Personal Security Accounts and Mandatory Annuitization In a Dynastic Framework

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1 Introduction

The aging of the populations in the OECD countries has prompted renewed academic research and political discussions of existing social insurance programs. Old age, disability, health and unemployment insurance programs have become the most expensive items on government budgets and demographic projections suggest a significant increase in old age and health portions of social insurance in the next few decades.

Several European Union countries are already implementing reforms to minimize the fiscal burden of the demographic shock on individuals. Similar reform proposals are also being discussed for the United States economy. In 2001, a President’s Commission to Strengthen Social Security was appointed to formulate proposals that would help maintain the current PAYG system’s benefits for current retirees while improving the future pensions of current workers through ‘personal security accounts’. The Commission proposed three plans. The first plan only recommended that workers make a voluntary contribution out of their social security payroll taxes into a Personal Security Account (PSA), owned and managed by the worker and invested in a well-diversified portfolio. The second plan extended this idea further and recommended diverting 4 percentage points of the social security payroll tax to PSAs while reducing the indexation of current pension by following
price increases as opposed to wage increases. The third plan introduces a mix of add-on and carve-out approach such that a worker who chooses to voluntarily invest an additional 1 percent of earned income may divert 2.5 percent of social security payroll taxes, up to $1,000 annually, to PSAs. Although all three plans are clearly partial privatization plans, plans 2 and 3 also guarantee 30-year minimum-wage workers a retirement income above the poverty line. All 3 plans allow for individual ownership of the accounts that can be bequeathed to heirs.

The debate on social security reform in the U.S. has focused on the relative merits of the existing PAYG system versus some form of partial privatization with a safety net for low income workers. In its key features, the 2001 Commission’s proposals are similar to the reform recommendations made by the 1997 Advisory Council on Social Security, according to which there were three approaches formulated to solve the long-term financial imbalance in the program:

- Maintain Benefits: keep them essentially as is, but tax them.
- Individual Accounts: an additional and mandatory contribution of

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1The organization of the ‘Council’ changed after 1997. At the time of this writing, its successor is the Social Security Advisory Board (SSAB), an independent, bipartisan board created by Congress and appointed by the President and the Congress to advise on matters related to the Social Security and Supplemental Security Income Programs. Its 2003 report urged rapid reform.
1.6% of payroll.

- Personal Security Accounts: Convert the current system to a basic, flat benefit program, and redirect 5% of the existing payroll tax to PSAs.

In the current defined benefit system, individuals are given insurance against life span risk as they receive retirement benefits as long as they are alive. A switch to a defined contribution retirement program can be designed to provide the same insurance if the funds in the individuals’ accounts are required to be annuitized. However, as Diamond (1998) points out, political feasibility may necessitate that the accounts’ of the deceased be transferred to their estates.²

Recent proposals for public, defined contribution programs address withdrawals in several, different ways: 1) the accumulations in PSAs are used to finance real annuities, 2) lump-sum withdrawals are made, or 3) the benefi-

²Boskin (1986) proposed a defined contribution plan where individuals’ accounts are mandatorily annuitized. His suggestion was to separate retirement policies into two parts; the annuity part would provide actuarily equivalent insurance for retirement annuities, disability, and catastrophic hospital care, and the welfare (or transfer) part, which would guarantee a minimally adequate level of retirement income to all citizens. Huggett and Ventura (1999) studied the steady-state effects of a Boskin type reform. Their treatment of the reform mimics the Boskin proposal and is quite different from the PSAs suggested above. First, the annuity part is financed in a PAYG basis. Second, benefits are set equal to the maximum of the floor benefit and the annuity part of the Boskin proposal. The floor benefit is proportional to output per worker, which is similar to the PSA reform. The social security tax rate that finances the annuity part is set equal to the actual U.S. social security tax rate.
ciary is allowed to choose some options, making phased withdrawals accord-
ing to some formula and mandatorily annuitizing the remaining amount. Designing regulations to govern the withdrawal of funds during retirement is further complicated if individuals face binding borrowing constraints. Clearly, distribution rules are an integral part of the architecture of a suc-
cessful social security reform program.

**Annuity Markets**

From Yaari (1965)'s work we know that individuals facing mortality risk in a pure life cycle model find it optimal to annuitize their wealth. More recentely, Mitchell, Poterba, Warshawsky, and Brown (1999) estimate that individuals facing mortality risk in an overlapping generations model without bequests are willing to give up more than 20-25% of their wealth to purchase actuarially fair annuities rather than follow the optimal consump-
tion plan without annuities, even in the presence of significant pre-retirement social security wealth. However, private annuity markets are very small.  

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This estimate is probably an upper bound on the welfare benefit of annuities for several reasons. First, the utility calculations that yield these estimates only take into account consumption and annuities after retirement, taking as given wealth at retirement. If individuals were to make their decisions at the beginning of their working life, which is what is assumed in almost all life cycle models, then they could plan better for mortality risk, where as in the above calculations they are surprised with mortality at retirement. Second, these calculations abstract from general equilibrium effects. Mortality risk tends to raise aggregate saving which in turn lowers the return to capital. Finally, they assume that individuals have no bequest motives.
Diamond (1977) suggests that one possible reason may be the presence of asymmetric information and adverse selection and the accompanying failure in the market for annuities. Friedman and Warshawsky (1990) and Mitchell, Poterba, Warshawsky, and Brown (1999) document how unattractively private annuity contracts are priced in the U.S. and argue that adverse selection might lead to this outcome. Indeed, Feldstein (2005) emphasizes that the problem of asymmetric information and adverse selection could “...weaken the functioning of private insurance markets”.4

**Modeling the Costs and Benefits of Social Security**

The large literature that attempts to quantify the overall welfare effects of unfunded social security using applied general equilibrium models can be summarized as follows. The welfare costs of social security arise because the public provision of retirement insurance leads to:

1. a reduction in saving,

2. a distortion in labor supply,

3. a distortion on the retirement decision (early retirement)5.

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4 Starting with Hubbard and Judd (1987), quantitative models have been used to evaluate the role of social security in providing insurance against mortality risk.

5 Social security has become the last stop in the transition from unemployment insurance to disability and finally to social security for many individuals in EU countries. Although at a relatively smaller scale, the U.S. has also started to experience a similar transition from one public insurance program to another.
Social security potentially raises individual welfare by improving risk sharing in the population\(^6\). In particular, social security provides partial insurance against:

1. mortality risk,
2. individual income risk,
3. investment income risk.

