\omega)$ over p. If this interpretation is used, we call M a (direct) *mechanism*. Let $\mathbb{E}^{M(\omega)}$ denote the expectation operator associated with lottery $M(\omega)$.

[5]Our regularity condition is connected with conditions used in AM and MS. It is also related to the sufficient condition for the optimality of deterministic mechanisms in the principal-agent problem with monetary transfers as found in Strausz [38]. We discuss the relationship between our results and those in the existing literature in detail in Section 5.

A function $r : \Omega \to \Omega$ that maps the agent's information into a report is an *equilibrium* in a direct mechanism M if it maximizes the agent's expected payoff, i.e.,

$$r(\omega) \in \arg\max_{s\in\Omega} \mathbb{E}^{M(s)} u_a(p,\omega) \quad \text{for all } \omega \in \Omega.$$

An allocation M is *incentive-compatible* if truth-telling, i.e., $r(\omega) = \omega$ for all $\omega \in \Omega$, is an equilibrium in the mechanism M. By the Revelation Principle we can restrict attention to incentive-compatible allocations.

Consider an allocation M and let $\mu^M(\omega) = \mathbb{E}^{M(\omega)} p$ and $\tau^M(\omega) = \mathrm{Var}^{M(\omega)}\, p$ denote the expected decision and the variance of the lottery $M(\omega)$. Allocation M is deterministic if and only if $\tau^M(\omega) = 0$ for all $\omega \in [0,1]$. Since the agent's loss function is quadratic, his payoff in a state ω from a report ω' in the mechanism M can be expressed using μ^M and τ^M:

$$U_a^M(\omega,\omega') = \mathbb{E}^{M(\omega')} u_a(p,\omega) = -[\mu^M(\omega') - \omega]^2 - \tau^M(\omega'). \tag{1}$$

In addition, let $V_a^M(\omega) = U_a^M(\omega,\omega)$ denote the agent's expected payoff from truth-telling if the state is ω. The following lemma provides a characterization of incentive-compatible allocations in terms of $(\mu^M,\ \tau^M)$.

Lemma 1. *Functions μ^M and τ^M represent an incentive-compatible allocation M if and only if:*

(IC_1) *μ^M is non-decreasing,*

(IC_2) *for all $\omega \in \Omega$:*

$$V_a^M(\omega) - V_a^M(0) = 2\int_0^\omega [\mu^M(s) - s]\,\mathrm{d}s,$$

(VAR) *$\tau^M(\omega) \geq 0$ for all $\omega \in \Omega$.*

Proof. See Appendix A.

Condition (IC_1) is a standard monotonicity condition in mechanism design. Condition (IC_2) follows from the *Integral form of Envelope Theorem* (Milgrom [32]). Condition (VAR) is a feasibility condition that requires variance to be non-negative.

We now provide a geometric interpretation of Lemma 1. Consider, for example, an allocation M in which the expected decision and the variance of the lottery are given by

$$\big(\mu^M(\omega),\tau^M(\omega)\big) = \begin{cases} (\mu_1,\tau_1)\,, & \omega < \omega_1; \\ (\mu_2,\tau_2)\,, & \omega_1 \leq \omega \leq \omega_2; \\ (\mu_3,\tau_3)\,, & \omega > \omega_2; \end{cases}$$

where $(\mu_1,\tau_1) = (0.2, 0.07)$, $(\mu_2,\tau_2) = (0.5, 0.1)$, and $(\mu_3,\tau_3) = (0.9, 0)$, and $\omega_1 = 0.4$ and $\omega_2 = 0.575$. This allocation is incentive-compatible. In Figure 1, we depict the agent's payoff in this allocation (the axes represent the agent's type, ω, and the agent's payoff, $U_a^M(\omega,\omega')$). First, observe that the agent's payoff is given by one of

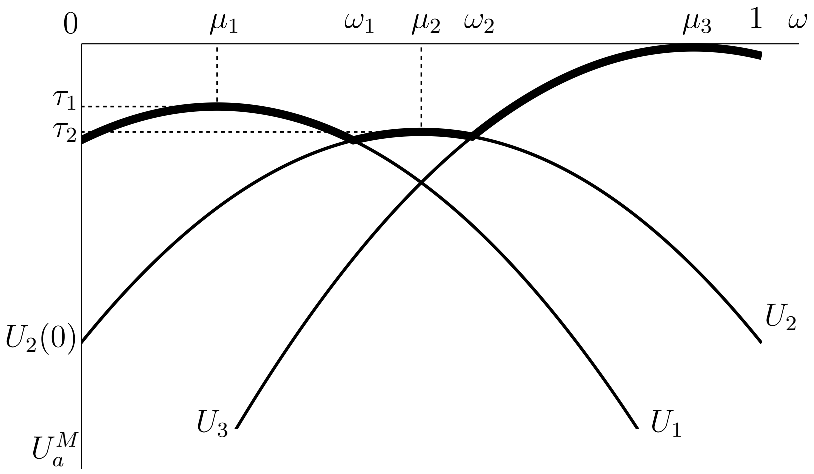

Figure 1: Agent's payoff in an incentive-compatible allocation.

the three parabolas, U_1, U_2, and U_3, which achieve their maximum at (μ_1, τ_1), (μ_2, τ_2), and (μ_3, τ_3) respectively. For example, if the agent's type is $\omega = 0$ and he reports ω' such that $\omega_1 \leq \omega' \leq \omega_2$, his payoff is $U_2(0) = -\mu_2^2 - \tau_2$.

In the (truth-telling) equilibrium, the agent chooses a report that maximizes his expected payoff. Thus, the payoff of the agent, $V_a^M(\omega)$, is given by the bold curve in Figure 1, the upper envelope of the three parabolas.

We are now ready to interpret the conditions set forth in Lemma 1. First, consider two different agent's types, $\underline{\omega}$ and $\overline{\omega}$, where $\underline{\omega} < \overline{\omega}$. It follows then that the agent's payoff is given either (i) by the same parabola or (ii) by two different parabolas, in which case the parabola corresponding to $\underline{\omega}$ achieves its maximum at a lower ω than the the parabola corresponding to $\overline{\omega}$. This is condition (IC_1), which requires $\mu^M(\omega)$ to be non-decreasing. Second, for almost all ω, the slope of the agent's payoff is equal to the slope of the parabola corresponding to his type and is linear in $\mu^M(\omega)$. This is condition (IC_2). Finally, the feasibility condition (VAR) implies that the parabolas cannot have values above the horizontal axis. In a deterministic allocation, all parabolas are tangent to the horizontal axis.

Let $\mathcal{M}^c$ denote the set of all incentive-compatible allocations M where both $\mu^M(\omega)$ and $\tau^M(\omega)$ are continuous from the right at $\omega = 0$ and continuous from the left at $\omega = 1$. In what follows, without loss of generality, we restrict our analysis to incentive-compatible allocations in $\mathcal{M}^c$.[6]

We would like to characterize incentive-compatible allocations that maximize the expected payoff of the principal. An allocation M is *optimal* if it is a solution of the following program

$$\text{(E)} \max_{M \in \mathcal{M}^c} \mathbb{E} u_p(p, \omega),$$

where $\mathbb{E}$ denotes the expectation operator associated with the cumulative distribution

[6]It is straightforward to show that if M is an incentive-compatible allocation, then so is allocation M^c in which $\left(\mu^{M^c}(0), \tau^{M^c}(0)\right) = \left(\mu^M(0^+), \tau^M(0^+)\right)$, $\left(\mu^{M^c}(1), \tau^{M^c}(1)\right) = \left(\mu^M(1^-), \tau^M(1^-)\right)$, and $\left(\mu^{M^c}(\omega), \tau^{M^c}(\omega)\right) = \left(\mu^M(\omega), \tau^M(\omega)\right)$ for all $\omega \in (0, 1)$.

function F. An optimal allocation maximizes the principal's ex ante payoff on the set of incentive-compatible allocations. As illustrated by Proposition 4 in Section 4, an optimal allocation might fail to exist in some settings.

