Topic 1: Measuring market power

Part 1: intro - what can be done without behavioral assumptions

- Bresnahan (1982): How can the data identify market power?
- Corts (1999): As-if interpretation of conjecture parameter is incorrect

Part 2: with behavioral assumptions

- Porter (1983): make functional form assumption on pricing regime and check reasonableness
- Bresnahan (1987): estimate model using different behavioral assumptions and compare the R^2’s
- Borenstein et al. (2003): assume perfect competition (PC) and compare predicted prices with actual prices

Lecture 1: overview

Bresnahan (1989): Alternative treatments of firm conduct

Different theories of firm behavior generate different supply relationships. Notice that the first order conditions for a firm can be written in a similar form for different assumptions on market structure — demand: \( P = D(Q, Y, \alpha, \epsilon) - \frac{\partial P}{\partial Q} = D_1 \leq 0: \)

\[
P_t = C_1(Q_{it}, W_{it}, Z_{it}, \beta, \epsilon^c_{it}) - D_1(Q_t, Y_t, \alpha, \epsilon^d)Q_{it} \times \text{SOMETHING}
\]

Straightforward examples are:

Cournot: \( P_t = C_1(Q_{it}, W_{it}, Z_{it}, \beta, \epsilon^c_{it}) - D_1(Q_t, Y_t, \alpha, \epsilon^d)Q_{it} \times 1 \)
Bertrand/PC: \( P_t = C_1(Q_{it}, W_{it}, Z_{it}, \beta, \epsilon_{it}) - D_1(Q_{it}, Y_t, \alpha, \epsilon^d)Q_{it} \times 0 \)

Monopoly/Cartel: \( P_t = C_1(Q_{it}, W_{it}, Z_{it}, \beta, \epsilon_{it}) - D_1(Q_{it}, Y_t, \alpha, \epsilon^d) \frac{Q_{it} \times N}{Q_t} \)

A more involved example for the newspaper market is Rosse (1970). Firms set the circulation price of the paper, marking up the circulation MC with a premium that not only depends on the circulation MR \((D^c_2 \leq 0)\), but also the effect of reduced circulation on the advertising MR. \((D^a_2 \geq 0)\)

Rosse (1970): \( P^c_t = MC^c - D^c_2(Q^a_t, Q^c_t, \ldots)Q^c_t - D^a_2(Q^a_t, Q^c_t, \ldots)Q^a_t \)

Two examples that we will study in more detail in the next lecture are

Bresnahan (1981): \( Q_i = (P_i - MC_i)\left(\frac{\alpha_0}{X_j - X_i} - \frac{\alpha_0}{X_i - X_h}\right) + \frac{P_j\alpha_0d_{ij}}{X_j - X_i} + \frac{P_h\alpha_0d_{ih}}{X_i - X_h} + \epsilon^i \)

Porter (1983) \log P_t = ... + \alpha^{pw} \) \( \text{with probability } \pi \)
\log P_t = ... + \alpha^{pw} + \alpha^c \) \( \text{with probability } 1 - \pi \)

Finally, two more examples have one firm maximizing profits making an assumption about the behavior of other firms. This prepares us for the conjectural variations framework.

In Stackelberg, the leading firm knows that the followers will compete in Cournot fashion taking its output \((Q_1)\) as given. All firms \(i\) will have a f.o.c. as above and solving that simultaneously gives their optimal output \(Q_i(Q_1)\). In setting \(Q_1\), the leader takes the sum of the followers reaction into account \((\theta_S \leq 0)\). The more responsive the followers, the lower the market-up and price and the higher \(Q_1\).

Stackelberg: \( P_t = C_1(Q_{it}, W_{it}, Z_{it}, \beta, \epsilon^e_{it}) - D_1(Q_{it}, Y_t, \alpha, \epsilon^d)Q_{it} \times (1 + \theta_S) \)
where \(\theta_S = \sum_i \frac{\partial Q_i}{\partial Q_{1t}}\)

In the dominant firm framework, the dominant firm knows that the fringe will simply put \(P = MC\). A higher price will increase the fringe output, for upwardsloping MC, according to the slope of their supply \(S_1 \geq 0\). The more responsive the fringe, high \(S_1\), the more elastic the residual demand is (note the denominator of \(S_D\)), and the smaller the markup will be \((S_D \leq 0)\):

Dominant firm: \( P_t = C_1(.) - D_1(.)Q_{1t} \times (1 + S_D) \)
Given that all these models take the same form we can try to nest them all and replace “SOMETHING” with a parameter that will be estimated from the data.
Conjectural variations ($\theta_{it} \in [0, N]$)

$$P_t = C_1(Q_{it}, W_{it}, Z_{it}, \beta, \epsilon_{it}) - D_1(Q_t, Y_t, \alpha, \epsilon^d)Q_{it} \times \theta_{it}$$

Now we will ask three questions with respect to such a conjectural variations model:

1. Is the $\theta$ parameter identified?
2. Does it make theoretical sense?
3. Does it make empirical sense?

**Bresnahan (1982): Identification of market power**

The full model of the industry is described by 2 equations (Note the entire analysis is now done at the industry level, i.e. all firms face the same $\theta/\lambda$ parameter and we only look at the market quantity $Q_t = \sum_i Q_i$):

Demand function: $Q = D(P, Y, \alpha) + \epsilon^d$

Supply relation: $P = c(Q, W, \beta) - \lambda h(Q, Y, \alpha) + \epsilon^c$

where $Y$ (demand shifter) and $W$ (cost shifter) are exogenous, and $P$ and $Q$ are endogenous. If firms are price takers, the supply relation simplifies to the supply function $P = MC = c(.)$. $P + \lambda h(.)$ is MR as perceived by the firm.