Using an overlapping generations setting without bequests, previous research has largely found that social security imposes an overall welfare cost on the society. As Auerbach and Kotlikoff (1987) argue, the main reason for this negative finding is the large decrease in the capital stock which in turn leads to a significant reduction in lifetime consumption. Hubbard and Judd (1987) allow social security to provide longevity insurance but its partial insurance aspect is not sufficient to overturn the negative lifetime consumption effect of social security. İmrohoğlu, İmrohoğlu, and Joines (1995, 1999) and Storesletten, Telmer, and Yaron (1999) introduce individual income risk as an additional benefit of social security but obtain a similarly

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\(^6\)Following Kehoe and Levine (1993, 2001), Attanasio and Rios-Rull (2000) use a model with limited enforceability of private contracts to study the effectiveness of social insurance programs in distributing risk across individuals. They find that public insurance, designed to yield more risk-sharing (such as more redistributive taxes), hinders the ability of individuals for insuring against the idiosyncratic shock so that overall it leads to a decline in risk-sharing and ex-ante welfare.
negative outcome for social security.

These findings suggest that we might expect to observe widespread privatizations in the U.S., EU countries and elsewhere. There are at least three explanations for the persistence of PAYGO systems, inspite of the steady-state burdens associated with them.

First, the transitional costs from a pay-as-you-go social security system to a fully privatized retirement system may be large. Huang, İmrohoroğlu, and Sargent (1997) and Kotlikoff, Smetters, and Walliser (1999) consider several alternative policy transitions from the current U.S. system to a privatized one. De Nardi, İmrohoroğlu, and Sargent (1999) examine various alternative transitional policy responses to the aging of the U.S. population.7 These studies and Conesa and Krueger (1999) find large transitional costs, with a majority of the currently alive population suffering welfare losses, and, thus, blocking any reform proposal. Second, there may be political-equilibrium considerations that allow the introduction and maintenance of an unfunded social security system, as Cooley and Soares (1999) and Boldrin and Rustichini (2000) argue. Finally, it is possible that not all potential

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benefits of social security have been carefully modeled. In particular, the insurance role of social security in shielding the participants from investment income risk is only recently being carefully analyzed. In a recent paper, Krueger and Kuebler (2002) use an overlapping generations model with a stochastic production technology and incomplete markets to evaluate the welfare consequences of introducing a small unfunded social security system. Their main quantitative finding is that the ‘capital crowding-out’ effect mentioned above outweighs the insurance role of social security against realistic investment income risk.

**This Paper**

The studies summarized above make the extreme assumption that consumers have little or no altruism. However, there is a substantial literature that suggests that at least a fraction of individuals behave in a way consistent with an operative bequest motive. Laitner and Juster (1996) and

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8 Another justification for social insurance programs is the possibility that a fraction of the population might lack the foresight to accumulate sufficient assets for their old age consumption. For example, Diamond (1977) and Feldstein (1985) argue that many individuals will simply not follow an ‘optimal’ saving program on their own. Following the time-inconsistent preferences approach suggested by Strotz (1956), Akerlof (1998) argues that the quasi-hyperbolic model might justify the existence of social security as it acts as a commitment device in an economy populated with individuals who potentially value such an institution. However, Imrohoroglu, Istehoroglu, and Joines (2003) examine the welfare effects of social security on individuals with time inconsistent preferences and find i) that social security is a poor substitute for a perfect commitment technology in maintaining old-age consumption, and, ii) that unfunded social security generally does not raise welfare for short-term discount rates of up to 15 percent.
Wilhelm (1996) are recent examples of empirical papers that provide some evidence on the strength of altruism. This paper assumes individuals have two-sided altruism, they care both about their parents and their children.

Fuster (1999) and Fuster, Ímrohoğlu and Ímrohoroğlu (2003, 2005) explore the role of social security in a dynastic setting with incomplete markets and distorting taxes, and show that there are qualitative and quantitative differences between the pure lifecycle and the dynastic environments. In particular, they find 1) that the ‘capital crowding-out’ effect of unfunded social security is quantitatively much smaller in the dynastic model due to the strong additional motive to save, and 2) that ‘family insurance’ largely makes up for the loss of publicly provided insurance in privatization experiments. Furthermore, they highlight the importance of a ‘flexible’ labor market as one of the key conditions for a successful social security reform. When the PAYGO system is eliminated, there is a sizable reduction in the labor income tax rate, which raises labor supply and generates large welfare gains from privatization. This is especially true in an economy in which the link between contributions and benefits in the PAYGO system is small. There are three features of an economy that enhance the success of social security reform: i) operative bequest motives, ii) flexible labor markets, and, iii) weak linkage between contributions and benefits.
In this paper, we quantitatively evaluate the welfare effects of reforming social security by introducing a PSA with and without mandatory annuitization in an economic environment with uninsurable individual income shocks, two-sided bequests, borrowing constraints, and missing annuity markets. Our setup allows us to assess whether mandatory saving or mandatory annuitization of accumulated PSA wealth at retirement is welfare enhancing, and if so, for what type of households.

In our model, family insurance takes the form of intervivos transfers and bequests. There are three general types of households depending on the number of surviving household members. In type 1 households, the parents have died and only the children survive. In type 2 households, the children have died and only the parents survive. Type 3 households constitute the majority and have both the parents and the children in the household. In this type, we have further heterogeneity. Since the parents and the children receive the realization of generationally-persistent ‘ability’ shocks, type 3 households are further divided into four ability combinations depending on the high-low ability realizations of the parents and the children.9

This framework is well suited to consider the annuity role of social security for single individuals versus for households where families also provide

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9Type 1 and type 2 households are of two ability/longevity types, ‘high’ and ‘low’.
annuity insurance to their members. Our goal is to evaluate quantitatively PSA reforms with or without mandatory annuitization of PSA wealth at retirement. We do this in a setup where some households have higher desire to annuitize wealth than others.

We calibrate our economy to the U.S. economy, use numerical, discrete state-space methods to solve the households’ recursive decision problem, and restrict attention to steady-states under various pension systems.

The paper is organized as followed. Section 2 presents the model in detail. Section 3 contains calibration. Numerical results are given in Section 4, and concluding remarks are presented in Section 5.

2 The Model

The economic environment in this paper follows Fuster, İmrohoroğlu, and İmrohoroğlu (2003). It is an applied general equilibrium model with overlapping generations facing lifespan and ability uncertainty, borrowing constraints, and exhibiting two-sided altruism. The cost of the model’s richness in demographic and productivity dynamics is that we simplify the macroeconomic context. There is no uncertainty to the return to capital in this model. In order for this paper to be self-contained, we will describe the model although some of the details can be obtained in Fuster, İmrohoroğlu,
and İmrohoroğlu (2003).

2.1 Demographics and Endowments

Our setup is a stationary overlapping generations model where every period $t$ a generation of individuals is born. Individuals face random lives and some live through the maximum possible age $2T$. If the individual survives, then his lifetime support overlaps during the first $T$ periods with the lifetime support of his father and during the last $T$ periods with the lifetime support of his children. The total population in the economy consists of $2T$ overlapping generations of individuals with total measure one.

At birth, individuals make a draw from a distribution of abilities. An individual’s ability can be high or low and it determines both the individual’s lifetime labor productivity and his life expectancy. If the ability is high, $z = H$, the individual enjoys a permanently higher labor productivity throughout his life-span than an individual with low ability, $z = L$. The labor productivity of an individual of ability $z$ and age $j$ is denoted by $\varepsilon_j(z)$. We assume that from age $R$ to age $2T$ the labor productivity is zero so $R$ represents the exogenous retirement age. At any other age the individual supplies inelastically labor to firms.

