Elimination of Social Security in a Dynastic Framework *

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March 3, 2005

Abstract

Much of the existing literature on social security has taken the extreme assumption that individuals have little or no altruism; this paper takes an opposite but equally extreme assumption that there is full two-sided altruism. We show that this assumption is both qualitatively and quantitatively important, suggesting that future work on social security must pay close attention to whether and how generations are altruistically linked. More generally, the important message of the paper is that analyses of social security must take into account the extent to which the family provides insurance against mortality and income risk, the responsiveness of aggregate labor supply to tax changes, and the extent to which contributions are linked to benefits.

Keywords: Social security reform; Altruism; Heterogenous agents; Welfare.

JEL Classification: E6; D52; C68; H55

*We would like to thank Andrés Erosa, Tom Sargent and seminar participants at UCLA, UC Irvine, University of Maryland, University of Pittsburgh, the Wharton School of the University of Pennsylvania, SUNY at Buffalo, CESifo Conference on Social Insurance, Munich, the Stern School of Business of NYU, and an anonymous referee for helpful comments. Luisa Fuster would like to thank the Ramón Areces Foundation and the Ministerio de Ciencia y Tecnologia (SEC2001-0674) for financial support.

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1. Introduction

Issues surrounding social security reform in the United States and elsewhere continue to generate attention from both economists and policy makers. The unfunded public pension system provides insurance against mortality and individual income risks for which insurance through private markets is either unavailable or difficult due to moral hazard and other reasons. At the same time, the unfunded system distorts the saving and labor supply decisions and imposes a deadweight cost on the society. When these two sets of effects of social security are evaluated in economic models, it is almost always the case that the unfunded system has an overall welfare cost on the households.

While most research in this area considers pure life-cycle models, in this paper we study the effects of eliminating social security in a dynastic framework. We think that analyzing social security in a dynastic framework is relevant for several reasons. First, it is important to evaluate the insurance role played by the social security system in an economy that, unlike the pure life-cycle model, allows for family insurance. This is particularly important since social security may crowd-out family insurance. Second, the effect of social security on capital accumulation in a dynastic framework is quite different from that in a pure life-cycle model, as emphasized by Barro (1974) in a seminal contribution, and by Fuster (1999) and Fuster, Imrohoroglu and Imrohoroglu (2003) in quantitative studies examining social security. Third, since the dynastic and the life-cycle frameworks are the two workhorses in macroeconomic analyses, it is important to compare the short-run and long-run effects of social security reform in these two frameworks. In this way, we can check the robustness of the results of privatizing social security in the literature that are obtained using pure life-cycle economies.

Our economy is populated by overlapping generations of individuals with two-sided altruism. Because parents care about their descendents, they save in order to insure their children against lifetime earnings risk. Because children are altruistic towards their parents, they may insure their parents against living longer than expected. While both social security and the family provide insurance against lifespan and earnings risks, social security can pool risks across different families at a point in time. Social security also affects saving for retirement and for bequests since our framework nests the life-cycle and altruistic models. Retirement benefits are partially linked to contributions and financed with a payroll tax that distorts labor supply decisions and may also hurt borrowing constrained individuals.

In Fuster et al. (2003), we use a similar setup, but with exogenous labor supply, to study the steady state impact of eliminating social security. Our findings indicate that steady-state welfare is lower in an environment without social security for most households. In this paper, we incorporate an endogenous labor/leisure choice and take into account the short-run welfare effects by computing equilibrium transition paths across steady states. We evaluate alternative schemes for eliminating the U.S. social security system that differ in the compensation of past social security claims and on the fiscal policy used to finance such compensation.

Our benchmark reform is a sudden and uncompensated elimination of the unfunded system. According to this plan, the government sets the payroll tax and retirement benefits
to zero from the initial period and maintains them at these values forever. While this may be an unlikely scheme for the elimination of the unfunded social security system, it provides a useful benchmark because of the ease with which one can define the losses and the gains. Overall, 54.9% of the individuals in this economy are in favor of this elimination scheme. There are two major reasons why most individuals are in favor of the elimination of social security. First, when the unfunded system is removed, intervivos transfers within family members adjust so as to minimize the loss of public insurance for mortality and income risks. Second, households raise their labor supply in response to the reduction in the total labor income tax rate. Conesa and Krueger (1999) who study a life-cycle model with an uncompensated elimination scheme report that support for such a reform in their model ranges between 40% to 21%. Welfare losses for the older generation range between 20% to 60% (in equivalent consumption). In Kotlikoff (1996), an uncompensated elimination scheme causes the oldest members of the economy to suffer a reduction in welfare that is equivalent to a 26% decrease in life-time consumption and leisure. Our results indicate that in this framework with two sided altruism the results of uncompensated elimination are qualitatively and quantitatively different for a majority of households compared to a pure life-cycle model. There is more support for this elimination scheme, and the welfare losses for households that are against the reform are much smaller in this model, about 3%, than in a life-cycle framework.