We will stick to a linear form:

Demand function: $Q = \alpha_0 + \alpha_1 P + \alpha_2 Y + \epsilon^d$

Supply relation: $P = \beta_0 + \beta_1 Q + \beta_2 W - \lambda P'(Q)Q + \epsilon^c$

The demand equation is identified no matter what: one included endogenous variable as explanatory variable ($P$), one excluded exogenous variable ($W$).

Note that OLS will not be sufficient. For example, using the logit transformation in Berry (1994) $\epsilon^D = \xi_j$, a product-specific unobserved quality index. This is undoubtedly correlated with price and using OLS might even lead to positive price coefficient. In a homogenous goods example, under Cournot competition, the price will still be correlated with $\epsilon^D$, because the demand function (including the error) will enter the supply relation which determines pricing. Assuming $E[\xi_j|Y, P] = 0$ is unreasonable, so we need an instrument that shifts prices...
but does not enter the demand otherwise, e.g. $W$ and use IV (or 2SLS): $E[\xi_j | Y, W] = 0$. More generally:

in a method of moment’s estimator:  

$$E[Z' \epsilon^D] = 0$$

sample analog:  

$$\frac{1}{N} \sum_{n=1}^{N} Z'(Q - \alpha_0 - \alpha_1 P - alpha_2 Y) = G_N(\alpha)$$

GMM: pick $\alpha$ that minimizes:  

$$G_N(\alpha)' A_N G_N(\alpha)$$

(Note: on production side, just as unreasonable to estimate the production function with OLS, because factor inputs are correlated with unobserved productivity. Solution: using fixed effects might be enough, or use instruments, e.g. local demand shocks as in Syverson(2004))

The supply relation is identified, but not the degree of market power. In the linear case, the supply relation can be written as

$$P = \beta_0 + (\beta_1 - \frac{\lambda}{\alpha_1})Q + \beta_2 W + \epsilon^c.$$ 

The slope wrt $Q$ is identified if we have one instrument, e.g. a demand shock $Y$. However, it is impossible to uncover $\lambda$. (we treat the $\alpha$ coefficients as known because they can be estimated separately from demand) FIGURE 1

To identify the degree of market power,

1. Assume that marginal costs is constant: $\beta_1 = 0$ FIGURE 2
2. extend the demand equation to

\[ Q = \alpha_0 + \alpha_1 P + \alpha_2 Y + \alpha_3 PZ + \alpha_4 Z + \epsilon^d, \]

where \( Z \) is a new exogenous demand-side variable that shifts not only the demand intercept, but also the price sensitivity (e.g. price of a substitute). The new supply relation becomes

\[ P = \beta_0 + \beta_1 Q - \frac{\lambda}{\alpha_1 + \alpha_3 Z} Q + \beta_2 W + \epsilon^c. \]

In this case \( \lambda \) is identified as the coefficient on the new variable \( Q^* \) \((= Q/(\alpha_1 + \alpha_3 Z))\). Now there are two included endogenous variables \((Q\text{ and } Q^*)\) and two excluded exogenous variables \((Z \text{ and } Y)\).\(^1\) FIGURE 3

\(^1\)Note the two typos in the paper.
3. One example is a different technology (see tomato harvest example)

4. Other example shows that estimated p-cost margins are reasonable

**Conjectural variations games**

Write the conjectural variations f.o.c. alternatively:

\[ P_t = C_1(.) - D_1(.)Q_{it} \times (1 + r_i(Q_{it}, Q_{jt}, Z, \psi)) \]

In the single-firm profit maximization quantity choice problem, the first order condition for firm \( i \) boils down to the equation above if we assume:

\[ Q_t = Q_{it} + \sum_{j \neq i} Q_{jt} = Q_{it} + R(Q_{it}) \]

then \( \frac{\partial Q_t}{\partial Q_{it}} = 1 + \frac{\partial R(Q_{it})}{Q_{it}} \) and

\[ 1 + r_i(.) = \frac{dQ_{it}}{dQ_{it}} + \sum_{j \neq i} \frac{dQ_{jt}}{dQ_{it}} \]

One should clearly distinguish the theoretical use of the word “conjecture” from its empirical content.

Firms maximize profits by choosing output and making a conjecture about their rivals’ response. Different firms can make different conjectures, which are specified exogenously. This conjecture can be consistent or not (the only consistent equilibrium boils down to the Cournot equilibrium: both firms play best response relative to the other firm’s EQUILIBRIUM strategy). Of little theoretical interest.

**Corts (1999): Interpretation of the conduct parameter**

He takes issue with the as-if interpretation of the conduct parameter: The “degree of competitiveness” in the real world is as if firms play a conjectural variations game with the estimated conjecture applied to their expectation of their rivals’ responses. This is wrong if marginal responses differ from average responses.