An individual with high ability also enjoys a permanently higher condi-
tional survival probabilities throughout his life-span than an individual with low ability. With this assumption we want to capture the fact that in the U.S. economy, survival rates are higher among individuals with relatively high education level. We will use $\psi_j(z)$ to denote the probability of surviving to age $j + 1$ conditional on having survived to age $j$ for an individual with ability $z$ for age $j = 1, 2, \ldots, 2T$, where $\psi_{2T}(z) = 0$ and $z \in \{H, L\}$.

We assume that abilities are correlated across generations of the same family line. In particular, ability, $z$, is a two-state, first-order Markov process with the transition probability matrix

$$
\Pi(z', z) = [\pi_{ij}], \quad i, j \in \{H, L\},
$$

where $\pi_{ij} = \Pr\{z' = j \mid z = i\}$, $z$ is the labor ability of the father and $z'$ is the labor ability of the new born in the dynasty. In order to develop economic intuition about how the model works, it is important to note that there are no private insurance markets in the economy to diversify the risk of being born as a low ability-type individual. However, the informal family structure and some partial annuities, public or privately administered if they exist, do provide some partial insurance against this type of shock.

Cohorts shares are time invariant due to our assumptions of constant
conditional survival probabilities and population growth rate \( n \). Using 
\[
\mu_1(z) = \lambda(z)(1 + n)^T
\]
to indicate the size of cohort 1 (newborns) with ability \( z \), relative to that of cohort \((T + 1)\) (parents), where \((1 + n)^T\) is the number of children per parent and \( \lambda(z) \) is the measure of newborn individuals with ability \( z \), we can obtain the relative sizes of the other generations recursively:
\[
\mu_{j+1}(z) = \frac{\psi_j(z)\mu_j(z)}{(1 + n)}, \quad j = 1, \ldots, 2T - 1.
\]

2.2 Technology

There is a continuum of identical firms that produce a final good using capital and labor. The technology is represented by a constant returns Cobb-Douglas production function 
\[
Y_t = K_t^\alpha(A_tN_t)^{1-\alpha},
\]
where \( \alpha \in (0, 1) \) is the output share of capital, \( Y_t \) is output at time \( t \), \( K_t \) is aggregate capital input at time \( t \), \( N_t \) is aggregate labor input at time \( t \), and \( A_t \) is an exogenous labor-augmenting technological progress growing at a constant rate \( \gamma \). Capital depreciates at a constant rate \( \delta \in (0, 1) \). Since all markets are competitive, the profit maximizing conditions imply that factor prices are set equal to
marginal products, that is,

\[
\tilde{r}_t = \alpha K_t^{\alpha-1} (A_t N_t)^{1-\alpha}, \\
\omega_t = (1 - \alpha) K_t^\alpha (A_t N_t)^{-\alpha}
\]  

(1) 

(2)

where \(\tilde{r}_t\) is the rental price of capital and \(\omega_t\) is the wage per effective labor.\(^{10}\)

2.3 Social Security and Fiscal Policy

2.3.1 The Benchmark Social Security System: Pay-as-you-go.

In the benchmark economy, we assume that the social security system finances current pensions with the contributions of current workers. That is, the system transfers income from workers to retirees. In the U.S. economy, retirement benefits depend on individuals’ average lifetime earnings via a concave, piecewise linear function. The marginal replacement rate decreases with average lifetime earnings indexed to productivity growth. It is equal to 0.9 for earnings lower than 20% of the economy’s average earnings. Above this limit and below 125% of the economy’s average earnings the marginal replacement rate decreases to 0.33. For income within 125% and 246% of the economy’s average earnings the marginal replacement rate is 0.15.

\(^{10}\) In what follows, we express all variables per effective units of labor.
Additional income above 246% of the economy’s average earnings does not provide any additional pension payment.

In our benchmark economy, the function that relates the pension \( B_j(z) \), with the individual’s ability, \( z \), mimics the progressivity of the US social security benefits.\(^{11}\) In particular, at retirement, the pension payment for each ability group is calculated as follows:

\[
B_R(z) = \begin{cases} 
0.9 \cdot (0.2\overline{M}) + 0.33 \cdot (M(z) - 0.2\overline{M}), & \text{for } z = L, \\
0.9 \cdot (0.2\overline{M}) + 0.33 \cdot (1.25\overline{M} - 0.2\overline{M}) + 0.15 \cdot (M(z) - 1.25\overline{M}), & \text{for } z = H, 
\end{cases}
\]

where \( M(z) \) denotes the average lifetime earnings of an individual of ability \( z \) and \( \overline{M} \) denotes the economy’s average earnings.

Note that an individual’s pension remains constant during retirement while technology grows at the rate \( \gamma \). Thus the pension per effective labor decreases during retirement at rate \( \gamma \), that is, \( B_j(z) = B_R(z)/(1 + \gamma)^{j-R} \).

Pensions are financed by taxing earnings at a rate \( \tau \) and the budget of the

\(^{11}\)This function captures the differential in pension across the average college and non college worker observed in the US economy. Individuals without college education have average lifetime earnings between 20% and 125% of the economy’s average earnings. The average lifetime earnings of individuals with college education is between 125% and 246% the economy’s average earnings.
social security system is balanced at every period, that is,

\[
\sum_{z}^{2T} \sum_{j=R} \mu_j(z) B_j(z) = \tau \omega N. \tag{3}
\]

2.3.2 Reform 1: Personal Security Accounts without Annuitzation

Under a PSA system, retirement benefits come from two distinct sources. The first tier is a flat pension benefit equal to 18 percent of per capita GDP (a monthly payment of $410 in 1996). This portion of the total benefit is financed in a pay-as-you-go fashion by taxing current labor income. Hence, the social security tax rate that finances the first-tier system is set such that its aggregate revenue equals the aggregate first-tier benefits:

\[
\sum_{z}^{2T} \sum_{j=R} \mu_j(z) b = \tau_a \omega N, \tag{4}
\]

where \( b \) denotes the flat benefit, \( \tau_a \) is the social security payroll tax to finance the first-tier benefits. We can obtain the following close form solution for the equilibrium tax rate by substituting \( b = 0.18y \) and \( y = \omega N/(1 - \alpha) \) in
the above equation:

\[ \tau_s = \frac{0.18}{1 - \alpha} \sum_{z} \sum_{j=R}^{2T} \mu_j(z). \]

The second tier of retirement benefits is financed by forced saving. Every period, the individual deposits 5 percent of earnings in PSAs. These funds are owned and managed by the individuals, invested in the capital market where they earn the rate of return on capital, and cannot be withdrawn until the individual retires. The capital income accumulated in the PSA is not taxed during the individual’s working life. The amount of second tier benefits is determined by the wealth accumulated in these tax-favored personal security accounts.\(^{12}\) These assumptions define the following law of motion of the PSA (for an individual of ability \(z\))

\[
(1 + \gamma)s_{j+1}(z) = (1 + \bar{r})s_j(z) + \kappa \omega z_j(z),
\]

where \(\kappa = 0.05\) is the social security tax that finances the second-tier benefits, \(s_{j+1}(z)\) denotes the PSA funds of an age-\(j + 1\) individual with ability \(z\) and \(s_1(z) = 0\). At retirement the individual gets a lump-sum transfer of the wealth accumulated in his account which amounts to \((1 + \bar{r})s_{R}(z)\).