3 Quadratic payoffs

In this section we consider a principal with a quadratic loss function

$$u_p(p, \omega) = -[p - z(\omega)]^2.$$

Let M be an incentive-compatible allocation. Because the principal's loss function is quadratic, her payoff given the allocation in a state ω can also be expressed as a function of the expected decision $\mu^M(\omega)$ and the variance $\tau^M(\omega)$ of the lottery $M(\omega)$,

$$U_p^M(\omega) = \mathbb{E}^{M(\omega)} u_p(p, \omega) = -[\mu^M(\omega) - z(\omega)]^2 - \tau^M(\omega).$$

As a result, the difference between the payoffs of the principal and the agent is independent of τ^M and can be written in the form:

$$U_p^M(\omega) - V_a^M(\omega) = 2[z(\omega) - \omega]\mu^M(\omega) + \omega^2 - [z(\omega)]^2 =: \delta^M(\omega). \tag{2}$$

Hence, the payoff of the principal in a given state in an incentive-compatible allocation can be thought of as consisting of two terms, $U_p^M(\omega) = V_a^M(\omega) + \delta^M(\omega)$, where $V_a^M(\omega)$ represents the common interest of the parties and $\delta^M(\omega)$ represents the conflict of interest between the parties. By taking the expectation we obtain the principal's ex ante expected payoff from allocation M:

$$V_p^M = \int_0^1 U_p^M(\omega) f(\omega)\, \mathrm{d}\omega = \int_0^1 [V_a^M(\omega) + \delta^M(\omega)] f(\omega)\, \mathrm{d}\omega. \tag{3}$$

3.1 Example: uniform distribution and constant bias

In this section, we illustrate the main ideas behind our analysis in a setting with uniform distribution, $f(\omega) = 1$ on $[0, 1]$, and a constant agent's bias $\omega - z(\omega) = b$, where $b \in (0, \frac{1}{2})$. This setting is the leading example in Crawford and Sobel [10] and has been used extensively in the literature on communication and delegation. To facilitate the presentation, we skip some technical details; they are found in Section 3.2, where we present our results for more general environments.

Let us denote $\beta_0 = 1 - 2b$. We derive the optimal allocation in three steps:

1. We split the principal's ex ante expected payoff in two parts — by computing the integral on intervals $[0, \beta_0]$ and $(\beta_0, 1]$.

2. On interval $(\beta_0, 1]$: We show that both $\mu^M(\omega)$ and $\tau^M(\omega)$ are necessarily constant in optimal allocation.

3. On interval $[0, \beta_0]$: We show that $\mu^M(\omega) = \omega$ and $\tau^M(\omega) = 0$ in optimal allocation.

We now describe these steps in greater detail.

Step 1. In the first step, we observe that the principal's ex ante expected payoff in an incentive-compatible allocation can be expressed as

$$V_p^M = bV_a^M(0) + bV_a^M(\beta_0) + \int_0^{\beta_0} V_a^M(\omega)\,\mathrm{d}\omega + \int_{\beta_0}^1 (1-\omega-b)\mu^M(\omega)\,\mathrm{d}\omega + C, \qquad (4)$$

where $C = -(1-\frac{4}{3}b)b^2$ is a constant that does not depend on allocation M.

More precisely, on interval $[0,\beta_0)$ we use (IC_2) to evaluate

$$\int_0^{\beta_0} \delta^M(\omega)\,\mathrm{d}\omega = -2b\int_0^{\beta_0}[\mu^M(\omega)-\omega]\,\mathrm{d}\omega - \beta_0 b^2 = \\ = b\left[V_a^M(0) - V_a^M(\beta_0)\right] + C_2,$$

where $C_2 = -(1-2b)b^2$. This computation can also be expressed as $b\int_0^{\beta_0}\mathrm{d}V_a^M(\omega) = bV_a^M(\beta_0) - bV_a^M(0)$.

On interval $[\beta_0, 1]$, we use integration by parts $\int_{\beta_0}^1 V_a^M(\omega)\,\mathrm{d}\omega = V_a^M(1) - \beta_0 V_a^M(\beta_0) - \int_{\beta_0}^1 \omega\,\mathrm{d}V_a^M(\omega)$ and (IC_2) to obtain

$$\int_{\beta_0}^1 [V_a^M(\omega) + \delta^M(\omega)]\,\mathrm{d}\omega = 2bV_a^M(\beta_0) + 2\int_{\beta_0}^1 (1-b-\omega)\mu^M(\omega)\,\mathrm{d}\omega + C_3,$$

where $C_3 = -\frac{2}{3}b^3$.

Step 2. In the second step, we observe that $\int_{\beta_0}^1 (1-b-\omega)\,\mathrm{d}\omega = 0$. Thus, $\int_{\beta_0}^1 (1-b-\omega)\mu^M(\omega)\,\mathrm{d}\omega$ is equal to zero when μ^M is constant on $(\beta_0, 1]$. Furthermore, it is negative otherwise. To see this, note that $1-b-\omega$ is positive on $[\beta_0, 1-b)$ and negative on $(1-b, 1]$. Hence, due to monotonicity of μ^M we obtain

$$\int_{\beta_0}^1 (1-b-\omega)\mu^M(\omega)\,\mathrm{d}\omega < \int_{\beta_0}^1 (1-b-\omega)\mu^M(1-b)\,\mathrm{d}\omega = 0.$$

Therefore, an optimal allocation implements a constant lottery on $(\beta_0, 1]$. Indeed, let M be non-constant on this interval and define $\overline{M}$ to be a truncated allocation that coincides with M for $\omega \in [0,\beta_0]$ and implements a lottery characterized by $\left(\mu^M(\beta_0), \tau^M(\beta_0)\right)$ for $\omega \in (\beta_0, 1]$. The allocation $\overline{M}$ is then incentive-compatible. Moreover, it follows from (4) that the principal obtains a higher payoff in $\overline{M}$.

Step 3. Let M be an incentive-compatible allocation that is constant on $(\beta_0, 1]$. Then, from (4) the principal's ex ante expected payoff in M is a weighted sum of the agent's expected payoff in states $\omega \in [0,\beta_0]$, in which all weights are positive,

$$V_p^M = bV_a^M(0) + bV_a^M(\beta_0) + \int_0^{\beta_0} V_a^M(\omega)\,\mathrm{d}\omega + C \le C,$$

where the inequality follows from the fact that $V_a(\omega) \le 0$. Moreover, the value C can be achieved if $V_a^M(\omega) = 0$ for all $\omega \in [0,\beta_0]$.

Thus, the ex ante payoff of the principal is maximized by the deterministic allocation that implements the agent's most preferred decision on interval $[0,\beta_0]$ and is constant on interval $[\beta_0, 1]$:

$$\mu^M(\omega) = \max\{\omega, 1-2b\}, \qquad \tau^M(\omega) = 0.$$

3.2 General case

The approach in the previous section can be extended to more general environments. Let

$$g(\omega) = 1 - F(\omega) + [z(\omega) - \omega]f(\omega).$$

Note that function g depends only on the primitives of the model and is absolutely continuous.

Now consider some $\alpha, \beta \in [0,1]$. Using calculations similar to the ones in the previous section, we can express the payoff of the principal in an incentive-compatible allocation M on intervals $[0,\alpha]$, $[\alpha,\beta]$, and $[\beta,1]$ as

$$\int_0^\alpha U_p^M(\omega)f(\omega)\,\mathrm{d}\omega = F(\alpha)V_a^M(\alpha) + 2\int_0^\alpha [g(\omega)-1]\mu^M(\omega)\,\mathrm{d}\omega + C_1(\alpha), \tag{5}$$

$$\begin{aligned}\int_\alpha^\beta U_p^M(\omega)f(\omega)\,\mathrm{d}\omega = {}& [g(\beta)+F(\beta)-1]V_a^M(\beta) - [g(\alpha)+F(\alpha)-1]V_a^M(\alpha)\\ &- \int_\alpha^\beta g'(\omega)V_a^M(\omega)\,\mathrm{d}\omega + C_2(\alpha,\beta),\end{aligned} \tag{6}$$

$$\int_\beta^1 U_p^M(\omega)f(\omega)\,\mathrm{d}\omega = [1-F(\beta)]V_a^M(\beta) + 2\int_\beta^1 g(\omega)\mu^M(\omega)\,\mathrm{d}\omega + C_3(\beta), \tag{7}$$

where $C_1(\alpha) = \int_0^\alpha \left[\alpha^2 - \big(z(\omega)\big)^2\right]\mathrm{d}\omega$, $C_2(\alpha,\beta) = -\int_\alpha^\beta [z(\omega)-\omega]^2 f(\omega)\,\mathrm{d}\omega$, and $C_3(\beta) = \int_\beta^1 \left[\beta^2 - \big((z(\omega)\big)^2\right]f(\omega)\,\mathrm{d}\omega$ are constants, which do not depend on allocation M.

Formulas (5) and (7) are generalizations of the expression of the principal's payoff on interval $(\beta_0, 1]$ in the previous section. They can be obtained directly through the substitution of the value of $V_a(\omega)$ from (IC_2) into (2) and taking the expectation over ω. These formulas express the principal's ex ante expected utility as a linear combination of the expected decision and the agent's payoff at a boundary of the interval.