A second reform scenario we study is one in which individuals who have paid into the social security system are fully compensated and the transition is financed by a labor income tax. The government announces that individuals, from the reform date onwards, will not accumulate any more social security claims, and that retired individuals and others who have paid into the system will receive a pension corresponding to the social security claims that they accumulated in the past. Initially these pensions are financed by a labor income tax only. This tax is eliminated in 60 years. Overall, our results indicate that there is very small support for this elimination plan with 93% of individuals against it. Since there is full compensation by the government, intervivos transfers are essentially unchanged, but the higher labor income tax needed to finance the transition distorts the labor supply significantly.

Next, we compute the results of a third reform assuming that existing social security claims are financed by a new consumption tax and public debt. In this case, the percent of individuals in favor of reform is 60.4%. The significant increase in favor of reform, for this elimination scheme, is due to the use of the less distorting consumption tax to finance the compensation of social security claims.

Overall, our findings indicate the importance of the policies that are used in the elimination of the existing social security system. While payroll taxes that are used to finance a pay-as-you-go system are distortionary, the links that exist between social security contributions and the retirement benefits reduce the severity of these distortions. In this environment, the benefits of eliminating social security are mainly due to the effects of reform on labor supply. Consequently, to generate welfare gains, real-world reforms need to focus on institutions and tools that minimize distortion on labor supply. Our findings indicate that consumption taxes are far less distortionary than labor income taxes, resulting in a larger welfare gain and larger overall support for reform.
Much of the existing literature on social security has taken the extreme assumption that individuals have little or no altruism; this paper takes an opposite but equally extreme assumption that there is full two-sided altruism. We show that this assumption is both qualitatively and quantitatively important, suggesting that future work on social security must pay close attention to whether and how generations are altruistically linked. More generally, the important message of the paper is that analyses of social security must take into account the extent to which the family provides insurance against mortality and income risk, the responsiveness of aggregate labor supply to tax changes, and the extent to which contributions are linked to benefits.

The remainder of the paper is organized as follows: Section 2 describes the model and Section 3 describes the calibration of the benchmark economy. Section 4 presents the results, Section 5 the sensitivity analysis, and Section 6 concludes.

2. The Model

2.1. Demographics and Endowments

The economy is populated by overlapping generations of households that are linked through altruistic transfers. Every period a generation of individuals is born. They face random lives and some live through the maximum possible age $2T$. Conditional on survival, an individual’s lifetime support overlaps during the first $T$ periods with the lifetime support of his parent, and during the last $T$ periods with the lifetime support of his children. At any point in time, the economy is populated by $2T$ overlapping generations of individuals with total measure one.

Individuals are endowed with one unit of time. In each period until they reach the mandatory retirement age of $R$, they supply labor services to the firms.

At birth, each individual receives the realization of a random variable $z \in Z = \{H, L\}$ that determines his permanent lifetime labor ability. $z$ is a two-state, first-order Markov process with the transition probability matrix

$$\Pi(z', z) = [\pi_{ij}], \quad i, j \in \{H, L\},$$

where $\pi_{ij} = \Pr\{z' = j \mid z = i\}$, $z'$ is the labor ability of the new born in the dynasty, and $z$ is the labor ability of his parent. The transition probabilities are consistent with the existence of a unique stationary measure of abilities $\lambda(z)$.

Labor ability affects two features of an individual’s lifetime opportunities. First, labor ability determines an individual’s life expectancy. Let $\psi_j(z)$ denote the probability of surviving to age $j + 1$ conditional on having survived to age $j$ for an individual with ability $z$ for age $j = 1, 2, \ldots, 2T$, where $\psi_{2T}(z) = 0$ and $z \in \{H, L\}$. Second, $z$ determines the individual’s (expected) age-efficiency profile $\{\varepsilon_j(z)\}_{j=1}^{2T}$. If $z = H$, an individual has a higher (expected) labor productivity throughout his life-span than an individual with $z = L$.

1In this framework fertility is exogenous. In recent work Ehrlich and Kim (2003) argue that the pay-as-you-go system may have had an adverse effect on the total fertility rate in a panel of 57 countries.

2We assume that there are no insurance markets in the economy to diversify the risk of living too long and the risk of labor income shocks.
In addition, individuals face idiosyncratic shocks on their labor productivity at each age. In particular, the labor productivity of an individual of ability $z$ and age $j$ is $\varepsilon_j(z)e^{u_j}$ where $u_j$ follows an AR(1) process. For expositional reasons, we denote by $z$ the realization of the shock on the individual’s labor productivity and we do not distinguish between the ability component correlated within the family and the shock on labor productivity $u$.\textsuperscript{3} In the calibration section we explain how these two variables are distributed in more detail.