\(^{12}\)We do not analyze whether these tax-favored accounts produce new saving. See Imrohoroglu, Imrohoroglu, and Joines (1998) for the impact of tax-favored Individual Retirement Accounts on saving in the United States.
If the individual does not survive to his retirement, his PSA funds are transferred to his estate. Note that this reform is essentially a partial privatization achieved by a mandatory saving program.

Summarizing, the pension benefits under PSA (Reform 1) are described as follows:

\[
B_j(z) = \begin{cases} 
  b + (1 + \bar{\gamma})sR(z) & \text{for } j = R \\
  b/(1 + \gamma)^{j-R} & \text{otherwise}
\end{cases}
\]

and the tax rate on earnings used to finance pensions, \(\tau\), is the sum of \(\kappa\) and \(\tau_s\).

### 2.3.3 Reform 2: Personal Security Accounts with Mandatory Annuitization

In a modified version of the above reform, we consider the case of partial privatization in which the funds accumulated in PSA are annuitized by the institution managing the PSAs. At retirement, individuals are entitled to an annuity payment, \(b(z)\), which we assume to be proportional to the wealth accumulated in the PSA at retirement, that is, \(b(z) = p(1 + \bar{\gamma})sR(z)\), where the proportion \(p\) is determined endogenously as we explain below. The annuity payment remains constant during retirement, just like the first-tier flat PAYG benefit and, thus, the annuity payment per effective units of labor
decreases at the rate $\gamma$.\footnote{At any age $j$, the annuity payment per effective units of labor equals $b(z)(1 + \gamma)^{R-j}$.}

In order to compute the annuity payment of each individual, the social security system has to determine the proportion $p$ of accumulated wealth that the individual receives during retirement. This variable $p$ is set such that, the expected present value of the aggregate annuity payments of the generation that retires today equals the aggregate wealth held in PSA (the funds of individuals that are alive at retirement plus the funds of individuals that die before reaching retirement).\footnote{Notice that, $p$ is computed for the generation that retires in a given period and that we do not index $p$ by time because we are assuming a stationary equilibrium.} The expected present value of the aggregate annuity payments of the generation that retires today is given by

$$
\sum_z p(1 + \bar{r}) s_R(z) \lambda(z) \sum_{j=R}^{2T} (1 + \bar{r})^{R-j} \prod_{i=1}^{j-1} \psi_i(z).
$$

The aggregate wealth accumulated in PSA by individuals in this cohort who survived to retirement is given by \(\sum_z (1 + \bar{r}) s_R(z) \lambda(z) \prod_{i=1}^{R-1} \psi_i(z)\), and the aggregate wealth accumulated by individuals in this cohort who died before retirement is

$$
\sum_z (1 + \bar{r}) \sum_{j=2}^{R} \left( \frac{1 + \gamma}{1 + \bar{r}} \right)^{R-j} s_j(z) \lambda(z)(1 - \psi_{j-1}(z)) \prod_{i=1}^{j-2} \psi_i(z).$$

As a result,
proportion $p$ must satisfy the following condition:

$$p = \frac{\sum_z s_R(z) \lambda(z) \prod_{i=1}^{R-1} \psi_i(z) + \sum_z \sum_{j=2}^R \left( \frac{1 + \gamma}{1+\tilde{\gamma}} \right)^{R-j} s_j(z) \lambda(z)(1 - \psi_{j-1}(z)) \prod_{i=1}^{j-2} \psi_i(z)}{\sum_z s_R(z) \lambda(z) \sum_{j=R}^{2T} (1 + \tilde{\tau})^{R-j} \prod_{i=1}^{j-1} \psi_i(z)}.$$

The above condition implies that the return of annuities is linked to the average mortality rate across individuals with differential mortality (high and low ability individuals) and, as a result, the return of annuities is not fair. This return could not be offered by private annuities since it would not be accepted by individuals with high mortality rate (low ability). In contrast, the government can provide this return because annuities are mandatory. In other words, the government can overcome an adverse selection problem (private information on differential mortality) in the annuity market because PSA are mandatorily annuitized.

The law of motion of the aggregate wealth held in PSA by the social security system is

$$(1 + n)(1 + \gamma)W_{t+1} = (1 + \tilde{\tau})W_t + \kappa \omega N - \mathcal{B}$$

where $\kappa = 0.05$ and $\mathcal{B} = \sum_z \sum_{j=R}^{2T} b(z)(1 + \gamma)^{R-j} \mu_j(z)$ denotes the aggregate annuity payments at period $t$. At each period $t$, the aggregate funds in PSA,
$W_t$ are invested in the capital market.

Summarizing, the pension benefits under Reform 2 are described as follows:

$$B_j(z) = \begin{cases} b + b(z) & \text{for } j = R \\ \frac{(b + b(z))(1 + \gamma)^{j-R}}{1 + \gamma} & \text{for } j > R, \end{cases}$$

and the tax rate on earnings used to finance pensions, $\tau$, is the sum of $\kappa$ and $\tau_s$.

### 2.3.4 Government Budget

In addition to the administration of the pension system, the government taxes labor income, capital income and consumption in order to finance exogenously given government purchases. We assume that the government’s budget is balanced each period. Since tax rates and government expenditure are exogenous, the budget is balanced by an endogenous lump-sum transfer to the individuals. The government also collects the asset holdings and capital income of individuals that die without descendents. These resources are transferred in a lump-sum fashion to all survivors.\[^{15}\]

\[^{15}\text{In previous work, we have experimented with other distribution schemes for unintended bequests. Since the flow of these is only a small fraction of per person income, our quantitative results are robust to other schemes.}\]


2.4 Preferences

The preference structure in our setup follows Laitner (1992)’s two-sided altruistic specification in which individuals derive utility from their own lifetime consumption and from the felicity of their predecessors and descendants. An important feature of Laitner (1992)’s model is that parents and children have the same objective function during the periods when their lifetime overlaps. Due to this commonality of interest, strategic behavior between the father and the children does not arise and, thus, father and children constitute a single decision unit by pooling their resources. We call this decision unit a ‘household’ which is constituted by an adult male, the father, of generation \( j \) and age \( T + 1 \), and his \( m = (1 + n)^T \) adult children of generation \( j + T \) and age 1.