Two steps in applying the conduct parameter approach to measure market power (while maintaining an agnostic stance toward the behavioral model that generates the data):

- estimate the slope of the supply relation to measure “equilibrium variation”.
  This is identified if
    - MC is flat and the instrument that shifts demand (laterally)
We have an instrument that shifts the elasticity of demand
If we know about multiple pricing regimes and use functional form assumptions

• The equilibrium variation is implicitly mapped into the inferred “equilibrium value” of the elasticity-adjusted price-cost margin

\[ P = c'_i(q_i) - \theta_i P'(Q)q_i \]  

(1)

Each firm \( i \) anticipates that its rivals’ aggregate output is some function \( R_i(q_i) \) and firm \( i \)’s first order condition will be equal to equation (1) with \( \theta_i = 1 + R'_i(q_i) = 1 + r_i \).

Interpretation:

• \( r_i = -1 \) or \( \theta_i = 0 \): competitive model, output expansions are neutralized.

• \( r_i = 0 \) or \( \theta_i = 1 \): Cournot Nash: no response in equilibrium

• \( r_i = N - 1 \) or \( \theta_i = N \): Monopoly: output expansions are matched

Reorganize (1):

\[ \hat{\theta}_i = \frac{P - c'_i}{-P'q_i} = \frac{P - c'_i}{P} Ne^P = \frac{1}{-P'} (P - c')/x - q_i/x \]

The THEORETICAL interpretation of the CPM is an elasticity-adjusted Lerner index, or the AVERAGE price-cost margin. In this equation expressed by the average change induced by changes in \( x \), the demand shifter.

Now, what do we estimate:

Demand: \( P_t = \alpha_0 + \alpha_1 x_t + \alpha_2 Q_t + \epsilon_t \)

Supply: \( P_t = \beta_0 + \beta_1 w_t + \beta_2 q_{it} + \eta_{it} \)

\( x \) are demand shifters, \( w \) are cost shifters. (for simplicity we assume MC is flat so the parameter is identified and \( beta_2 \) is the conduct parameter of interest)

Note from the definition of (1):

\[-\theta_i P' = \beta_2 \]

\[ \theta_i = \frac{\beta_2}{-P'} \]
Estimate Demand by 2SLS, we have cost shifters as instruments for $Q_t$. Similarly, estimate Supply by 2SLS using demand shifters as instruments (In first state we predict $\hat{q}$ from the regression $q_{it} = \gamma x_t + \delta w_t + nu$, inverted this gives $x_t = \frac{1}{\gamma} q_{it} + \delta / \gamma w_{it} \ldots$, which is needed later.) Also, $M - w = I - w(w'w)^{-1}w'$, to first project dependent and explanatory variables on $w$, the other variable in the regression.

$$\hat{\beta}_2 = (\hat{q}'_t M w \hat{q}_t)^{-1} (\hat{q}'_t M w P)$$
$$= (\hat{q}'_t M w \hat{q}_t)^{-1} (\hat{q}'_t M w xa_1 + \hat{q}'_t M w \overline{Q} a_2 + \hat{q}'_t M w \epsilon)$$

and plim

$$\hat{\beta}_2 = (\hat{q}'_t M w \hat{q}_t)^{-1} \hat{q}'_t M w xa_1 + Na_2$$
$$= \frac{a_1}{\gamma} + Na_2$$

Finally, the asymptotic estimate of the conduct parameter $\theta_i$ is then:

$$\hat{\theta}_i = \frac{\hat{\beta}_2}{-\alpha_2} = - \frac{a_1}{a_2 \gamma} - N$$

The estimated CPM is a function only of demand parameters and $\gamma$, i.e. it is identified, and the mechanism that provides this is the responsiveness of equilibrium quantity to the demand shifter $x$.

More generally we can write for the estimated quantity:

$$\hat{\theta}_i = \frac{1}{-P_i^{n'}} \frac{dP^{n'}}{dq^{n'}} \hat{\beta}_2$$
$$= \frac{1}{-P_i^{n'}} \frac{d(P - c')/dx}{dq^{n'}/dx}$$

What we estimate is the MARGINAL response of price wrt quality, induced by changes in $x$. If the marginal and average responses are not equal, the estimated CPM does not coincide with the $\theta$ from the conjectural variations model and interpretations have to be modified accordingly.

On a graph, the equilibrium in a conjectural variations game represented by supply relation $S^1$ is about equally competitive as the observed equilibrium variation in $S^2$; i.e. prices are equally high. However, estimation will yield a much smaller $\theta$ based on the observed data, leading to an interpretation of more competition than there really is.
Fig. 1. Supply relationships under different models of conduct.

show that the supply relation for conjectural variations models is always a ray through the marginal cost intercept; one possible conjectural variations supply relation is depicted as $S_1$. Higher values of $h$ rotate the conjectural variations supply relation toward $S_M$. Some other (non-conjectural variations) model might generate supply relation $S_2$, which should be considered approximately as competitive as the model generating $S_1$ since prices at points A and B are on average as high as prices at C and D. However, the estimated conduct parameters for these two industries might diverge significantly, since supply curves $S_1$ and $S_2$ have different slopes. Thus, implementation of the CPM is problematic even when the system (5) — (6) is correctly specified, which is the case when the firm’s supply rule $q^*_t$ is linear in $(w_t, x_t)$. The problem is that $b_0 \neq b_1w$ need not be the marginal cost $c\partial/\partial q$ (even if marginal cost is independent of $q$ and $x$), and hence $b_2$ need not be equal to $(P - c\partial/\partial q)/q$. Therefore, a consistent estimate of $b_2/\hat{\alpha}_2$, as delivered by 2SLS, will not provide a consistent estimate of the conjectural variations parameter $h$ defined in Eq. (2), except of course when firms do behave according to the conjectural variations model (1).

