NOTES ON THE IS-LM MODEL

Based on various textbooks, the following macroeconomic model of the closed economy can be built:

- **Consumption:** \( C = a + b \cdot YD \)
- **Disposable Income:** \( YD = Y - T \)
- **Taxes:** \( T = tY \)
- **Investment:** \( I = e - dr \)
- **Government Spending:** \( G = G \)

\[ b, t, d > 0 \quad \text{Note: } b = \text{MPC out of disposable income} \]

1. **Focus: product market equilibrium.**

Using the national income equation \( Y = C + I + G \), we can derive the IS curve (= a locus of \( (Y,r) \) points at which the product market is in equilibrium), and we obtain:

\[
Y_{IS} = \frac{a + e + G}{1 - b(1-t)} \cdot \frac{d}{1 - b(1-t)} r
\]

\[
r_{IS} = \frac{a + e + G}{d} \cdot \frac{1 - b(1-t)}{d} Y
\]

1. **Slope of the IS** \( = \Delta r / \Delta Y = - \frac{[1 - b(1-t)]}{d} < 0 \) \quad \text{Note: } b(1-t) = \text{MPC out of income before taxes} \)

If \( d \) goes to zero (i.e. investment is interest-rate insensitive), the slope of the IS curve goes to minus infinity, and the IS curve is vertical. Note that the investment function is then defined as \( I = e \), i.e., it is also vertical. If \( d \) goes to infinity (i.e. investment is perfectly elastic with respect to interest-rate changes), the slope of the IS curve goes to zero, and the curve is horizontal (and so is the investment function).

An increase in \( d \) makes the IS curve flatter.

Similarly, an increase in the product market multiplier \( = 1 / [1 - b(1-t)] \) makes the IS curve flatter.

2. **Intercepts:**
   
   a. with the Y axis (horizontal intercept):
   \( \{(a + e + G) / [1 - b(1-t)] ; 0\} \)

   b. with the r axis (vertical intercept):
   \( \{0; (a + e + G) / d\} \)

Change in the product-market multiplier makes the IS curve rotate around the vertical intercept.

Change in the interest-rate sensitivity of investment \( (d) \) makes the IS curve rotate around the horizontal intercept.

An increase (or a decrease) in \( a, e, \) or \( G \), i.e., an autonomous increase (decrease) in the consumption, investment or government component of the aggregate spending) shifts the IS curve horizontally \( (\Delta r = 0) \) by the distance given by:

\[ \Delta Y_{IS} = \Delta a \cdot 1 / [1 - b(1-t)] = \Delta a \cdot \text{product-market multiplier} \]
II. **Focus: money market equilibrium:**

Real Money Demand: \[ L = kY - hr \]
Real Money Supply: \[ M/P = M/P \]

\[ k, h > 0 \]

From the money-market equilibrium condition, we can derive the LM curve (= a locus of \((Y, r)\) points at which real money demand equals real money supply):

\[ Y_{LM} = \frac{1}{k} M + \frac{h}{k} r \]

\[ r_{LM} = -\frac{1}{h} M + \frac{k}{h} Y \]

1. **Slope of the LM curve:** \( \Delta r / \Delta Y = k/h > 0 \)

If \( h \) goes to zero (zero interest-sensitivity of money demand) OR \( k \) goes to infinity (infinite income elasticity of money demand), the slope of the LM curve goes to infinity, and the curve is vertical.

If \( h \) goes to infinity or \( k \) goes to zero, the LM curve becomes horizontal, i.e., its slope is zero.

Decreases in \( k \) or increases in \( h \) make the LM curve flatter.

2. **Intercepts:**
   a. with the Y axis (horizontal intercept):
      \{(1/k) M/P; 0\}
   b. with the r axis (vertical intercept):
      \{0; -(1/h) M/P\}

Change in the income-elasticity of money demand \((k)\) makes the LM curve rotate around the vertical intercept. Change in the interest-rate sensitivity of money demand \((h)\) makes the LM rotate around the horizontal intercept.

A change in either \( M \) or \( P \) shifts the LM curve **horizontally** \((\Delta r = 0)\) by the distance:

\[ \Delta Y_{LM} = (1/k) \Delta(M/P) = \Delta (M/P) * \text{money-market multiplier} \]

III. **Focus: equilibrium for the whole IS-LM system:**

The IS and LM equations solved simultaneously provide the equilibrium levels of \( Y \) and \( r \) consistent with the equilibrium in both the product and money markets (IS and LM curves, respectively).

The equilibrium levels of \( Y \) and \( r \) can be obtained in either of the following two ways:

1. solve for \( r^* \) from \( Y_{IS} = Y_{LM} \), substitute this \( r^* \) back into the \( Y_{IS} \) (or \( Y_{LM} \)) equation, thus solving for \( Y^* \);
2. solve for \( Y^* \) from \( r_{IS} = r_{LM} \), substitute this \( Y^* \) back into the \( r_{IS} \) (or \( r_{LM} \)) equation, thus solving for \( r^* \).