We model altruism as in Laitner (1992) but in a model with uncertain lifetimes. This framework allows us to evaluate the annuity role of various social security institutions when families can also provide annuity insurance to their members. Because we assume uncertain lifetimes, the composition of the household evolves stochastically in our framework. At each period there are three types of households.\(^{16}\) Type-1 households are those where

\(^{16}\)We are assuming that, In a given household, all children are born at the same period and all of them die at the same period. Furthermore, we take all children in a given household to be identical regarding their labor abilities and vector of conditional survival
the father has died. Type-2 households consist only of the father since the
m children have died. Households of type-3 are those where both the father
and the children are still alive. Moreover, households are heterogeneous with
respect to their asset holdings, age, and abilities.\textsuperscript{17}

The budget constraint facing an age-$j$ household, where $j = 1, 2, \ldots, T-1$ is the age of the youngest member(s), is given by

\[
[\phi_s(h) + \phi_f(h)](1 + \tau_c)c_j + (1 + \gamma)a_j = [1 + r(1 - \tau_k)]a_{j-1}
\]

\[
+ e_j(h, z, z') + [\phi_s(h) + \phi_f(h)](\xi_1 + \xi_2),
\]

where $\phi_s$ is an indicator function which takes the value $m$ if the children are
alive and 0 otherwise, while $\phi_f$ is an indicator function that takes the value
unity if the father is alive and 0 otherwise; $h \in \{1, 2, 3\}$ is an indicator of
household composition, $r = \bar{r} - \delta$, $e_j(h, z, z')$ are the after tax earnings, $c_j$ is
the consumption of each household member, $a_j$ denotes the asset holdings
to be carried over to age $j + 1$, $\xi_1$ is the lump sum redistribution of unin-

\textsuperscript{17}A household survives $T$ periods or until the father and the children have died. If
the children survive to age $T + 1$, each of them becomes a father in the ‘next-generation
household’ of the same dynasty. Otherwise, the family line is broken, and this dynasty
is over. Since the population experiences broken dynasties every period, we assume that
these dynasties are replaced by new dynasties to maintain our assumption of a stationary
demographic structure. Since mortality rates are higher for low ability individuals, the
number of new dynasties of low ability is higher than the number of dynasties of high
ability. A new dynasty begins with an individual of age 1 that holds zero assets.
tended bequests left behind by fathers without sons and confiscated by the government, \( \xi_2 \) is a lump sum transfer to balance the government’s budget, and \( \tau_c \) and \( \tau_k \) denote the consumption and capital income tax rates, respectively. All per capita aggregate quantities reported in the paper are divided by the level of the technology, \( A_t \), and therefore represented in efficiency units. As we restrict attention to steady-states, consumption, asset holdings, lump-sum transfers, and earnings are in efficiency units and constant over time.

We represent the net of tax earnings of an age-\( j \) household with the function \( e_j(h, z, z') \):

\[
e_j(h, z, z') = \begin{cases} 
\phi_s(h)\omega(1 - \tau - \tau_\ell)e_j(z') + \phi_f(h)B_{j+T}(z) & \text{if } j \geq R - T, \\
\phi_s(h)\omega(1 - \tau - \tau_\ell)e_j(z') + \phi_f(h)\omega(1 - \tau - \tau_\ell)e_{j+T}(z) & \text{if not},
\end{cases}
\]

where \( \tau \) is the social security tax, \( \tau_\ell \) is the personal tax rate on labor income, and \( B_{j+T}(z) \) denotes the pension at age \( j + T \).
For \( j = T \), the budget constraint of the household is given by

\[
[\phi_s(h) + \phi_f(h)](1 + \tau_c)c_T + (1 + n)^T(1 + \gamma)a_T
\]

\[
= [1 + r(1 - \tau_k)]a_{T-1} + e_T(h, z, z') + \\
[\phi_s(h) + \phi_f(h)](\xi_1 + \xi_2).
\]

If the children survive to age \( T \), \((1 + n)^T\) new households are constituted in the dynasty and each of them will hold \( a_T \) assets. If the children do not survive to age \( T \), the family line breaks.

It is assumed that households face borrowing constraints and cannot hold negative assets at any age:

\[
a_j \geq 0, \quad \forall j.
\]

Individuals obtain utility from their consumption and from their predecessors and descendents consumption. We restrict the utility function to the CRRA class because we assume a balanced growth path for our economy. We will use the language of recursive economic theory to describe the household’s decision problem.

Let \( V_j(a, h, z, z') \) denote the maximized value of expected, discounted lifetime utility of an age-\( j \) household with the state vector \((a, h, z, z')\). For a
household of age \( j \leq T \),

\[
V_j(a, h, z, z') = \max_{\{c, a_j\}} \left\{ \left[ \phi_s(h) + \phi_f(h) \right] \frac{c^{1-\sigma}}{1-\sigma} + \beta(1 + \gamma)^{1-\sigma} \tilde{V}_{j+1}(a', h', z, z') \right\}
\]

subject to (6)-(9),

where \( \sigma \) is the coefficient of relative risk aversion and

\[
\tilde{V}_{j+1}(a', h', z, z') = \begin{cases} 
\sum_{h''=1}^{3} \chi_j(h, h'; z, z') V_{j+1}(a', h', z, z') & \text{for } j = 1, 2, \ldots, T-1, \\
\psi_T(z')(1 + n)^T \sum_{z'' \in \{H, L\}} \pi_{z'z''} V_1(a', 3, z', z'') & \text{for } j = T,
\end{cases}
\]

\( \chi_j(h, h'; z, z') \) is the probability that a household of age \( j \) and type \( h \) becomes type \( h' \) the next period given that the father is of ability \( z \) and the children of ability \( z' \).\(^{18}\)

### 2.5 Steady State Equilibrium

A fiscal policy is a set \( \{G, B, \tau_\ell, \tau_k, \tau_c, \tau_s, \kappa, \tau\} \). Given fiscal policy, a stationary recursive competitive equilibrium is a set of value functions \( \{V_j(a, h, z, z')\}_{j=1}^{T} \), households’ decision rules \( \{c_j(\cdot), a_j(\cdot)\}_{j=1}^{T} \), time invariant measures of households \( \{X_j(a, h, z, z')\}_{j=1}^{T} \), relative prices of labor and capital \( \{\omega, r\} \), a lump

\(^{18}\)We describe the computation of the measures of households in detail in the Appendix. For a description of the solution method see Fuster (1999).
sum distribution of unintended bequests $\xi_1$, and a lump-sum government transfer $\xi_2$ such that the following conditions are satisfied:

1. given fiscal policy, prices and lump-sum transfers, households’ decision rules solve households’ decision problem (10);

2. firms maximize profits, i.e. (1) and (2) hold;