In the conjectural variations model, the average relationship of price—cost margin to quantity (the LHS of Eq. (11), or the slope of the ray from the marginal cost intercept through the observed data) is identical to its marginal relationship to demand-driven variation in quantity (the RHS of Eq. (11), or the slope of the line defined by the observed data). Equilibrium values of the price-cost margin can therefore be inferred from observed equilibrium variation. Outside of the conjectural variations model, this need not hold and inferences based on this relationship may be invalid. Put another way, the CPM is valid only if the true process underlying the observed equilibrium generates behavior that is identical on the margin, and not just on average, to a conjectural
lecture 2: Making behavioral assumptions

Porter (1983): Joint Executive Committee: 1880-1886

A formal test rejecting that prices and quantities in the railroad industry 2 centuries ago can be explained by exogenous shifts in supply and demand. Instead, he concludes that occasional breakdowns in the cartel leads to price wars. After a period of low prices, the firms revert to the collusive pricing. While the analysis is very different from Bresnahan (1987), a Princeton contemporary, he reaches similar conclusions.

It is one of the first papers to integrate theory with estimation and data collection to investigate incidence and extent of market power. Discussed at length in both ?? and ?). A stylized model that allows very careful study what can identify market power. (In detail here as it will underly one of the problem sets).

Again part of a long series of papers. Including:

- a “structural” i.o. implementation of a theory to the railroad cartel at the end of 19th century. (?)
- a theory piece on how to construct a sub-game perfect collusion strategy under uncertainty (?)
- A second theory piece investigating optimal trigger strategies, where the trigger price and optimal length of revisionary period is derived (?).
- A purely empirical piece where the incidence of price wars is documented (?)
- ?) is an econometric piece on the use of switching regressions.

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The Model

N-firm railroad oligopoly: entry is taken to be exogenous and modeled as supply shifts.

Homogeneous good, single-product, grain shipped from Chicago to the East Coast, by railroad. \( Q_t = \sum_i Q_{it} \) is well-defined. No interperiod demand linkages.

\( t \) indexes time, from first week of 1880 to 16th week of 1986. Model is derived for firms, \( i \), but is aggregated for estimation.

Demand in inverse form: CES

\[
\log Q_t = \alpha_0 + \epsilon \log P_t + \gamma L_t + \epsilon_{Dt}
\]
Only exogenous demand shifter observed by the econometrician is $L$, dummy for whether great lakes are open to navigation, because shipping provides seasonal competition for railroads.

i.i.d error term, unobserved demand shifter that was observable to all firms (otherwise they would have optimized against the parameters of the distribution of $\epsilon_D$, and the endogenous variables of the model would not depend on actual realizations of $\epsilon_D$.) Distributional assumptions are added because estimation is with ML: the demand and supply errors are assumed to be i.i.d. normally distributed.

Supply curve doesn’t exist because other firms will take competitor’s actions into account. We will have to derive an industry supply relationship, given an equilibrium concept (subgame perfect NE). Costs are as follows:

$$C_i(Q_{it}) = F_i + a_i Q_{it}^\delta$$

and the f.o.c. give:

$$P_t = \frac{\partial C}{\partial Q_{it}} - \frac{\partial D}{\partial Q_t} Q_t \theta_{it}$$

$$P_t + \frac{\partial D}{\partial Q_t} Q_t \theta_{it} = a_i \delta Q_{it}^{\delta-1}$$

The lefthand side is the perceived MR for a firm. It depends on the $\theta_{it}$, a firm’s conjecture on how competitive oligopoly conduct is. If firms have a low discount factor, the highest price that is enforceable will be lower than the monopoly joint-profit maximizing price. $\theta$ captures this.

If all firms have the same $\theta$, then the market share would be determined solely by the cost efficiency

$$s_{it} = \frac{a_i^{1/(1-\delta)}}{\sum_j a_j^{1/(1-\delta)}}$$

independent of the value of the collusion parameter $\theta$. Multiplying each firm’s supply relationship by $s_i$ and summing over firms gives Porter’s supply equation:

$$P_t (1 + \frac{\theta_t}{\epsilon}) = D Q_t^{\delta-1}$$

with $$D = \delta (\sum_i a_i^{1/(1-\delta)})^{1-\delta}$$

and $$\theta_t = \sum_i s_{it} \theta_{it}$$

An unfortunate side effect of assuming constant $\theta$ is that the conjectures about rival’s responses have to vary inversely with a firm’s market share. There is no apparent economic reason for that and it makes the model inconsistent with Cournot competition.
To get Porter’s supply equation, we only have to take logs and add cost shifters.

\[
\log P_t = \beta_0 + \beta I_t + (\delta - 1) \log Q_t + \sum_j \gamma_j S_j + \epsilon_{St}
\]

Where \( \beta = -\log(1 + \frac{\theta}{2}) \). \( I_t \) is an indicator random variable which takes on the value 1 when the industry is in a cooperative equilibrium and 0 otherwise. The higher \( \beta \) is estimated, the closer the regime is to collusion in the cooperative regime. Based on the theoretical model in Green-Porter, the highest sustainable \( \beta \) is expected to be lower than joint-profit maximization.