Formula (6) is a generalization of the expression of the principal's payoff on interval $[0,\beta_0]$ in the previous section. It can be obtained by evaluating the term representing the conflict of interests, δ^M, using the following integration by parts:[7]

$$\begin{aligned}\int_\alpha^\beta \delta^M(\omega)f(\omega)\,\mathrm{d}\omega &= 2\int_\alpha^\beta [z(\omega)-\omega]f(\omega)[\mu^M(\omega)-\omega]\,\mathrm{d}\omega + C_2(\alpha,\beta) =\\ &\overset{(\mathrm{IC}_2)}{=} [g(\beta)+F(\beta)-1]V_a^M(\beta) - [g(\alpha)+F(\alpha)-1]V_a^M(\alpha)\\ &\quad - \int_\alpha^\beta [g'(\omega)+f(\omega)]V_a^M(\omega)\,\mathrm{d}\omega + C_2(\alpha,\beta).\end{aligned}$$

This formula expresses the principal's ex ante expected payoff as a linear combination of the agent's utilities in different states of the world.

[7] Note that this computation is nothing other than integration by parts for the *Riemann-Stieltjes integral*: $\int_\alpha^\beta [g(\omega)+F(\omega)-1]\,\mathrm{d}V_a^M(\omega) = [g(\beta)+F(\beta)-1]V_a^M(\beta) - [g(\alpha)+F(\alpha)-1]V_a^M(\alpha) - \int_\alpha^\beta V_a^M(\omega)\,\mathrm{d}[g(\omega)+F(\omega)-1]$.

Adding up equalities (5)–(7), we arrive at a generalization of formula (4) obtained in Step 1 in Section 3.1:

$$V_p^M = g(\beta)V_a^M(\beta) + [1-g(\alpha)]V_a^M(\alpha) - \int_\alpha^\beta g'(\omega)\, V_a^M(\omega)\,\mathrm{d}\omega \qquad (8)$$
$$+ 2\int_0^\alpha \mu^M(\omega)[g(\omega)-1]\,\mathrm{d}\omega + 2\int_\beta^1 \mu^M(\omega)g(\omega)\,\mathrm{d}\omega + C(\alpha,\beta),$$

where $C(\alpha,\beta) = C_1(\alpha) + C_2(\alpha,\beta) + C_3(\beta) = \alpha^2 + 2\int_\alpha^\beta \omega g(\omega)\,\mathrm{d}\omega - \mathbb{E}[z(\omega)]^2$.

In order to characterize an optimal allocation, we impose a regularity condition on function g, which relates the distribution of the agent's private information to the conflict of preference between the parties.

Assumption 1. *If $0 \leq g(\omega) \leq 1$, then g is decreasing at point ω.*[8]

Before we interpret Assumption 1, observe that it is satisfied in a broad class of environments. For example, it holds in the settings in which the agent's bias $b(\omega) = \omega - z(\omega)$ is positive and non-decreasing and the distribution function F has an increasing hazard rate $f(\omega)/[1-F(\omega)]$.[9] In particular, it is satisfied in the setting with uniform distribution and a constant bias considered in the previous section.

Similarly, Assumption 1 holds if the agent's bias is negative and non-increasing and the distribution function F is strictly log-concave (or, equivalently, $f(\omega)/F(\omega)$ is decreasing).[10] Observe also that Assumption 1 is satisfied if the agent's bias is zero, i.e., $z(\omega) = \omega$ (in this case, $g(\omega) = 1 - F(\omega)$). In addition, as we show in Proposition 2 below, it also holds if the preferences of the agent and the principal are sufficiently close. In this case, $z(\omega)$ may not necessarily be monotone and the agent's bias may change sign at several points.

Assumption 1 is somewhat similar to the regularity condition in the optimal auction setting in Myerson [35] that requires $[1-F(\omega)]/f(\omega) + z(\omega) - \omega = g(\omega)/f(\omega)$ to be decreasing. In Myerson's setting ω is the valuation of the buyer and $z(\omega)$ is a revision effect function (Myerson [35], p. 60) that captures the effect of ω on the payoffs of other players. Finally, Assumption 1 is related to some of conditions used in AM and MS. The relationship between our analysis and that in Myerson, AM, and MS will be discussed in more detail in Section 5.

We now provide a geometric interpretation of Assumption 1. A possible shape of function g is shown in Figure 2. First, Assumption 1 implies a single-crossing property: the graph of g intersects the line $y = 0$ and the line $y = 1$ once at the most.[11] In order to observe this, imagine that the graph of g intersects, for example, the line $y = 1$ more than once. Then, by continuity of g, it is impossible that g be decreasing at all points of the intersection, contradicting Assumption 1. A consequence of the

[8]We say that function g is *decreasing at point* ω if there exists some open neighborhood O of ω such that for all $\omega' \in \Omega \cap O$: If $\omega' < \omega$, then $g(\omega') < g(\omega)$, and if $\omega' > \omega$, then $g(\omega') > g(\omega)$. An alternative, stronger, definition would be to require the function g to be decreasing in some neighborhood of this point.

[9]In order to observe this, write $g(\omega) = [1-F(\omega)]\big[1 - b(\omega)f(\omega)/\big(1-F(\omega)\big)\big]$.

[10]Similar to the previous case, we may write $g(\omega) = 1 - F(\omega)[1 + b(\omega)f(\omega)/F(\omega)]$.

[11]More precisely, function g attains the values of 0 and 1 at the most once.

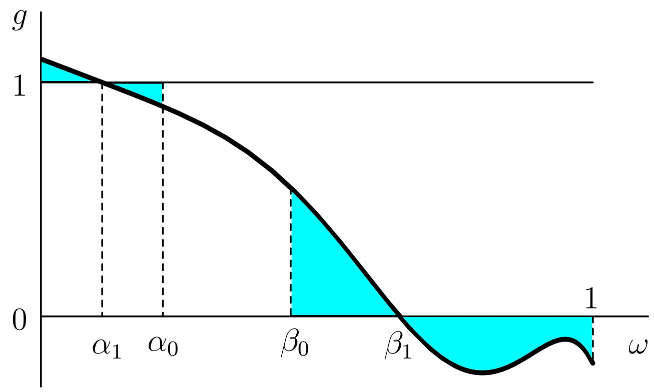

Figure 2: Interpretation of the regularity condition.

single-crossing property is that the set of states in which $0 \leq g(\omega) \leq 1$ is convex. Furthermore, by Assumption 1 function g is decreasing on this set.

The regularity condition allows us to derive an optimal allocation using a procedure analogous to the one in the example in the previous section. Let

$$\alpha_1 = \min\big(\{\omega \in [0,1] : g(\omega) \leq 1\} \cup \{1\}\big),$$
$$\beta_1 = \max\big(\{\omega \in [0,1] : g(\omega) \geq 0\} \cup \{0\}\big).$$

The value of β_1 is the point of intersection of the graph of g and the horizontal line $y = 0$, if this point exists. If g is always greater than 0, then $\beta_1 = 1$. If g is always less than 0, then $\beta_1 = 0$. Similarly, α_1 is the point of intersection of the graph of g and the horizontal line $y = 1$, if this point exists. If g is always less than 1, then $\alpha_1 = 0$. If g is always greater than 1, then $\alpha_1 = 1$.

In addition, let

$$\alpha_0 = \max\{\omega \in [0,1] : G(\omega) \geq \omega\},$$
$$\beta_0 = \min\{\omega \in [0,1] : G(\omega) \geq G(1)\},$$

where $G(\omega) = \int_0^\omega g(s)\,\mathrm{d}s$. Figure 2 illustrates the meaning of α_0 and β_0. If the graph of g and the line $y = 1$ intersect and $\alpha_0 < 1$, then by definition of α_0 the areas between these two functions on intervals $[0, \alpha_1]$ and $[\alpha_1, \alpha_0]$ are the same. Similarly, if the graph of g and the line $y = 0$ intersect and $\beta_0 > 0$, then the areas between these two functions on intervals $[\beta_0, \beta_1]$ and $[\beta_1, 1]$ are the same.[12]

The following lemma lists two implications of the regularity condition that are of particular interest to us.