3. aggregation holds,

$$\tilde{K} = \sum_{j,a,h,z,z'} a_{j-1}(a,h,z,z') \chi_j(a,h,z,z')(1+n)^{1-j} + W,$$

$$\tilde{N} = \sum_{j=1}^{R-1} \sum_{z \in \{H,L\}} \varepsilon_j(z) \mu_j(z),$$

$$C = \sum_{j,a,h,z,z'} \left[ \phi_s(h) + \phi_f(h) \right] c_j(a,h,z,z') \chi_j(a,h,z,z')(1+n)^{1-j},$$

where $W = \sum_{z} \sum_{j=1}^{R} s_j(z) \mu_j(z)$ is the aggregate PSA wealth in the economy, adjusted for growth;

4. the set of age-dependent measures of households satisfies

$$X_{j+1}(a', h', z, z')$$

$$= \sum_{\{a, h: a' = a_j(a,h,z,z')\}} X_j(a, h, z, z') \chi_j(h, h'; z, z'), \text{ for } j = 1, \ldots, T - 1;$$

29
the invariant distribution of age-1 households is given by conditions

\[ X_1(a', 3, z', z'') = \pi_{z'z''} \sum_{\{a, h, z: a' = a_T(a, h, z, z')\}} X_T(a, h, z, z') \chi_T(h, 3; z, z'), \]

and

\[ X_1(0, 1, z', z'') = \lambda(z') \pi_{z'z''} - \sum_{a'} X_1(a', 3, z', z''), \]

that is, new dynasties, holding zero assets, substitute for the family lines broken during any given period;

5. the lump-sum redistribution of unintended bequests satisfies

\[ \xi_1 = (1+r) \sum_{j=1}^{T} a_j(a, h, z, z') X_j(a, h, z, z') \left[ 1 - \sum_{h'=1}^{3} \chi_j(h, h'; z, z') \right] (1+n)^{1-j}, \]

6. the government’s budget is balanced

\[ \xi_2 = \tau_k r \left[ \bar{K} - \frac{\xi_1}{1+r} \right] + \tau_\delta \omega \bar{N} + \tau_C C - G; \]

7. the budget of the social security system is balanced, i.e. (3), or (4) hold depending on the social security system in place;
8. the goods market clears

\[ C + \left[ (1 + n)(1 + \gamma)\tilde{K} - (1 - \delta)\tilde{K} \right] + G = \tilde{K}^\alpha \tilde{N}^{1-\alpha}. \]

3 Calibration and Solution

We are going to calibrate our model economy to the long run quantities in the U.S. economy in order to conduct our counterfactual experiments for reforming social security. Our main calibration target is the average capital-output ratio in the U.S. economy over the last fifty years, 2.5. Table 1 shows the major modeling and calibration choices made.
Table 1: List of Parameters

<table>
<thead>
<tr>
<th>Demographics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$2T$</td>
<td>= 14</td>
</tr>
<tr>
<td>$R$</td>
<td>= 10</td>
</tr>
<tr>
<td>$n$</td>
<td>= 0.012</td>
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</table>

<table>
<thead>
<tr>
<th>Maximum lifetime (70 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retirement age (45 years)</td>
</tr>
<tr>
<td>Population growth rate (annual)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Preferences</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>= 2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>= 0.988</td>
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</table>

<table>
<thead>
<tr>
<th>Coefficient of relative risk aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual discount factor</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Technology</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>= 0.0165</td>
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<tr>
<td>$\alpha$</td>
<td>= 0.31</td>
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<tr>
<td>$\delta$</td>
<td>= 0.044</td>
</tr>
<tr>
<td>$\lambda(H)$</td>
<td>= 0.28</td>
</tr>
<tr>
<td>$\pi_{LL}$</td>
<td>= 0.83</td>
</tr>
<tr>
<td>$\pi_{HH}$</td>
<td>= 0.57</td>
</tr>
</tbody>
</table>

| Annual rate of growth of technology   |
| Capital share of GNP                  |
| Annual depreciation rate              |
| Measure of individuals with high ability |
| Transition probability matrix of abilities |

<table>
<thead>
<tr>
<th>Fiscal Policy</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_l$</td>
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</tr>
<tr>
<td>$\tau_k$</td>
<td>= 0.4</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>= 0.05</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>= 18%</td>
</tr>
</tbody>
</table>

| Labor income tax rate                 |
| Capital income tax rate               |
| Consumption tax rate                 |
| Government expenditure to GDP ratio  |
A newborn in our setup is a 21 year old individual; a model period is five years. Retirement is mandatory at age 65, and maximum lifespan is 90 years. The population growth rate is 1.2% per year, and the productivity growth rate is 1.65% annually. Again, these are the averages from the U.S. economy over the last fifty years. The depreciation rate is taken as 4.4%, and capital’s share of GNP is 31%.

Fiscal policy is captured by a constant annual government-GNP ratio of 18%, and taxes on labor income, capital income, and consumption, at 20%, 40%, and 5%, respectively.

The coefficient of relative risk aversion $\sigma$ is set equal to 2, and the subjective discount factor $\beta$ is taken as 0.988 in order to obtain a capital-output ratio of 2.5.$^{19}$

4 Numerical Findings

4.1 Aggregate Long Run Effects

We summarize the long-run effects of various types of reform in Table 2. The benchmark steady-state describes the current U.S. economy where pensions are provided by a PAYG system with a replacement rate of 44% and a payroll

---

$^{19}$For details of the calibration choices, see Fuster, İmrohoroğlu, and İmrohoroğlu (2003).
tax rate of 9.44%. We set the values of aggregate variables in this steady-state equilibrium equal to 100 for easy comparison with other steady-state equilibria under alternative reform proposals.

Table 2: Aggregate Effects of Reforms

<table>
<thead>
<tr>
<th>τs(%)</th>
<th>κ(%)</th>
<th>K</th>
<th>Y</th>
<th>r(1−τk)(%)</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>9.44</td>
<td>0</td>
<td>100.0</td>
<td>100.0</td>
<td>4.62</td>
</tr>
<tr>
<td>PSA+Annuity</td>
<td>4.38</td>
<td>5</td>
<td>109.5</td>
<td>102.8</td>
<td>4.24</td>
</tr>
<tr>
<td>PSA</td>
<td>4.38</td>
<td>5</td>
<td>108.9</td>
<td>102.7</td>
<td>4.26</td>
</tr>
<tr>
<td>Elimination</td>
<td>0</td>
<td>0</td>
<td>106.1</td>
<td>101.8</td>
<td>4.37</td>
</tr>
</tbody>
</table>

The steady-state labeled ‘Elimination’ is an equilibrium where there is no unfunded system at all and all individuals are left free to save the optimal amounts desired to support old-age consumption. The steady-states labeled ‘PSA+Annuity’ and ‘PSA’ enforce mandatory saving at the rate of 5% of individuals’ gross wage income in each of the years during their working life. Not surprisingly, all three alternative steady-states exhibit higher capital stock than the benchmark economy since the distortion on saving is reduced with the decline of the payroll tax from 9.44% to 4.38%.