There are no observable exogenous cost shifters (at the industry level). The error term can be interpreted as a common cost shifter for all firms, again known to them but not the econometrician and assumed to be normally distributed. A number of structural dummies are added \( S_j \) that capture both changes in MC when some firms merge, and others enter or exit the industry AND they have to absorb the potential effect on industry conduct (degree of competitiveness) from changes in industry structure.

In practice, Porter assumes that \( \theta_t \) can take only two values, which gives him two possible intercepts to the supply relationship. Each supply regime has a certain probability to occur, with the intercepts and the probabilities to be estimated from the data. The full supply model is:

\[
\log P_t = \underbrace{\beta_0 + (\delta - 1) \log Q_t + \sum_j \gamma_j S_j + \epsilon_{St}}_{\beta_{NC}} \quad \text{with probability } \lambda
\]

\[
\log P_t = \underbrace{\beta_0 + \beta + (\delta - 1) \log Q_t + \sum_j \gamma_j S_j \epsilon_{St}}_{\beta_{C}} \quad \text{with probability } (1 - \lambda)
\]

The difference between the constant terms in both equations is the main parameter of interest in the study.

The overparametrization of \( \theta_t \) is solved by aggregation. \( \theta_t \) is the average of all conduct parameters. The crucial question to Porter is whether \( \theta_t \) is constant or variable over time. High \( \theta_t \) periods will be interpreted as successful cartel cooperation, while low \( \theta_t \) times are price wars or general breakdowns in cooperation.

**Estimation**

The model can be summarized as

\[
B y_t = \Gamma X_t + \Delta I_t + U_t
\]

where

\[
y_t = \left( \begin{array}{c}
\log Q_t \\
\log p_t
\end{array} \right), \quad X_t = \left( \begin{array}{c}
1 \\
L_t \\
S_t
\end{array} \right), \quad U_t = \left( \begin{array}{c}
\epsilon_{Dt} \\
\epsilon_{St}
\end{array} \right)
\]
\[
B = \left( \begin{array}{cc} 1 & -\epsilon \\
1 - \delta & 1 \end{array} \right), \quad \Delta = \left( \begin{array}{c} 0 \\ \beta \end{array} \right), \quad \Gamma = \left( \begin{array}{ccc} \alpha_0 & \gamma & 0 \\ \beta_0 & 0 & \gamma_j \end{array} \right)
\]

If the regime variable \( I \) (the sequence \( \{I_1, ..., I_T\} \)) were observable, estimation would be straightforward with 2SLS. As in \( ? \), use cost shifters as instruments for \( \log Q_t \) in the demand function, and demand shifters as instruments for \( \log p_t \) in the supply relationship.

Alternatively, if \( I \) was observable the conditional likelihood could be maximized.

\[
h(y|I_t) = (2\pi)^{-1/2}||B|| \exp\{ -1/2 (By_t - \Gamma X_t - \Delta I_t)'\Sigma^{-1}(By_t - \Gamma X_t - \Delta I_t) \}
\]

and the likelihood function is

\[
LH(I_1...I_T) = \prod_{t} h(y_t|I_t)
\]

However, if \( I_t \) is unknown, more assumptions are needed. Porter assumes that \( I \) takes the Bernoulli distribution:

\[
I_t = \begin{cases} 1 \text{ with probability } \lambda \\ 0 \text{ with probability } 1 - \lambda \end{cases}
\]

This produces a simultaneous regression model with unknown switch. This can be estimated with the ML or with the E-M algorithm. In case the regime is unobservable, the distribution of the endogenous variables is:

\[
f(y_t) = (2\pi)^{-1/2}||B|| \times [ \lambda \exp\{ -1/2 (By_t - \Gamma X_t - \Delta)'\Sigma^{-1}(By_t - \Gamma X_t - \Delta) \} \\
\quad + (1 - \lambda) \exp\{ -1/2 (By_t - \Gamma X_t)'\Sigma^{-1}(By_t - \Gamma X_t) \} ]
\]

And the likelihood function can be constructed as:

\[
LH = \prod_{t} f(y_t)
\]

Estimation can proceed using ML, but in those days computing was much slower and he adopted the E-M algorithm. In the Markov switching version of the model, by Ellison, straight ML becomes impossible. There is an Estimation stage, where for each observation the probability for each regime is assessed. This generates an estimate of \( I_t: w_t \). Given an initial estimate \( w_t^0 \), an initial estimate \( \lambda^0 \) can be obtained as

\[
\lambda^0 = \frac{1}{T} \sum_{t} w_t^0
\]

In later iterations, this \( w_t^0 \) will be updated by Bayes’ law:

\[
w_t^1 = \text{Prob}\{I_t = 1|y_t, X_t, \Omega^0, \lambda_0\} \\
= \frac{\lambda_0 h(y_t|X_t, \Omega^0, I_t = 1)}{\lambda_0 h(y_t|X_t, \Omega^0, I_t = 1) + (1 - \lambda_0) h(y_t|X_t, \Omega^0, I_t = 0)}
\]
Conditional on the estimate of $w_t$, there is a Maximization stage where the parameters in the demand and supply equation are maximized knowing the regime using the conditional LH function. Iteration produces the ML estimates. In practice the E-M algorithm is an iterated variant of the ML equation with observable regimes, where the regime classifications are updated with Bayes’ law.

The cartel model of Green-Porter underlies the analysis but it does not figure very prominently. It is simply taken that firms will occasionally enter price wars if prices fall below a threshold, because they cannot distinguish cheating on the cartel from drops in costs (which would make firms produce more, depending on the cartel arrangement) or from low demand (which would depress the aggregate price even if firms adhered to their cartel quota). Note that under most cartel theories cheating will not take place, because strategies are specifically designed to avoid that.