Lemma 2. *Assumption 1 implies that:*

(R$_1$) $\alpha_1 \leq \alpha_0$ *and* $\beta_0 \leq \beta_1$*. In addition,* $g(\omega) > 1$ *on* $[0, \alpha_1)$ *and* $g(\omega) < 1$ *on* $(\alpha_1, \alpha_0]$*. Similarly,* $g(\omega) > 0$ *on* $[\beta_0, \beta_1)$ *and* $g(\omega) < 0$ *on* $(\beta_1, 1]$*.*

(R$_2$) *If* $\alpha_0 < \beta_0$*, then* $g(\beta_0) > 0$*,* $1 - g(\alpha_0) > 0$*, and* $-g'(\omega) > 0$ *almost everywhere on* (α_0, β_0)*.*

[12] For the special case in Section 3.1, we obtain $\alpha_1 = \alpha_0 = 0$, $\beta_0 = 1 - 2b$, and $\beta_1 = 1 - b$.

Proof. See Appendix A.

The first property in Lemma 2, (R_1), can be used to show that we can restrict attention to allocations that are constant on $[0, \alpha_0)$ and $(\beta_0, 1]$. The second property, (R_2), then implies that the optimal allocation maximizes the agent's payoff on $[\alpha_0, \beta_0]$.

We now make this argument more precise. Let M be an incentive-compatible allocation. If $\alpha_0 < \beta_0$, we define a new allocation

$$\overline{M}(\omega) := \begin{cases} M(\alpha_0), & \text{if } 0 \leq \omega < \alpha_0; \\ M(\omega), & \text{if } \alpha_0 \leq \omega \leq \beta_0; \\ M(\beta_0), & \text{if } \beta_0 < \omega \leq 1. \end{cases}$$

On the other hand, if $\alpha_0 \geq \beta_0$, we define $\overline{M}(\omega) = M(\beta_0)$ for all $\omega \in [0, 1]$.

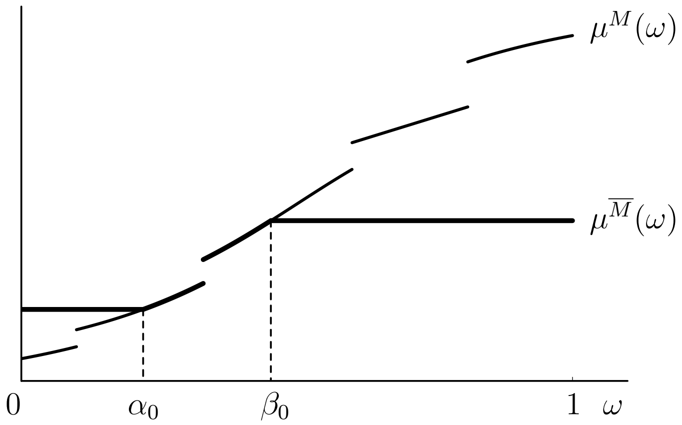

Figure 3: Expected decisions in allocations M and $\overline{M}$

The values of μ in allocations M and $\overline{M}$ (when $\alpha_0 < \beta_0$) are shown in Figure 3. In this case, allocation $\overline{M}$ is a truncated allocation that coincides with M for $\omega \in [\alpha_0, \beta_0]$ and implements a lottery characterized by $\big(\mu^M(\alpha_0), \tau^M(\alpha_0)\big)$ for $\omega \in [0, \alpha_0)$ and a lottery characterized by $\big(\mu^M(\beta_0), \tau^M(\beta_0)\big)$ for $\omega \in (\beta_0, 1]$. Observe that the truncation points α_0 and β_0 are the same for any incentive-compatible allocation M and depend only on the principal's prior beliefs and the agent's bias.

On the other hand, when $\alpha_0 \geq \beta_0$, allocation $\overline{M}$ is independent of ω.

Lemma 3. *If allocation $M \in \mathcal{M}^c$ is incentive-compatible, then allocation $\overline{M}$ is incentive-compatible and $V_p^{\overline{M}} \geq V_p^M$. If, in addition, $\mu^M(0) < \mu^M(\alpha_0)$ or $\mu^M(\beta_0) < \mu^M(1)$, then $V_p^{\overline{M}} > V_p^M$.*

Proof. See Appendix A.

Let $\overline{\mathcal{M}}$ be the set of all incentive-compatible allocations with μ^M constant on $[0, \alpha_0]$ and constant on $[\beta_0, 1]$. The above lemma establishes that we may restrict attention to allocations in $\overline{\mathcal{M}}$.

Moreover, Lemma 3 immediately implies that in an optimal allocation $\mu^M(\omega)$ is constant if $\alpha_0 \geq \beta_0$. Now let $\alpha_0 < \beta_0$. It follows from (8) and the definitions of α_0

and β_0 that the payoff in an allocation from $\overline{\mathcal{M}}$ is a linear combination of the agent's payoffs,

$$V_p^M = g(\beta_0)V_a^M(\beta_0) + [1 - g(\alpha_0)]V_a^M(\alpha_0) - \int_{\alpha_0}^{\beta_0} g'(\omega)\, V_a^M(\omega)\, \mathrm{d}\omega + C, \tag{9}$$

where $C = C(\alpha_0, \beta_0)$ is a constant, which does not depend on allocation M. By Lemma 2, the regularity condition implies that the coefficients $g(\beta_0)$, and $1 - g(\alpha_0)$ are positive, and the coefficient $-g'(\omega)$ is positive almost everywhere on (α_0, β_0). The following proposition states that an optimal allocation exists, is unique, and maximizes the agent's payoff on $[\alpha_0, \beta_0]$.

Proposition 1. *An optimal allocation in $\mathcal{M}^c$ exists and is unique. If $\alpha_0 < \beta_0$, then the optimal allocation M from $\mathcal{M}^c$ is deterministic. It implements the decision*

$$\mu^M(\omega) = \begin{cases} \alpha_0, & \text{if } 0 \le \omega < \alpha_0; \\ \omega, & \text{if } \alpha_0 \le \omega \le \beta_0; \\ \beta_0, & \text{if } \beta_0 < \omega \le 1. \end{cases} \tag{10}$$

If $\alpha_0 \ge \beta_0$, then the optimal allocation in $\mathcal{M}^c$ is deterministic and is independent of ω. It implements the decision $\mu^M(\omega) = \mathbb{E}z(\omega')$ for all $\omega \in [0, 1]$.

Proof. See Appendix A.

The optimal allocation in Proposition 1 is well-known to be optimal among *deterministic* allocations (Propositions 3–5 in AM, Proposition 3 in MS, and Proposition 3 in Melumad and Shibano [31]). It is also known to be optimal among stochastic allocations in the case of a uniform distribution and a constant bias (Theorem 1 in GHPS).

If $\alpha_0 \ge \beta_0$, the optimal allocation involves no communication and gives the principal the ex ante payoff of $-\operatorname{Var} z(\omega')$. In this case, the conflict of preferences between the parties is so severe that it is optimal for the principal to disregard the agent and make a decision based on her prior beliefs.

If $\alpha_0 < \beta_0$, the optimal allocation gives the principal the payoff of C. In this allocation, the implemented decision depends on the agent's information. It is equal to the agent's most preferred decision if $\omega \in (\alpha_0, \beta_0)$ and is independent of ω otherwise. Thus, the principal follows the agent's report for intermediate values and truncates it for extreme values. For the settings in which the regularity condition is satisfied, the following corollaries describe the conditions such that truncation occurs only from one side, or equivalently, when $\alpha_0 = 0$ or $\beta_0 = 1$.

Corollary 1 (no truncation from above). *The optimal allocation M in $\mathcal{M}^c$ implements $\mu^M(\omega) = \max\{\alpha_0, \omega\}$ for all $\omega \in [0, 1]$ if and only if $z(1) \ge 1$.*

Corollary 2 (no truncation from below). *The optimal allocation M in $\mathcal{M}^c$ implements $\mu^M(\omega) = \min\{\omega, \beta_0\}$ for all $\omega \in [0, 1]$ if and only if $z(0) \le 0$.*

Corollary 3 (full delegation). *The optimal allocation M in $\mathcal{M}^c$ implements $\mu^M(\omega) = \omega$ for all $\omega \in [0,1]$ if and only if $z(0) \leq 0$ and $z(1) \geq 1$.*

All corollaries follow directly from Proposition 1 and Lemma 2.

The next proposition demonstrates that Assumption 1 is satisfied if the parties' preferences are sufficiently aligned. It also provides comparative statics results for α_0 and β_0. In order to state the proposition, consider an absolutely continuous function $\tilde{z} : [0,1] \to \mathbb{R}$. Now let us analyze the principal's maximization problem (E) under the assumption that her optimal ideal decision is $z^\lambda(\omega) = \lambda\tilde{z}(\omega) + (1-\lambda)\omega$, where $\lambda \in [0,1]$.[13] In this case, $g^\lambda(\omega) = 1 - F(\omega) + \lambda[\tilde{z}(\omega) - \omega]f(\omega)$.