In either way, we obtain:
\[
    r^* = \frac{k}{h [1 - b(1 - t)] + dk} \left[ a + e + G \right] - \frac{1 - b(1 - t)}{h [1 - b(1 - t)] + dk} \frac{M}{P} \\
    Y^* = \frac{h}{h [1 - b(1 - t)] + dk} \left[ a + e + G \right] + \frac{d}{h [1 - b(1 - t)] + dk} \frac{M}{P} 
\]

Note that \( Y^* \) can be written as:
\( Y^* = \text{system's fiscal multiplier} \times [a + e + G] + \text{system's monetary multiplier} \times \frac{M}{P} \)

1. **System's fiscal multiplier**

\[
    \frac{h}{h [1 - b(1 - t)] + dk} = \frac{1}{[1 - b(1 - t)]} \frac{M}{P} \leq \frac{1}{1 - b(1 - t)}
\]

Note: The system's fiscal multiplier never exceeds the product-market multiplier.

Changes in fiscal policy change \( Y \) which, in turn, changes \( L \), causing a disequilibrium in the money market. The restoration of the equilibrium in the money market requires changes in the real interest rate \( r \). Changes in \( r \) affect investment \( I \) and hence \( Y \). Investment changes in the direction opposite to the initial change in the government budget deficit (*crowding-out effect*). The interaction between the product and the money market has therefore an adverse effect on the effectiveness of the fiscal policy.

This adverse effect is zero (i.e., no crowding out) if:

a. \( d = 0 \) vertical IS
b. \( k = 0 \) or \( h \to \infty \) horizontal LM.

Fiscal policy cannot change the level of \( Y \) (i.e., it is completely ineffective) if:

a. \( d \to \infty \) horizontal IS
b. \( k \to \infty \) or \( h = 0 \) vertical LM.

An increase in \( d \) or an increase in \( k \) or a decrease in \( h \) reduce the system's fiscal multiplier. It follows that, given the same product-market multiplier, the fiscal policy is more effective the steeper the IS curve is. And the steeper the LM curve is, the less effective fiscal policy is.

**CHALLENGE**: How do changes in the product-market multiplier affect fiscal policy effectiveness?

Note the apparent ambiguity: an increase in this multiplier shifts the IS curve by a greater distance horizontally for a given change in \( G_s \) (i.e., it increases fiscal policy effectiveness) but, at the same time, it makes the IS curve flatter (i.e., it decreases fiscal policy effectiveness).

Is the outcome really ambiguous? Why?
2. **System's Monetary Multiplier**

\[ \frac{d}{h(1-b(1-t)) + dk} = \frac{1}{k} \frac{dk}{h(1-b(1-t)) + dk} = \frac{1}{k} \frac{h}{dk} \frac{1}{1-b(1-t) + \frac{1}{k}} \leq \frac{1}{k} \]

Note: The system's monetary multiplier never exceeds the money-market multiplier.

The interaction between the product and money markets weakens the response of \( Y \) to changes in real money supply (\( M/P \)). For example, an increase in money supply increases money income. The equilibrium in the product market requires therefore higher aggregate spending. This increase in aggregate spending will occur partly because of increased endogenous consumption (\( \Delta C = MPC*\Delta Y, \ 0<MPC<1 \)). The remaining part has to be met by higher investment. However, investment will rise only if the real interest rate falls. Falling interest rates lead to an increase in money demand, weakening thus pressure on money demand to adjust to the increased money supply entirely through an increase in income.

No "weakening" takes place if:

a. \( d \to \infty \) horizontal IS
b. \( k \to \infty \) or \( h = 0 \) vertical LM.

Monetary policy cannot change the level of \( Y \) (i.e., it is completely ineffective) if:

a. \( d = 0 \) vertical IS
b. \( h \to \infty \) or \( k = 0 \) horizontal LM.

The steeper the IS curve is (i.e., the lower \( d \) is or the lower the product-market multiplier is), the less effective monetary policy is. Given the same money-market multiplier, the monetary policy is more effective the steeper the LM curve is (i.e., the lower \( h \) is).

**CHALLENGE**: How do changes in the money market multiplier affect the monetary policy effectiveness? Note the apparent ambiguity: an increase in this multiplier (i.e., a decrease in \( k \)) shifts the LM curve by a greater distance horizontally for a given change in money supply (i.e., it increases monetary policy effectiveness) but, at the same time, it makes the LM curve flatter as \( k \) goes down (i.e., it decreases monetary policy effectiveness). Is the outcome really ambiguous? Why?

IV. **Aggregate Demand**

The solution for \( Y^* \) implies the following relationship between prices \( P \) and income \( Y \):

\[ P = \frac{M \ast \text{system's monetary multiplier}}{Y - [a + e + G] \ast \text{system's fiscal multiplier}} \]

The above equation describes the aggregate demand curve (AD). Note that the curve is negatively sloped: \( \Delta P / \Delta Y < 0 \).

It shifts in response to the same shocks which shift the IS or the LM curves with the exception of changes in the price level. The latter shift the LM curve, whereas they only cause a movement along the AD curve. **Recall**: AD curve is derived from shifts in LM triggered by \( P \) changes.