Estimation of $\lambda$ can tell us when cartels break down. They cannot tell us why – the area where theories differ. The few papers that attempt to investigate the time series pattern of regimes are less conclusive. Investigations of the question “Do there seem to be price wars?” can take advantage of the data in all periods. Investigating “What sets off price wars?” can only draw on the number of price wars as observations, necessarily a much smaller sample.

Results

With PO (trade press report indicating when there is a price war - straightforward ML maximization), $\beta = 0.382$ and $R^2 = 0.32$ on the supply equation.

With PN (unobserved $I - t$ dummy - E-M algorithm) $\beta = 0.545$ $R^2 = 0.863!!$ Demand is about equal, all coefficients make sense.

In latter case, $\hat{\theta} = 0.336$, larger than 0: some collusion, smaller than 1 (not enforced) lower price than full profit maximization (would not be sustainable), is about equally competitive as Cournot would be.

LR test for $\beta = 0$, estimate model with $I = 0$, gives 554.2 statistic, reject.

Time periods for PN ($w^*_t, 0.5$ or prob($I=1), .5$) coincide to large extent with PO and coincide with price drops. A reasonable and simple model that can explain huge variation (+50%) of $Q$ and $P$ by week.
What is there in the data that lets us tell the regimes apart or even lets us believe there are two regimes?

Identification is solely by functional form.

The extent to which the shape of the joint distribution of \( P \) and \( Q \) on exogenous variables differs from a normal one, leads us to the interpretation that there are different regimes in the data and that these regimes can be usefully interpreted as shifts in the supply relationship. In Porter’s case, the joint distribution has two local modes. The \( P \) and \( Q \) that solve the demand function and supply relationship with \( \alpha_{NC} \) — call them \( P_{NC} \) and \( Q_{NC} \) are random variables (they depend on both errors) with a different mean than the \( P \) and \( Q \) that solve the demand and supply equation with \( \alpha_C \). The first \( P \) will be lower and \( Q \) higher if collusion is successful. The random variables will have different means (but otherwise the same distribution). Conditional on the regime shift, the \( P \) and \( Q \) are distributed differently — different mean. Unconditionally, they will have a distribution with two modes. What makes the regime observable are assumptions to the distribution of the regime: it only takes two values and there are constant probabilities.

A robustness check is provided by comparing the predicted regime states with data collected from archives. Porter’s model also provides an appealing way to explain the 50% period to period changes in prices and quantities.

? argue much more generally that the best a researcher can do without functional form assumptions is to uncover the conditional joint density of \( P \) and \( Q \) conditional on (exogenous) cost and demand shifters. Because prices and quantities are only observed in equilibrium pairs, functional form assumptions on demand and costs are necessary to say anything on market power (which requires separate identification of demand and supply relationships).

It is possible to rewrite Porter’s model with a new constant term \( \beta_0 + \beta E[I_t] = \lambda + \beta \tau \) and add the de-meaned part of \( I_t \) to the error term \( \epsilon'_s = \epsilon_S + \beta(I_t - \tau) \). This illustrates that it is impossible to separately estimate \( \beta_0 \) and \( \tau \).

From the likelihood function we can also see that

\[
\frac{\partial E(y_t | x_t)}{\partial x_{it}} = \tau \frac{\partial E(y_t | I_t = 1, x_t)}{\partial x_{it}} + (1 - \tau) \frac{\partial E(y_t | I_t = 0, x_t)}{\partial x_{it}}
\]

Any partial derivative of the conditional mean is the weighed sum of the partial derivatives of the conditional means under both regimes. Signing the sign of such comparative statics or evaluating their plausibility is much harder if the structure of behavior were ignored. It is possible, but the quantities obtained would be much less informative if Porter’s model were the true DGP.
Bresnahan (1987): Price war in 1955 automobile market

Fact to explain: in 1955 U.S. automobile market: quantity +45% price -2.5%. Looks like a supply shock (shift along the D)
Modeled as a very specific shock: 1 year breakdown in collusion

**Intuition:** in vertical product differentiation model competition is localized (only prices of neighboring products are relevant).

If collusion: P-MC margin will not be very sensitive to the identity of the owner of neighboring products (whether these are sold by the same firm or not) because externality of price on neighbors is taken into account anyway under joint profit maximizing behavior.

If competitive: while each firm still has some market power (consumers in the middle of your market-interval don’t have good substitutes available) this will be lower if close by products are produced by rival firms.

So the test is whether taking the identity of owner of closely located products (good substitutes) is important for the fit of the model. (FIGURE: if 2 and 3 are produced by different firm: small p-mc margin only under Bertrand Nash)

Note: data shows higher profits in 1955, but this is misleading. Accounting profit is determined by taxation rules that spread fixed costs (which are shared over year, e.g. design and tooling) across years, so high volume years automatically look more profitable.
Demand: highly stylized, not too much detail.

utility for consumer of buying a car (will be different at end points, but ignored here):

$$U(x, Y, \nu) = \nu x + Y - P$$

$\nu$ is uniformly distributed over $[0, V_{max}]$ with density $\delta$. (\# of consumers in the market links these two parameters, only 1 free one)