Proposition 2. *If both functions f and $\tilde{z}$ are differentiable and, moreover, have bounded derivatives on $[0,1]$, then:*

(i) *There exists some $\bar{\lambda} > 0$ such that g^λ satisfies Assumption 1 for all $\lambda \in (0, \bar{\lambda})$.*

(ii) *If $\tilde{z}(0) > 0$ and $0 < \lambda < \bar{\lambda}$, then α_0^λ is increasing in λ.*

(iii) *If $\tilde{z}(1) < 1$ and $0 < \lambda < \bar{\lambda}$, then β_0^λ is decreasing in λ.*

(iv) *If $\lambda \to 0$, then $\alpha_0^\lambda \to 0$ and $\beta_0^\lambda \to 1$.*

Proof. See Appendix A.

For the case of deterministic mechanisms, the result in part (i) of this proposition has been obtained in Proposition 4 in AM. Figure 4 illustrates the optimal allocation and the comparative statics. The thin dashed and solid lines represent the principal's ideal decision $\tilde{z}(\omega)$ and $\tilde{z}^\lambda(\omega)$. The thick dashed and solid lines represent the corresponding optimal allocations.

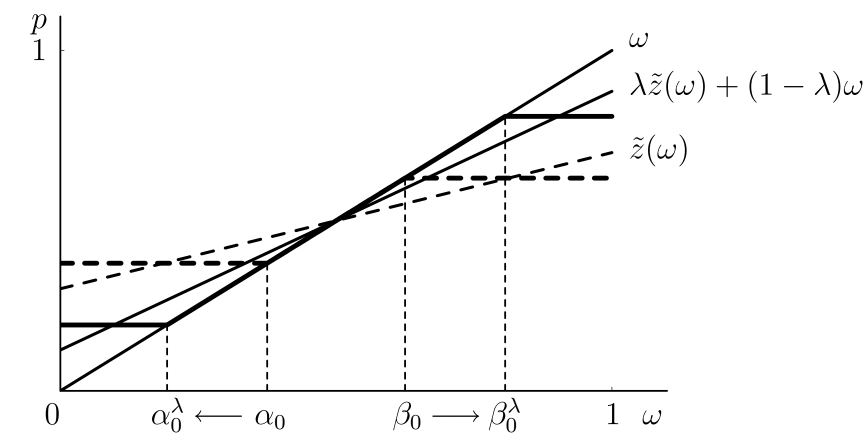

Figure 4: Optimal allocation and comparative statics

Our method of characterizing the optimal stochastic allocation relies on the regularity condition and cannot be directly extended to the settings in which the condition does not hold. First, without the regularity condition we cannot ensure the properties

[13]For this problem we will modify our notation by adding the superscript λ.

of function g listed in Lemma 2. This invalidates the argument behind the truncation result in Lemma 3. Furthermore, if the regularity condition fails, the principal might potentially benefit from truncating allocations inside interval $[0, 1]$. Because of the constraint of non-negative variance, it is not straightforward to define an incentive-compatible truncated allocation in this case. Finally, even if we restrict our attention to allocations that cannot be improved by truncation, we cannot guarantee that the principal's payoff is a weighted sum of the agent's payoffs where all coefficients are *positive*. It is not immediate to us how to modify the method used in the proof of Proposition 1 to this situation.

We leave the problem of complete characterization of optimal stochastic allocations for future research. Although we do not know how the optimal allocation looks in general, it is not always deterministic. For instance, AM, Section 8.3, provide a setting in which the regularity condition fails and a stochastic allocation gives the principal a higher ex ante payoff than the optimal deterministic allocation.

4 Non-optimality of deterministic allocations

In this section, we provide an example in which stochastic allocations outperform deterministic ones and can achieve an outcome arbitrarily close to the first-best outcome for the principal. Consider a principal with an absolute value loss function $u_p(p, \omega) = -|p - (\omega - b)|$, where $b > 0$. The principal's ex ante payoff is maximized by the first-best allocation that implements $p = \omega - b$ with certainty for almost all $\omega \in \Omega$. However, this allocation is not incentive-compatible: condition (IC_2) gives $0 = b^2 - b^2 = 2\int_0^\omega (-b)\, \mathrm{d}s = b\omega$ for almost all $\omega \in [0, 1]$, which results in a contradiction. Alternatively, we may observe that in the direct mechanism corresponding to this allocation, the agent's payoff is maximized by the report $\omega' = \min\{\omega + b, 1\}$ for almost all $\omega \in [0, 1]$. Furthermore, this argument suggests that any incentive-compatible allocation implements decisions different from the principal's most preferred alternatives for a set of ω of positive measure. We have the following result.

Proposition 3. *The upper bound of the principal's ex ante payoff on the set of incentive-compatible deterministic allocations is negative.*

Proof. See Appendix A.

Observe that in this setting the variance of the lottery does not have any effect on the principal's payoff if all decisions in a lottery are higher than the principal's preferred decision $p = \omega - b$. This is not true for the agent. Consider two lotteries, one with an average decision close to the agent's preferred decision, $p_a = \omega$, and the other with an average decision close to the principal's preferred decision, $p_p = \omega - b$. If the variance of the first lottery is relatively high, then the agent prefers the second lottery. This suggests that the principal can use variance to implement decisions closer to her most preferred alternatives than would be possible in a deterministic allocation, without imposing any cost on herself. Proposition 4 shows that there exist stochastic incentive-compatible allocations in which the principal obtains a payoff arbitrarily close to the first-best payoff of zero. In these allocations, the agent selects a lottery that implements the principal's preferred decision with high probability; he avoids lotteries with more attractive decisions because they are associated with higher variance. In order to state this proposition, consider some $\varepsilon > 0$ and an allocation M such that

$$\mu^M(\omega) = \omega - b + \varepsilon, \qquad \tau^M(\omega) = 2(b-\varepsilon)\omega, \quad \text{and} \quad \operatorname{supp} M(\omega) \subseteq [\omega - b, \infty), \tag{11}$$

for all $\omega \in \Omega$.[14]

Proposition 4. *For any $\varepsilon \in (0, b)$, the allocation M satisfying (11) is incentive-compatible and yields the principal's ex ante payoff $-\varepsilon$.*

Proof. It is straightforward to verify that M satisfies (IC_1), (IC_2), and (VAR) and, hence, is incentive-compatible. Because the support of $M(\omega)$ belongs to $[\omega - b, \infty)$, the principal's expected payoff from this allocation equals $-\mathbb{E}[\mu^M(\omega) - (\omega - b)] = -\varepsilon$. □

Proposition 4 implies that, even if the regularity condition holds, the optimal allocation might be stochastic if the principal's preferences are not quadratic. Observe, however, that the result in this proposition is obtained for the preferences that are non-smooth in the principal's most preferred decision. By contrast, in those settings in which the principal's payoff function is smooth and the conflict of interest is not very large, one can approximate the principal's payoff by a quadratic loss function. This suggests that in this case the difference between the principal's payoffs in the optimal deterministic allocation and the optimal stochastic allocation might be relatively small whenever the regularity condition holds.

5 Conclusions

We conclude the paper with a discussion of the related literature. The first part of this section connects our results with results in AM, MS, and Strausz [38]. The second part of this section compares our approach with the approach in Krishna and Morgan [27] (henceforth, KM), who study optimal mechanisms in a setting with single-peaked preferences and monetary transfers, and the classical analysis of optimal auction in Myerson [35].

AM analyze optimal deterministic mechanisms for the environment in which the principal's preferences are quadratic, while the agent's preferences are described by a symmetric single-peaked payoff function. If, in addition, the preferences of the agent are quadratic and Assumption 1 is satisfied, then Proposition 1 in our paper implies Proposition 3 in AM. The analysis in AM is conducted using the concepts of the effective backward bias $T(\omega) = \omega - G(\omega)$ and the effective forward bias $S(\omega) = G(\omega) - G(1)$. The regularity assumption in this paper can be equivalently stated as the condition that $T(\omega)$ is convex if $T'(\omega) \geq 0$ and $S'(\omega) \geq 0$.

[14] Such an allocation does exist. For instance, an allocation that implements the decision $p = \omega - b$ with probability $q = 2(b-\varepsilon)\omega/[\varepsilon^2 + 2(b-\varepsilon)\omega]$ and the decision $p = \omega - b + [\varepsilon^2 + 2(b-\varepsilon)\omega]/\varepsilon$ with probability $1 - q$ for all $\omega \in [0, 1]$ satisfies (11).