The size of cohort 1 (newborns), with ability $z$, relative to that of cohort $(T+1)$ (parents) is $\mu_1(z) = \lambda(z)(1+n)^T$ where $(1+n)^T$ is the number of children per parent and $\lambda(z)$ is the measure of newborn individuals with ability $z$. The relative sizes of the other generations are obtained recursively as follows:

$$\mu_{i+1}(z) = \frac{\psi_i(z)\mu_i(z)}{(1+n)}, \quad i = 1, \ldots, 2T - 1.$$  

The population growth rate, $n$, and conditional survival probabilities, $\psi_i(z)$, are taken as constant which makes the cohort shares time-invariant.

\section*{2.2. Technology}

There are firms in this economy that use capital and labor to produce a single good according to the following production function: $Y_t = K_t^\alpha (A_tN_t)^{1-\alpha}$, where $\alpha \in (0, 1)$ is the output share of capital, $Y_t$ is output at time $t$, $K_t$ is aggregate capital input at time $t$, $N_t$ is aggregate labor input at time $t$, and $A_t$ denotes a labor augmenting productivity index that grows at a constant rate $g$. Capital depreciates at a constant rate $\delta \in (0, 1)$. Firms maximize profits renting capital and hiring labor from the households so that marginal products equal factor prices $\bar{r}_t$, the rental price of capital and $\omega_t$ the wage per effective labor.

\section*{2.3. Social Security and Fiscal Policy}

There is a pay-as-you-go social security system where pension benefits to retired individuals are financed by taxing earnings of the current workers. The payroll tax, $\tau$, is set to balance the budget of the social security system each period. An individual’s pension is a function of that individual’s average lifetime earnings via a concave, piecewise linear function. This function captures the progressivity of the U.S. benefit formula and it is described in Section 3. A progressive social security provides insurance against labor income risk.\textsuperscript{4} In our economy, the degree of progressivity of social security is affected by the assumption that low ability individuals live a shorter lifetime than high ability individuals. Notice also that social security provides insurance against the risk of living too long.

The government also taxes labor income, capital income and consumption in order to finance an exogenously given level of government purchases. The labor income tax is set such that the government budget is balanced.\textsuperscript{4}

\textsuperscript{4}In addition, the government collects the asset holdings and capital income left over by deceased individuals who do not have descendants. These resources are transferred in a lump-sum fashion to the entire population.
2.4. Altruistic Preferences and the Households’ Decision Problem

Individuals derive utility from their own lifetime consumption and leisure, and from the felicity of their predecessors and descendants. The formalization of preferences follows Laitner (1992) in the sense that the parent and the children maximize the same objective function. Because of this commonality of interests, during the periods when their lives overlap the parent and the children constitute a single decision unit by pooling their resources. This decision unit is called a household and is constituted by an adult male, the “parent”, of age $T+1$, and his $m = (1+n)^T$ adult children of age 1. A household lasts $T$ periods or until the parent and the children have died. $^5$ A dynasty is a sequence of households that belong to the same family line. If the children survive to age $T+1$, each of them becomes a parent in the next-generation household of the dynasty. Otherwise, the family line is broken, and this particular dynasty is over. Every period some dynasties disappear since there are individuals who do not reach age $T+1$. We assume that these dynasties are replaced by new dynasties to maintain our assumption of a stationary demographic structure. Since mortality rates are higher for low ability individuals, the number of new dynasties of low ability is higher than the number of dynasties of high ability. A new dynasty begins with an individual of age 1 that holds zero assets.

Households are heterogeneous regarding their asset holdings, age, abilities, and their composition. The composition of a household changes when either the parent or his $m$ children die. Since the life-span shock that affects each of the children are perfectly correlated, there are three types of households. Households of type-1 are those where the parent has died. Households of type-2 are those where the $m$ children have died. Households of type-3 are those where both the parent and the children are still alive.

The budget constraint facing an age-$j$ household, where $j = 1, 2, \ldots, T$ is the age of the youngest member(s), is given by

$$ [\phi_s(h)c_{s,j} + \phi_f(h)c_{f,j}](1 + \tau_c) + (1 + g)a_j = [1 + r(1 - \tau_k)]a_{j-1} + \epsilon_j(h, \eta, z_f, z_s) + [\phi_s(h) + \phi_f(h)]\xi, \tag{2.1} $$

where $\phi_s$ is an indicator function which takes the value $m$ if the children are alive and 0 otherwise, while $\phi_f$ is an indicator function that takes the value unity if the parent is alive and 0 otherwise; $h \in \{1, 2, 3\}$ is an indicator of household composition, $r$ is the interest rate $r = \tilde{r} - \delta$, $\epsilon_j(h, \eta, z_f, z_s)$ denotes the after tax earnings, $c_{s,j}$ and $c_{f,j}$ are the consumption of the child and the parent, $a_j$ denotes the asset holdings to be carried over to age $j+1$, $\xi$ is a lump sum redistribution of accidental bequests left behind by single individual households and confiscated by the government, and $\tau_c$ and $\tau_k$ denote the consumption and capital income tax rates, respectively. Consumption, asset holdings, lump-sum transfers, and earnings are transformed to eliminate the effects of labor augmenting, exogenous productivity growth. In particular, we have normalized those variables by the level of the technology, $A_t$, at any