According to Table 2, ‘PSA’ and ‘PSA+Annuity’ induce a higher capital stock than the elimination of social security. This is because individuals cannot borrow against the mandatory savings induced by Personal Security
Accounts. Also, the ‘PSA+Annuity’ reform generates a (slightly) higher capital stock than the ‘PSA’ only reform because the funds of the deceased are invested in the capital market in the former reform while they are transferred to the estates in the latter reform. As a consequence, aggregate ‘mandatory savings’ are larger in the case of annuitized Personal Security Accounts.

4.2 Welfare Effects

We compare the utility of a newborn household across alternatives steady states for different composition and ability types using a consumption equivalent variation measure. In particular, we compute the change in consumption in each future period and state of nature, $(1 + c(h, z, z'))$, relative to the benchmark consumption level that is necessary to make the household indifferent between being born in the benchmark economy or in the alternative steady state, that is

$$(1 + c(h, z, z'))^{1-\sigma}U_B(h, z, z') = U_R(h, z, z'),$$

where $U_B(h, z, z')$ is the average utility of a newborn household type $(h, z, z')$ at the benchmark economy and $U_R(h, z, z')$ is the average utility of such
household at the alternative steady state. The average utility of a newborn household type \((h, z, z')\) is computed by aggregating the utility across asset levels using the stationary distribution of assets. For instance,

\[
U_B(h, z, z') = \frac{\sum_a X_{1,B}(a, h, z, z')V_{1,B}(a, h, z, z')}{\sum_a X_{1,B}(a, h, z, z')},
\]

where \(X_{1,B}(a, h, z, z')\) denotes the invariant distribution of states \((a, h, z, z')\) across age-1-households and \(V_{1,B}(a, h, z, z')\) denotes the utility of an age-1 household with state \((a, h, z, z')\) at the benchmark economy.

Table 3 shows the consumption equivalent variation measure for newborn households for different composition and ability types. The second and third rows show the welfare measure under the partial privatization proposals, and the last row depicts the welfare measure of households under full privatization. Notice that a welfare measure higher that 1 means that the household would prefer to be born in the alternative steady state relative to the benchmark economy. The last column of the table shows the aggregate welfare measure across all newborn household types.\(^{20}\)

\(^{20}\) Since we are comparing the welfare effects on newborn households, we do not have any type 2 households in this category.
Table 3: Welfare of Newborns

<table>
<thead>
<tr>
<th></th>
<th>Type 3</th>
<th>Type 1</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HH</td>
<td>HL</td>
<td>LH</td>
</tr>
<tr>
<td>Benchmark</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>PSA+Annuity</td>
<td>1.008</td>
<td>0.98</td>
<td>1.037</td>
</tr>
<tr>
<td>PSA</td>
<td>0.980</td>
<td>0.97</td>
<td>1.025</td>
</tr>
<tr>
<td>Elimination</td>
<td>0.990</td>
<td>0.98</td>
<td>1.017</td>
</tr>
<tr>
<td>Measure of types</td>
<td>0.147</td>
<td>0.110</td>
<td>0.107</td>
</tr>
</tbody>
</table>

Type 1 households prefer the elimination of the PAYG system entirely. In particular, they prefer elimination over partial privatization, with or without mandatory annuitization. There are at least two reasons for their preference. First, these households have no fathers who would generate the demand for annuity insurance, and since these households are very young they would rather not be subject to forced saving. Second, they might be facing binding liquidity constraints and mandatory saving would then reduce their welfare. Another way to get the same intuition is to compare their lifetime welfare under the two PSA reforms. They prefer the one without annuitization for the same reasons listed above. In addition, type 1 households prefer the PSA reform to the PAYG benchmark.
For type 3 households, the proposed PSA reform benefits those with low preference for annuity insurance. In particular, the households with low life expectancy fathers (LL and LH households) are better off, where as households with long life expectancy fathers (HH and HL households) are worse off, relative to the benchmark PAYG social security system.

A great majority of type 3 households finds it best of all to be born into the steady-state labeled ‘PSA+Annuity’ where there is mandatory saving at the 5% rate and wealth generated by this partial privatization program is annuitized at retirement.\(^{21}\) This finding might seem surprising since our households care about leaving bequests to their relatives. However, since fathers are alive in these households, they like to hold annuities and therefore mandatory annuitization does not lower their welfare. Moreover, the fathers are very close to retirement age and the timing of annuitization is also in line with the households’ desire to hold annuities to insure the soon-to-be retirees. The HL households prefer the PAYG system. These households benefit from the progressivity of the PAYG system because of the low ability son, and the and they also receive a generous pension for the high ability father.

The PSA ‘only’ reform benefits LH and LL type households and hurts

\(^{21}\) Storesletten, Telmer, and Yaron (1999) also find annuitization welfare enhancing in their study of various social security reforms in a pure life cycle setting.
HH and HL type households, relative to the benchmark equilibrium. This may be due to the fact that PSA does not provide annuity insurance against lifespan uncertainty while PAYG social security does so in the benchmark equilibrium. In fact, under PSA and mandatory annuitization, all ability types but HL enjoy higher lifetime welfare than in the benchmark economy.

To see how type 1 households fare under the proposed reforms, Table 4 calculates the expected rate of return to ‘social security’ for H and L types.\footnote{With ‘social security’ we mean the sum of the PAYG payroll tax and the mandatory saving rate. See the Appendix for the computation of the overall rate of return on social security.}

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>2.70</td>
<td>2.90</td>
</tr>
<tr>
<td>PSA+Annuity</td>
<td>5.51</td>
<td>5.49</td>
</tr>
<tr>
<td>PSA</td>
<td>5.10</td>
<td>5.41</td>
</tr>
</tbody>
</table>

In both steady-states with PSAs, the expected return on social security at birth is higher than the after tax return on capital. In the case of PSA without annuitization, there is some progressivity in the benefits since there is a first-tier which is a flat benefit for every retiree. Indeed, the expected return of social security is higher for individuals with low ability (5.41%) than for the H types (5.10%). This difference between the returns for low
and high ability households disappears when part of the retirement wealth is annuitized. In this case, high ability households enjoy a higher rate of return (5.51%) than low ability households (5.49%) because H types have a longer life expectancy than L types. Therefore, when the part of wealth generated by mandatory saving is annuitized at retirement, a rationale for introducing a flat first tier pension is to compensate low ability individuals for the fact that their expected life is shorter.

**General Equilibrium Effects.**

The welfare effects of the different reforms of social security are partially due to general equilibrium effects on factor prices and on the lump-sum transfers used to balance the government’s budget and to redistribute accidental bequests. The increase of the capital stock due to the elimination of the pay-as-you-go system induces a decrease in the interest rate and an increase in the wage rate which favor relatively poor households. The two reforms with personal security accounts reduce the government revenue from capital income taxes (reducing the lump-sum transfer \( \xi_2 \)) which favors relatively rich households. Moreover, the reform with annuitized personal security accounts reduces the redistribution of accidental bequests (\( \xi_1 \)) which hurts relatively poor (and borrowing constrained) households.