MS consider a setting with a constant bias $\omega - z(\omega) = -\delta < 0$ for all $\omega \in [0, 1]$. In this setting, $\beta_0 = \beta_1 = 1$. Under the assumption that

$$f(\omega) - \delta f'(\omega) \geq 0 \text{ for almost all } \omega, \tag{12}$$

Proposition 2 in MS demonstrates that $\mu^M(\omega)$ is continuous in the optimal deterministic allocation. Under the additional assumption that

$$F \text{ is strictly log-concave,} \tag{13}$$

Proposition 3 in MS shows that the optimal deterministic allocation satisfies $\mu^M(\omega) = \max\{\omega, \alpha_0\}$ whenever $\alpha_0 < 1$. Observe that condition (12) is equivalent to requiring that $g(\omega)$ is non-increasing almost everywhere and, hence, is similar to Assumption 1. Similarly, as discussed in the paragraph following its definition, Assumption 1 is satisfied if $\omega - z(\omega) = -\delta < 0$ and (13) hold. Hence, Proposition 1 in this paper extends the result of MS to stochastic mechanisms and shows that either (12) or (13) can be relaxed.

The result that Assumption 1 is sufficient for the optimality of deterministic mechanism is analogous to the result in Strausz [38] for the principal-agent model with monetary transfers. Strausz demonstrates that if an optimal deterministic mechanism includes no bunching, then this mechanism is also optimal among stochastic mechanisms. In that environment, bunching does not occur if a monotonicity constraint similar to (IC_1) can be relaxed. In our setting, Assumption 1 guarantees that (IC_1) can be ignored for $\omega \in (\alpha_0, \beta_0)$.

Section 5 in Krishna and Morgan [27] studies optimal deterministic allocations in a setting with monetary transfers, single-peaked payoff functions, and a constant bias. They describe qualitative properties of the optimal allocation and characterize it explicitly for the case of quadratic preferences and a uniform distribution. The formal structure of our models is closely related. In their model, the principal's and agent's payoffs are given by

$$u_p(\mu, \omega) - \tau \quad \text{and} \quad u_a(\mu, \omega, b) + \tau,$$

where u_p and u_a are single-peaked, ω is the agent's private information, b is the agent's bias, μ is the implemented decision, and τ is a positive transfer from the principal to the agent. In Section 3 of our paper, the principal's and agent's payoffs are given by

$$u_p\big(\mu, z(\omega)\big) - \tau \quad \text{and} \quad u_a(\mu, \omega) - \tau,$$

where u_p, u_a are quadratic, ω is the agent's private information, $z(\omega)$ is the most preferred alternative of the principal, μ is the expected implemented decision, and τ is the variance of the implemented decision. Hence, in our model, τ is a cost imposed on both players, whereas in KM τ is a (positive) payment from the principal to the agent. KM demonstrate that payments to the agent may improve the principal's expected payoff. By contrast, Proposition 1 in this paper shows that under Assumption 1 and quadratic preferences the principal cannot improve her expected payoff if costs are imposed on both players.[15]

[15]It is known, however, that if the principal cannot commit to a mechanism, imposing costs only on the agent may improve the payoffs of both players (Austen-Smith and Banks [7] and Kartik [20]).

Finally, we remark on the applicability of the methods used in the optimal auction literature to our setting. Using (7) with $\beta = 0$, one can express the ex ante payoff of the principal as

$$V_p^M = V_a^M(0) + 2\int_0^1 g(\omega)\mu^M(\omega)\,\mathrm{d}\omega + C_3(0) =$$
$$= V_a^M(0) - 2\int_0^1 \left(\omega - \frac{1-F(\omega)}{f(\omega)} - z(\omega)\right) f(\omega)\mu^M(\omega)\,\mathrm{d}\omega + C_3(0),$$

which reminds of the expression for the expected payoff of the seller in an auction in Myerson [35]. Therefore, one might hope that a method similar to the one used in Myerson could be adopted to our model. Unfortunately, this is not the case. In the auction setting, there exist an upper and a lower bound on μ^M, which allows to solve for the optimal allocation both in the regular case in which the virtual valuation function is monotone and in the irregular case in which it is not. By contrast, there are no bounds on μ^M in our setting. Instead, the constraint of non-negative variance plays a crucial role.

A Proofs omitted in the text

Proof of Lemma 1. Let M be an incentive-compatible allocation. Select any $\omega, \omega' \in \Omega$. By incentive compatibility,

$$-[\mu^M(\omega) - \omega]^2 - \tau^M(\omega) \geq -[\mu^M(\omega') - \omega]^2 - \tau^M(\omega'),$$
$$-[\mu^M(\omega') - \omega']^2 - \tau^M(\omega') \geq -[\mu^M(\omega) - \omega']^2 - \tau^M(\omega).$$

Adding the above inequalities gives

$$[\mu^M(\omega) - \mu^M(\omega')]\,(\omega - \omega') \geq 0,$$

which implies (IC_1). Because μ^M is non-decreasing on Ω, the derivative of the agent's payoff with respect to ω,

$$\frac{\partial U_a^M(\omega, \omega')}{\partial \omega} = 2[\mu^M(\omega') - \omega]$$

is uniformly bounded.[16] Therefore, the *Integral form of Envelope Theorem* (Milgrom [32], Theorem 3.1) implies

$$U_a^M(\omega, \omega) = U_a^M(0, 0) + \int_0^\omega \frac{\partial U_a^M(s, s')}{\partial s}|_{s'=s}\,\mathrm{d}s \quad \text{for all } \omega \in \Omega. \tag{14}$$

We obtain (IC_2) by substituting (1) with $\omega' = \omega$ into (14). Finally, condition (VAR) means that variance of $M(\omega)$ must be non-negative.

[16]The lower bound is $2[\mu^M(0) - 1]$ and the upper bound is $2\mu^M(1)$.

Now assume that (IC_1), (IC_2), and (VAR) are satisfied. By substituting (IC_2) with $\omega = \omega'$ into (1), we obtain

$$U_a^M(\omega, \omega') = U_a^M(0,0) - \omega^2 + 2\mu^M(\omega')(\omega - \omega') + 2\int_0^{\omega'} \mu^M(s)\,\mathrm{d}s \quad \text{for all } \omega, \omega' \in \Omega.$$

Therefore,

$$U_a^M(\omega, \omega) - U_a^M(\omega, \omega') = 2\int_{\omega'}^{\omega} [\mu^M(s) - \mu^M(\omega')]\,\mathrm{d}s \quad \text{for all } \omega, \omega' \in \Omega. \tag{15}$$

By monotonicity of μ^M, the right hand side of (15) is non-negative. □

Proof of Lemma 2. It follows from Assumption 1 and from definitions of α_1 and β_1 that $\alpha_1 \leq \beta_1$ and that

$$\begin{array}{ll} g(\omega) < 0 & \text{if } \omega > \beta_1; \\ g(\omega) > 1 & \text{if } \omega < \alpha_1; \\ g(\omega) \in (0,1) & \text{if } \alpha_1 < \omega < \beta_1. \end{array}$$

Furthermore, $\alpha_1 \leq \alpha_0$ and $\beta_0 \leq \beta_1$ by construction. Both (R_1) and (R_2) then follow. To see, for example, the inequality $\beta_0 \leq \beta_1$, consider three cases. First, when $\beta_1 \in (0,1)$, then $g(\omega) < 0$ on $(\beta_1, 1]$. Thus, G is decreasing on this interval and $G(\omega) > G(1)$ for all $\omega \in [\beta_1, 1]$, implying $\beta_0 < \beta_1$. Second, when $\beta_1 = 1$, then $g(\omega) > 0$ for all $\omega \in [0, 1)$. As a result, G is increasing on $[0,1]$ and $\beta_0 = 1$. Third, when $\beta_1 = 0$, then $g(\omega) < 0$ for all $\omega \in (0,1]$. Thus, G is decreasing on $[0,1]$ and $\beta_0 = 0$. □

Proof of Lemma 3. First, we show that if M is incentive-compatible, then $\overline{M}$ is incentive-compatible. Let $\alpha_0 < \beta_0$. Because $M \in \mathcal{M}^c$, this allocation is incentive-compatible. By construction the allocation $\overline{M}$ satisfies (IC_1) and (VAR). In order to verify that $\overline{M}$ satisfies (IC_2) we rewrite it as

$$-[\mu^M(\omega)]^2 - \tau^M(\omega) + [\mu^M(0)]^2 + \tau^M(0) = 2\int_0^{\omega} [\mu^M(s) - \mu^M(\omega)]\,\mathrm{d}s. \tag{IC_2'}$$

First, let $\omega \leq \alpha_0$. In this case $\overline{M}$ satisfies (IC_2) because both sides of (IC_2') are equal to zero. Second, let $\alpha_0 < \omega \leq \beta_0$. Subtracting ($IC_2'$) for allocation M with the state ω and the state $\omega = \alpha_0$ and using $\overline{M}(0) = \overline{M}(\alpha_0)$, we find that ($IC_2'$) is satisfied for $\overline{M}$. Finally, if $\omega > \beta_0$, (IC_2') for allocation $\overline{M}$ is equivalent to (IC_2') for allocation M for $\omega = \beta_0$ and so is satisfied.