Take 3 cars, ranked by quality $x_h < x_i < x_j$ (price ranking has to be inverse, otherwise a car will be dominated and get no demand. In estimation, this is more or less guaranteed by letting $x$ be determined from characteristics and hedonic-like valuations on them)

Consumer will be indifferent between $h$ and $i$ if

$$P_h - x_h\nu_{hi} = P_i - x_i\nu_{hi}$$

$$\nu_{hi} = \frac{P_i - P_h}{x_i - x_h}$$

consumers with $\nu > \nu_{hi}$ will prefer $i$ over $h$ and vice versa.
Demand for car $i$ will be

$$q_i = \delta (\nu_{ij} - \nu_{hi})$$

Crucial to the intuition is that

- Own price elasticity for $i$ is
  $$\frac{-\delta}{x_i - x_h} - \frac{\delta}{x_j - x_i}$$
- Cross price elasticity with $h$ is
  $$\frac{\delta}{x_i - x_h} > 0$$

The closer $i$ is to $h$ in quality space ($|x_i - x_h|$), the more elastic the demand for $i$ is and the better substitutes they are.

**Supply:** equally stylized

$$C(x, q) = A(x) + mc(x)q$$

$$\frac{\partial X}{\partial q} = mc(x)$$

$$mc(t) = \mu e^t$$

Figure 1
The cost of producing additional quality grows exponentially specified exogenously as a function of characteristics: 
\[ x = \sqrt{\sum_j Z_j \beta_j} \]

behavior:

\[ \max_{p_i} \pi_i = p_i q_i - mc(x_i)q_i - A(x_i) \]

Note that \( q_i \) is linear in own price and price of two neighboring products.

1. Bertrand-Nash in prices: f.o.c. is

\[ q_i + (p_i - mc_i) \frac{\partial q_i}{\partial p_i} + H_{i,i+1} q_i + (p_{i+1} - mc_{i+1}) \frac{\partial q_{i+1}}{\partial p_i} + H_{i,i-1} \text{etc.} = H_{i,i+1} \]

\( H_{i,i+1} \) is equal to 1 if \( i \) and \( i + 1 \) are produced by the same firms. Only then is the demand externality taken into account. The addition to \( p_i \) will be larger the smaller \( |x_i - x_h| \) is.

Note: it can be shown that changing one \( H_{i,i+1} \) from 0 to 1 increases \( p_k - mc_k \ \forall k \)

2. Collusion: the only difference is that \( H_{i,i+1} = 1 \ \forall i \)

3. Two more models are estimated because the nonnested tests can reject both models against each other. Including more models might increase our belief in the robustness of the test results

estimation:

1. calculate \( x \) as a function of starting values for \( \beta \)
2. order all products according to \( x \)
3. calculate \( p^* \) and \( q^* \) as functions of other parameters (\( \delta, V_{max}, \gamma, \nu \))
4. plug in LH function to compare with actual \( p \) and \( q \) realizations: maximize over parameters

Evidence

• strike 1: nonnested Cox test: see if residuals under model \( H_0 \) can be explained to a statistically significant extent by model \( H_1 \). (do it both ways) (test statistic is \( N(0,1) \))

   table III: Collusion never rejected in '54 and '56, against all 3 models strongly rejected in '55. Inverse for Nash.

   Other intuition for this: \( R^2 \) for D and S equations highest with C in '54 and '56 and with N in '55
• strike 2: estimation is independent by year, but prediction coincides with higher level of output in '55 (strike 1 had to do with goodness of fit under two sets of assumptions on $H$ dummies). Level of production provides independent evidence.

• strike 3: parameters are most stable across years if behavior assumption is changed from C to N in '55. With constant behavioral assumption, we see large parameter changes to rationalize the data.


Principle is extremely straightforward (with some wrinkles), computation is very burdensome.

Context: The mother of all exercises of MP.

Firms signed document with California electricity regulation board that they did not posess unilateral MP. Final consumer D is completely inelastic in SR and MP could be very harmful. Firms might withhold capacity from the market (or bid very high in the auctions) to increase price for their operational units. Firms can always claim they shut down because of a breakdown (Duke: employee ordered to throw away spare parts) or they can perform preventative maintenance. Impossible to know whether Q-withholding is strategic or optimal.

MC: estimate an time (hour) specific MC for each plant. We know a lot from pre-liberalization cost filings. Will depend on

• fuel type because fuel prices move around

• heat rating: how efficient generator is. Known and constant

• emissions: right to emit pollution, traded and we use hourly price as opportunity cost

• maintenance and operating costs: observed

Aggregate horizontally (different each hour) to find market MC; step function with length of step capacity of each active plant, height the increment in MC; becomes relatively steep as less efficient units come online. FIGURE
exceeds the direct production cost. Must-take generation operates under a regulatory side agree-
ment and is always ... of the South Coast Air Quality Management District (SCAQMD) in southern California, we also include the cost of NO
emissions, which are regulated under a trade-
able emissions permit system within SCAQMD. The cost of NO
permits was not signiﬁcant in 1998 and 1999, but rose sharply during the summer of 2000. The generator cost ... to be must-take genera-
tion and shows how it increased between 1998 and 2000 due to higher fuel and environmental costs.

Note that because the higher-cost plants
\begin{itemize}
\item tend to be the least fuel efﬁcient and the heavi-
\item est polluters, increases in fuel costs and pollu-
tion permits ...
\end{itemize}
but also increase its slope since the costs of high-
cost plants increase by more than the costs of low-cost plants.