In this subsection we evaluate the importance of these general equilib-
rium effects. To this end, we fix the values of the factor prices and lump-sum transfers to the levels at the benchmark economy and compute the welfare effects of the alternative reforms. We find that, for all households types, the preference ordering for social security arrangements is as follows: 1) PSA+annuity, 2) PSA, 3) Elimination, and 4) Benchmark. In this partial equilibrium experiment, we conclude that the preference ordering of social security systems is consistent with the differential in social security returns across these economies.

Table 5: Welfare of Newborns (Fixed prices and transfers)

<table>
<thead>
<tr>
<th></th>
<th>Type 3</th>
<th>Type 1</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HH</td>
<td>HL</td>
<td>LH</td>
</tr>
<tr>
<td>Benchmark</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>PSA+Annuity</td>
<td>1.159</td>
<td>1.163</td>
<td>1.163</td>
</tr>
<tr>
<td>PSA</td>
<td>1.102</td>
<td>1.108</td>
<td>1.111</td>
</tr>
<tr>
<td>Elimination</td>
<td>1.041</td>
<td>1.036</td>
<td>1.042</td>
</tr>
<tr>
<td>Measure of types</td>
<td>0.147</td>
<td>0.110</td>
<td>0.107</td>
</tr>
</tbody>
</table>

Table 5 shows that all households would prefer to be born in the economy with personal security accounts and mandatory annuitization at retirement because the return of social security is the highest (about 6%) in this economy. Households of Type 1, and Type 3 of abilities HL, which are
relatively poor and likely to be borrowing constrained, benefit from the reform PSA+annuity because lump-sum transfers do not decrease due to this partial equilibrium reform. Households of types HL and HH, which are relatively rich, prefer the reform with PSA or the elimination of social security to the benchmark system because the interest rate does not decrease with the partial equilibrium reforms.

Table 5 also reveals that the percent loss in welfare from nonannuitization is about 5; the average welfare gains for PSA+Annuity and PSA ‘only’ are 15.8% and 10.8%, respectively. This value for annuities is much smaller than the Mitchell et al. (1999) estimates of about 20-25%. Our welfare benefits are not directly comparable with their’s because they compute the welfare effects of having access to an annuity market while we compute the welfare effects of reforming the PAYGO social security system. Still, there are several reasons why their welfare benefits from annuitization are much higher than our estimates. First, Mitchell et al. (1999) consider the value of annuities for individuals at retirement (65 years old). The closer an individual is to retirement, the higher is the ‘benefit of having an annuity’. Our welfare calculations reflect the value of annuitization for a 21 year old individual, taking into account longevity risk and individual income risk. Second, they consider selfish individuals for whom it is optimal to allocate
all wealth at retirement in actuarially fair annuities. This is not the case for altruistic individuals who populate our model.

5 Conclusions

Public pension programs have come under renewed scrutiny with the projected increase in the share of the elderly in the population. Economists have argued that existing PAYGO social security systems lead to a reduction in national saving and discourage labor supply. The insurance role of social security against longevity risk, individual income risk, and macroeconomic risks have been mentioned as benefits of PAYGO systems. Despite extensive research that evaluates the costs and benefits of unfunded social security systems, a consensus on their overall value has not emerged.

Most of the research has assumed that individuals have no bequest motives. In this paper, we examine some of the benefits and costs of social security in a model with two-sided altruism, borrowing costs, and, longevity and individual income risk. The households in our setup typically have parents and children coexisting and their lifetime utility functions include both parents’ and their children’s lifetime utilities, yielding a dynastic structure. Credit markets and private annuity markets are assumed to be closed. We calibrate the model to the U.S. data and numerically solve steady-states.
The welfare gains from three social security reforms are calculated, relative to steady-state which represents the current PAYGO system in the United States. One reform is a complete privatization. A second reform is a partial privatization where 5% of the payroll taxes are redirected to Personal Security Accounts that earn the rate of return to economy-wide capital. The third reform combines this PSA with mandatory annuitization of accumulated PSA wealth at retirement.

Our main findings can be summarized as follows:

• A majority of households prefer a PSA reform (with or without mandatory annuitization) over the current PAYG pension system. Aggregate capital, output, and consumption, as well as individuals’ lifetime welfare, are higher in the reformed pension system.

• Mandatory annuitization benefits most households.

• When we abstract from general equilibrium effects, all household types prefer PSA with mandatory annuitization.

These results suggest the importance of annuities at retirement, especially in small, open economies whose wage rate and interest rate closely follow the ‘world’ factor prices. Although the welfare benefits of annuitization are smaller in our altruistic framework, the combination of higher
returns to Personal Security Accounts and mandatory annuitization raises individuals’ welfare.
References


Appendix

Transition Probability Matrix:

This transition probability matrix is a function of the age of the household and of the abilities of the father and the son, and is given by

\[
[\chi_j(h, h'; z, z')]_{h,h'\in\{1,2,3\}} = \begin{bmatrix}
\psi_j(z') & 0 & 0 \\
0 & \psi_{j+T}(z) & 0 \\
\psi_j(z')(1 - \psi_{j+T}(z)) & (1 - \psi_j(z'))\psi_{j+T}(z) & \psi_j(z')\psi_{j+T}(z)
\end{bmatrix}.
\]

Computation of the Expected Rate of Return of Social Security:

In the paper, we report the expected rate of return of social security for a newborn individual. The expected return of social security is implicitly defined as the rate of return that equates the present value (at age 1) of expected tax payments to the present value (at age 1) of expected pension benefits. For a newborn individual of ability \(z\), the present value of expected tax payments is given by

\[
\sum_{j=1}^{R-1} \frac{\tau \omega (1 + \gamma)^{j-1} \varepsilon_j(z)}{(1 + \tau_{ss})^{j-1}} \prod_{i=1}^{j-1} \psi_i,
\]
where $r_{ss}$ denotes the expected rate of return of social security and $\prod_{i=1}^{j-1} \psi_i$ indicates the probability that the individual is alive at any age $j > 1$. The present value of expected pension benefits is given by

$$
\sum_{j=R}^{2T} \frac{(1 + \gamma)^{R-1} B_R(z)}{(1 + r_{ss})^{j-1}} \prod_{i=1}^{j-1} \psi_i,
$$

where the term $(1 + \gamma)^{R-1}$ in the numerator accounts for the growth of pension benefits in the next $R - 1$ periods.\(^{23}\) Thus, the expected rate of return of social security is defined as the value of $r_{ss}$ for which the following equation holds:

$$
\sum_{j=1}^{R-1} \frac{\tau \omega (1 + \gamma)^{j-1} \varepsilon_j(z)}{(1 + r_{ss})^{j-1}} \prod_{i=1}^{j-1} \psi_i = \sum_{j=R}^{2T} \frac{(1 + \gamma)^{R-1} B_R(z)}{(1 + r_{ss})^{j-1}} \prod_{i=1}^{j-1} \psi_i.
$$

\(^{23}\)We define $B_R(z)$ for each of the social security systems in Section 2.3.