If $\alpha_0 \geq \beta_0$, the allocation $\overline{M}$ is incentive-compatible, as $\mu^{\overline{M}}$ and $\tau^{\overline{M}}$ are constant.

Now, we demonstrate that $V_p^{\overline{M}} \geq V_p^M$, where inequality is strict if $\mu^M(0) < \mu^M(\alpha_0)$ or $\mu^M(\beta_0) < \mu^M(1)$. Let $\alpha_0 < \beta_0$. Then $\beta_0 > 0$ and $G(1) = G(\beta_0)$. Similarly, $\alpha_0 < 1$ and $G(\alpha_0) = \alpha_0$. Now consider expression (8) for the principal's ex ante expected payoff when $\alpha = \alpha_0$ and $\beta = \beta_0$. First observe that the first three terms and the last (constant) term are the same for allocations M and $\overline{M}$. Now consider the fifth term $\int_{\beta_0}^1 \mu^M(\omega)g(\omega)\,\mathrm{d}\omega$. For allocation $\overline{M}$, it is equal to $\mu^M(\beta_0)[G(1) - G(\beta_0)] = 0$. For allocation M, we split the integral into two parts:

$$\int_{\beta_0}^{\beta_1} \mu^M(\omega)g(\omega)\,\mathrm{d}\omega + \int_{\beta_1}^{1} \mu^M(\omega)g(\omega)\,\mathrm{d}\omega. \tag{16}$$

By incentive compatibility, $\mu^M(\omega)$ is non-decreasing. Furthermore, by Lemma 2, g is positive on $[\beta_0, \beta_1)$ and negative on $(\beta_1, 1]$. Therefore, the first integral in (16) is less than or equal to $\int_{\beta_0}^{\beta_1} \mu^M(\beta_1)g(\omega)\,\mathrm{d}\omega$, and the second integral in (16) is less than or equal to $\int_{\beta_1}^{1} \mu^M(\beta_1)g(\omega)\,\mathrm{d}\omega$. Adding up, we obtain

$$\int_{\beta_0}^{1} \mu^M(\omega)g(\omega)\,\mathrm{d}\omega \leq \int_{\beta_0}^{1} \mu^M(\beta_1)g(\omega)\,\mathrm{d}\omega = \mu^M(\beta_1)[G(1) - G(\beta_0)] = 0.$$

Moreover, if $\mu^M(\beta_0) < \mu^M(1)$, then $\beta_0 < 1$ and the above inequality is strict.

By the same procedure, we can show that

$$\int_{0}^{\alpha_0} \mu^M(\omega)[g(\omega) - 1]\,\mathrm{d}\omega \leq 0 = \int_{0}^{\alpha_0} \mu^{\overline{M}}(\omega)[g(\omega) - 1]\,\mathrm{d}\omega.$$

Again, we obtain a strict inequality when $\mu^M(0) < \mu^M(\alpha_0)$.

In the case when $\alpha_0 \geq \beta_0$, consider expression (8) for $\alpha = \beta = \beta_0$. Consequently, we have $G(\beta_0) \geq \max\{\beta_0, G(1)\}$. By an analogous procedure as above, we can show that

$$\int_{0}^{\beta_0} \mu^M(\omega)[g(\omega) - 1]\,\mathrm{d}\omega \leq \mu^M(\beta_0)[G(\beta_0) - \beta_0] = \int_{0}^{\beta_0} \mu^{\overline{M}}(\omega)[g(\omega) - 1]\,\mathrm{d}\omega,$$
$$\int_{\beta_0}^{1} \mu^M(\omega)g(\omega)\,\mathrm{d}\omega \leq \mu^M(\beta_0)[G(1) - G(\beta_0)] = \int_{\beta_0}^{1} \mu^{\overline{M}}(\omega)g(\omega)\,\mathrm{d}\omega.$$

The remainder of the proof is the same as in the case $\alpha_0 < \beta_0$. □

Proof of Proposition 1. Case $\alpha_0 < \beta_0$. Let M be an allocation in $\overline{\mathcal{M}}$. Then, the principal's payoff in this allocation is given by (9). Now recall that $V_a^M(\omega) \leq 0$ for all $\omega \in [0, 1]$, where equality holds if and only if $\mu^M(\omega) = \omega$ and $\tau^M(\omega) = 0$. By Lemma 2, $-g'(\omega) > 0$ almost everywhere on (α_0, β_0), $1 - g(\alpha_0) > 0$ and $g(\beta_0) > 0$. Therefore, we obtain

$$V_p^M \leq C \text{ for any } M \in \overline{\mathcal{M}},$$

where equality holds if and only if

$$\mu^M(\omega) = \omega, \qquad \tau^M(\omega) = 0, \tag{17}$$
$$\text{for } \omega = \alpha_0,\ \omega = \beta_0, \text{ and for almost all } \omega \in (\alpha_0, \beta_0).$$

It follows then that M is optimal in $\overline{\mathcal{M}}$ if and only if it satisfies (17).

We can now prove the statement of the proposition. First, the allocation given by (10) satisfies (17). Additionally, it satisfies (IC_1), (IC_2), and (VAR) and is, therefore, incentive-compatible. Thus, it is optimal.

Conversely, consider an allocation $M \in \overline{\mathcal{M}}$ that satisfies (17). We will show that it also satisfies (10). The monotonicity condition (IC_1) implies that $\mu^M(\omega) = \omega$ for all $\omega \in [\alpha_0, \beta_0]$. The constraint ($IC_2$) together with continuity imply that $\tau^M(\omega) = 0$ for all $\omega \in [\alpha_0, \beta_0]$. It remains to be shown that $\mu^M(\omega) = \alpha_0$ for all $\omega \in [0, \alpha_0)$ and $\mu^M(\omega) = \beta_0$ for all $\omega \in (\beta_0, 1]$. Because $M \in \overline{\mathcal{M}}$, the value of μ^M is constant on $[0, \alpha_0)$ and on $(\beta_0, 1]$. Let k_1 and k_2 denote these constants respectively. Then, for any $\omega \in [0, \alpha_0)$, we find from (IC_2) that

$$V_a^M(\alpha_0) - V_a^M(\omega) = \omega^2 - \alpha_0^2 + 2\int_{\omega}^{\alpha_0} \mu^M(s)\,\mathrm{d}s.$$

Since $V_a^M(\alpha_0) = 0$, this reduces to $\tau^M(\omega) = -(k_1 - \alpha_0)^2$, which implies that $\tau^M(\omega) = 0$ and $k_1 = \alpha_0$. Similarly, for $\omega \in (\beta_0, 1]$, we have

$$V_a^M(\omega) - V_a^M(\beta_0) = \beta_0^2 - \omega^2 + 2\int_{\beta_0}^{\omega} \mu^M(s)\,\mathrm{d}s,$$

which reduces to $\tau^M(\omega) = -(k_2 - \beta_0)^2$. Hence, $\tau^M(\omega) = 0$ and $k_2 = \beta_0$.

Case $\alpha_0 \geq \beta_0$. If either $\alpha_0 > \beta_0$ or $\alpha_0 = 1$ or $\beta_0 = 0$, then any allocation $M \in \overline{\mathcal{M}}$ has $\mu^M \equiv k$ being constant on $[0, 1]$. The principal's payoff from such an allocation is

$$V_p^M = -[k - \mathbb{E}z(\omega')]^2 + [\mathbb{E}z(\omega')]^2 - \mathbb{E}[z(\omega')]^2 - \tau^M(0).$$

It is maximized in the set $\overline{\mathcal{M}}$ if and only if

$$k = \mathbb{E}z(\omega') \quad \text{and} \quad \tau^M(0) = 0. \tag{18}$$

The remainder of the argument is analogous to the case $\alpha_0 < \beta_0$.