The supply curves illustrated in Figure 1 do not include any adjustments for "forced outages." Generation unit forced (as ... according to a probability speciﬁed by that unit’s forced outage factor. In our analysis, each generation unit,
\( i \), is assigned a constant marginal cost
\( mc_i \)— reflecting that unit’s average heat rate, fuel price, and its vari-
able O&M cost— as well as a maximum output capacity,
\( cap_i \). Each unit also has a forced out-
age factor,
\( fof_i \), which represents the probabil-
ity of an unplanned outage in any given hour. Because the aggregate marginal cost curve is convex, estimating aggregate marginal cost using the expected capacity of each unit,
\( cap_i z_1(1 - fof_i) \), would understate the actual expected cost at any given output level.

We therefore sim-
ulate the marginal cost curve that accounts for forced outages using Monte Carlo simulation methods. If the generation units
\( i = 1, \ldots, N \) are ordered according to increasing marginal cost, the aggregate marginal cost curve pro-
duced by the
\( j \)th draw of this simulation,
\( C_j(q) \), is the marginal cost of the
\( k \)th cheapest gener-
ating unit, where
\( k = \argmin \{ MC_x | \sum_{i=1}^{x} I(x)cap_i \geq q \} \).

This gives one estimate for this hour, draw 100 such samples.

if in doubt... make assumptions that raise P.C. market clearing price:

\begin{itemize}
\item outage is random, while a price taking firm would choose a low demand period to have its units available in peak demand, high price periods.
\item Hydro: mostly owned by utilities who have an incentive to lower p. We assume they cannot influence price by acting strategically with these assets
\item Cost info from RoR period: firms had an incentive to overstate costs
\item adjust import levels if P.C. price would have reduced price
\end{itemize}
Complications

- Nuclear: not in the wholesale market, get separate price
- startup costs: mostly relevant for nuclear, observed for other plants
- marginal unit not fossile: did not happen
- 2 markets: day-ahead (PX) is focus of analysis: 85% of volume. 5% on ISO balance/spot market, remainder in bilateral trade. Average prices are equal, arbitrage!
- could earn rents if price in neighboring states was higher than in CAL, could export: only happened in 17 of the 22681 observed hours
- imports: crucially need elasticity of net import demand. If there is MP, prices are higher and more imports flow in market than under P.C. Counterfactual P.C. price needs to take into account that at lower prices, less than the observed imports would flow into CAL, raising the amount that has to be produced locally and raising the competitive price estimate.
- utilities monopsony power: buy bulk of power on PX market. By withholding some D, price lower there for a large volume. Price higher on spot market, but until firms arbitrage difference away, they make some gains. Gradually the PX market unravelled, authors truncate the sample period.

Results

Mean $P_{PX} >$ mean MC (much larger in summer 2000)
aggregate $\frac{\Delta TC}{T}$ is enormous: cost increase

Fig 3: lerner index $(P-MC)/P$ much higher at high capacity utilization, where we would expect market power to be highest for inframarginal producers.

Table 3: some competitive rents: low-cost units earn scarcity rent on a desirable asset. Swamped by huge oligopoly rents (MP). Difference between actual and expected production cost is additional welfare loss because capacity withholding removes some low cost units from the market.
What may be surprising is that the resulting productive inefficiency is small because nearly all resources are running in any case. B.

With the calculation of deadweight loss due to productive inefficiency, we are now in a position to parse the market payments into costs, competitive rents, and rents due to the exercise of market power. In Figure 6, the quantity $q_{\text{tot}}^2$ represents the amount of power traded in the wholesale market in a given hour, $q_{\text{mt}}$ being must-take power that is not compensated at the market price. Total wholesale market payments are the sum of all the shaded areas. When the areas labeled Mkt. Power Rents and ImportLoss (together the area above $P_{\text{comp}}$) are removed from the total, the result is the total wholesale payments that would have resulted if the market were perfectly competitive. The quantity $q_{\text{rs}}$ represents hydro and geothermal production during the hour. For purpose of calculating the change in rents during summer 2000 and how those rents were divided, we assume that hydro and geothermal power have zero marginal cost, though this assumption has no effect on the calculation of the change in rents. Under competition, the quantity $q^*$ is produced by in-state generation units and the quantity $q_{\text{tot}}^2 - q^*$ is imported. The area labeled Comp. Total Cost is the variable production costs of in-state units (other than must-take production) and of imported power for their respective shares of production. Competition generates inframarginal rents equal to the sum of the areas labeled Comp. Rents 1 and Comp. Rents 2 for in-state fossil-fuel and reservoir generators, and the area Comp. Rents 3 for imports.

Together, these areas—Comp. Rents and Comp. Total Cost—account for all wholesale market payments under perfect competition. With market power, the quantity $q^*$ is produced by in-state generation units and the quantity $q_{\text{tot}}^2 - q^*$ is imported. The areas labeled Comp. Rents 2 and ImportLoss are the additional variable production costs of the imported power under the assumption that imports are bid competitively. In addition to the inframarginal rents

$\frac{\Delta a_{\text{imp}}}{p_{\text{comp}}}$

We have not calculated the import elasticity for prices below $P_{\text{comp}}$ so we assume that marginal costs decrease in a linear fashion and that the marginal costs of imports are zero when the price is zero and assume that all imports that made no (or negative price) adjustment bids had marginal costs equal to zero.

\textbf{Figure 6. Calculation of Division of Rents}