Finally, if $\alpha_0 = \beta_0 \in (0, 1)$, then $\mathbb{E}z(\omega') = G(1) = G(\beta_0) = G(\alpha_0) = \alpha_0$. The principal's expected payoff reduces to

$$V_p^M = V_a^M(\alpha_0) + \alpha_0^2 - \mathbb{E}[z(\omega')]^2.$$

This payoff is maximized by $V_a^M(\alpha_0) = 0$ or, equivalently, by $\mu^M(\alpha_0) = \alpha_0 = \mathbb{E}z(\omega')$ and $\tau^M(\alpha_0) = 0$. The remainder of the argument is analogous to the case $\alpha_0 < \beta_0$. □

Proof of Proposition 2. (i) We will prove a stronger statement, namely, that there exists $\bar{\lambda} > 0$ such that $\frac{\mathrm{d}}{\mathrm{d}\omega} g^\lambda(\omega) < 0$ for all $\lambda < \bar{\lambda}$ and $\omega \in [0, 1]$. Let m denote the minimum of function f on $[0, 1]$. Indeed, it exists and is positive. By the assumption $|\tilde{z}'(\omega)| \leq K_1$ and $|f'(\omega)| \leq K_2$ for all $\omega \in [0, 1]$ and some $K_1, K_2 > 0$. Next, function g^λ is differentiable and

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\omega} g^\lambda(\omega) &= -f(\omega) + \lambda\big[\big(\tilde{z}'(\omega) - 1\big)f(\omega) + \big(\tilde{z}(\omega) - \omega\big)f'(\omega)\big] \leq \\ &\leq -m + \lambda[|K_1 - 1|f(\omega) + |\tilde{z}(\omega) - \omega|K_2].\end{aligned}$$

The function $(K_1 - 1)f(\omega) + |\tilde{z}(\omega) - \omega|K_2$ is continuous on $[0,1]$ and, hence, is bounded; let $K_3 > 0$ be its upper bound. Then, $\frac{\mathrm{d}}{\mathrm{d}\omega} g^\lambda(\omega) < -\frac{1}{2}m + \lambda K_3$. Setting $\bar{\lambda} = \min\{1,\, m/(2K_3)\}$ completes the proof.

(ii) If $\tilde{z}(0) > 0$, then $z^\lambda(0) > 0$ for all $\lambda \in [0,1]$. Therefore, $\alpha_0^\lambda > 0$. Furthermore, α_0^λ solves $G^\lambda(\omega) = \omega$ by definition. For $\omega > 0$, this equation can be rewritten as

$$H(\omega,\lambda) = 0, \qquad \text{where} \qquad H(\omega,\lambda) = \frac{\int_0^\omega F(s)\,\mathrm{d}s}{\int_0^\omega [\tilde{z}(s) - s]f(s)\,\mathrm{d}s} - \lambda. \tag{19}$$

For $\omega = 0$, we define $H(0,\lambda) = -\lambda$. (This extension is continuous.[17]) Part (i) implies that (19) has a unique solution for $\lambda < \bar{\lambda}$. Next,

$$\frac{\partial}{\partial\omega} H(\omega,\lambda) = \frac{F(\omega)}{\int_0^\omega [\tilde{z}(s) - s]f(s)\,\mathrm{d}s} - \frac{[\tilde{z}(\omega) - \omega]f(\omega)\int_0^\omega F(s)\,\mathrm{d}s}{\left(\int_0^\omega [\tilde{z}(s) - s]f(s)\,\mathrm{d}s\right)^2}. \tag{20}$$

After substitution of $\omega = \alpha_0^\lambda$ and using $H(\alpha_0^\lambda, \lambda) = 0$, we obtain

$$\frac{\partial}{\partial\omega} H(\omega,\lambda)|_{\omega=\alpha_0^\lambda} = \frac{1 - g^\lambda(\alpha_0^\lambda)}{\int_0^{\alpha_0^\lambda} [\tilde{z}(s) - s]f(s)\,\mathrm{d}s}.$$

If $0 < \lambda < \bar{\lambda}$, the denominator equals $\frac{1}{\lambda}\int_0^{\alpha_0^\lambda} F(s)\,\mathrm{d}s > 0$. The numerator is positive by Lemma 2. Using the *Implicit function theorem* we obtain

$$\frac{\mathrm{d}\alpha_0^\lambda}{\mathrm{d}\lambda} = -\frac{\frac{\partial}{\partial\lambda} H(\omega,\lambda)|_{\omega=\alpha_0^\lambda}}{\frac{\partial}{\partial\omega} H(\omega,\lambda)|_{\omega=\alpha_0^\lambda}} > 0.$$

(iii) The proof is analogous to part (ii).

(iv) Since $\lambda = 0$ implies $\alpha_0^\lambda = 0$, it remains to be shown that α_0^λ is continuous in $\lambda = 0$. Applying *L'Hospital rule* to (20) verifies that $\frac{\partial}{\partial\omega} H(\omega,\lambda)\big|_{\substack{\lambda=0\\ \omega=0}} = 1/[2\tilde{z}(0)] \neq 0$. The remainder of the argument follows from the *Implicit function theorem*. The proof for β_0^λ is analogous. □

Proof of Proposition 3. Let M be an incentive-compatible deterministic allocation. Define $\bar{\omega} = \min\{b, 1\}$ and consider a function $\tilde{\varepsilon} : \mathbb{R} \times [0,\bar{\omega}] \to \mathbb{R}$ such that

$$\tilde{\varepsilon}(p,\omega) = -\int_0^\omega |p - (s - b)|f(s)\,\mathrm{d}s - \int_\omega^{\bar{\omega}} (b - s)f(s)\,\mathrm{d}s$$

for all $p \in \mathbb{R}$ and all $\omega \in [0,\bar{\omega}]$.

By (IC_1), the function μ^M is non-decreasing and the limit $m = \lim_{\omega\to 0+} \mu^M(\omega)$ exists. If $m \geq 0$, it follows from (IC_1) that $\mu^M(\omega) \geq 0$ for all $(0,\bar{\omega}]$. In this case, $u_p\big(\mu^M(\omega),\omega\big) = -|\mu^M(\omega) - (\omega - b)| \leq -(b - \omega)$ for all $\omega \in [0,\bar{\omega}]$. Therefore, the principal's expected payoff is bounded from above by $-\int_0^{\bar{\omega}} (b - s)f(s)\,\mathrm{d}s = \tilde{\varepsilon}(0,0)$.

[17] Using *L'Hospital rule* we obtain $\lim_{\omega\to 0} H(\omega,\lambda) = \lim_{\omega\to 0} F(\omega)/\big[\big(\tilde{z}(\omega) - \omega\big)f(\omega)\big] - \lambda = -\lambda$.

Let $m < 0$. Then, $\mu^M(\omega) < 0$ for some $\omega \in (0, \bar{\omega}]$. It follows from incentive compatibility that there exists some $\omega^* > 0$ such that $\mu^M(\omega) = m$ for all $\omega \in [0, \omega^*)$ and $\mu^M(\omega) > 0$ for all $\omega > \omega^*$.[18] Finally, if $\omega^* < \bar{\omega}$, then $\mu^M(\omega) \geq 0$ for all $\omega \in (\omega^*, \bar{\omega}]$. Therefore, the principal's expected payoff is bounded from above by

$$-\int_0^{\omega^*} |m - (s - b)| f(s)\, \mathrm{d}s - \int_{\omega^*}^{\bar{\omega}} (b - s) f(s)\, \mathrm{d}s = \tilde{\varepsilon}(m, \omega^*).$$

The function $\tilde{\varepsilon}$ is continuous and *negative* on $\mathbb{R} \times [0, \bar{\omega}]$. Let $\bar{\varepsilon}$ denote its maximum on the (compact) set $[-b, 0] \times [0, \bar{\omega}]$. Clearly, $\bar{\varepsilon} < 0$. Furthermore, $\tilde{\varepsilon}(p, \omega) < \tilde{\varepsilon}(-b, \omega) \leq \bar{\varepsilon}$ for all $p < -b$ and all $\omega \in [0, \bar{\omega}]$. Therefore, $\bar{\varepsilon} < 0$ is the maximum of $\tilde{\varepsilon}$ on $(-\infty, 0] \times [0, \bar{\omega}]$ and is an upper bound on the principal's expected payoff of the set of deterministic incentive-compatible allocations. □

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[18] If $\mu^M(\omega) < 0$, then we have $0 \geq 2\int_0^{\omega} [\mu^M(s) - \mu^M(\omega)]\, \mathrm{d}s = -[\mu^M(\omega)]^2 + [\mu^M(0)]^2 \geq 0$, where both inequalities follow from (IC_1) and the equality follows from (IC_2') introduced in the proof of Lemma 3. Thus, $\mu^M(\omega) = m$.

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