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$^5$In a given household, all children are born at the same period and all of them die at the same period. Children in a given household are identical regarding their labor abilities and vector of conditional survival probabilities.
period $t$.\footnote{For the sake of clarity, we will drop the time subscripts from now on although we do not restrict attention to steady-states.}

The function $e_j(h, \overline{e}_f, z_f, z_s)$ gives the net of tax earnings of an age-$j$ household:

$$e_j(h, \overline{e}_f, z_f, z_s) = \begin{cases} 
\phi_s(h)\omega(1 - \tau - \tau_\ell)\varepsilon_j(z_s)(1 - \ell_{s,j}) + \phi_f(h)B_{j+T}(\overline{e}_f) & \text{if } j \geq R - T, \\
\phi_s(h)\omega(1 - \tau - \tau_\ell)\varepsilon_j(z_s)(1 - \ell_{s,j}) + \phi_f(h)\omega(1 - \tau - \tau_\ell)\varepsilon_{j+T}(z_f)(1 - \ell_{f,j}) & \text{otherwise},
\end{cases} \tag{2.2}$$

where $\ell_{s,j}$ and $\ell_{f,j}$ are the leisure of the child and the parent, $\tau$ is the social security tax rate and $\tau_\ell$ is the tax rate on labor income. $B_{j+T}(\overline{e}_f)$ denotes the pension at age $j + T$ of which is a function of the parent’s average lifetime earnings ($\overline{e}_f$).\footnote{When the age of the son is $j$, the age of the father is $j + T$.} An individual’s pension remains constant during retirement while technology grows at the rate $g$. Thus, the pension per effective labor decreases during retirement at rate $g$. In other words, the retirement benefits of successive cohorts increase at the rate $g$.

For $j = T$, the budget constraint of the household is given by

$$[\phi_s(h)c_{s,T} + \phi_f(h)c_{f,T}](1 + \tau_o) + (1 + n)^T(1 + g)a_T = [1 + r(1 - \tau_k)]a_{T-1} + \epsilon_T(h, \overline{e}_f, z_f, z_s) + [\phi_s(h) + \phi_f(h)]\xi. \tag{2.3}$$

If the children survive to age $T + 1$, $(1 + n)^T$ new households are constituted in the dynasty and each of them will hold $a_T$ assets. If the children do not survive to age $T + 1$, the family line breaks.

It is assumed that households face borrowing constraints and cannot hold negative assets at any age: $a_j \geq 0$, $\forall j$.

The economic problem of a household is to choose a sequence of consumption, leisure, and asset holdings given a set of policies for social insurance. The state of a household is given by the age $j$, the assets $a$, the demographic type $h$, labor productivity of parent and children $z_f, z_s$ and the average lifetime earnings of the members of the household $\overline{e}_f, \overline{e}_s$. The last two variables are part of the state of the household because an individual’s pension is a function of the average lifetime earnings. We denote by $V_j(a, h, \overline{e}_f, \overline{e}_s, z_f, z_s)$ the steady state maximized value of expected, discounted lifetime utility of an age-$j$ household with the state vector $x = (a, h, \overline{e}_f, \overline{e}_s, z_f, z_s)$. For a household of age $j \leq T$,

$$V_j(x) = \max_{\{c_s,c_f,\ell_s,\ell_f,a_j\}} \left\{ [\phi_s(h)u(c_{s,j}, \ell_{s,j}) + \phi_f(h)u(c_{f,j}, \ell_{f,j})] + \beta\overline{V}_{j+1}(a', h', \overline{e}_f', \overline{e}_s', z_f', z_s') \right\} \tag{2.4}$$

subject to (2.1)-(2.3), $a_j \geq 0$,

$$\overline{e}_f' = \left[(T + j - 1)\overline{e}_f + \omega(1 - \tau - \tau_\ell)\varepsilon_j(z_f)(1 - \ell_{f,j})\right]/(T + j) \text{ and } \overline{e}_s' = \left[(j - 1)\overline{e}_s + \omega(1 - \tau - \tau_\ell)\varepsilon_j(z_s)(1 - \ell_{s,j})\right]/j$$

where

$$\overline{V}_{j+1}(a', h', \overline{e}_f', \overline{e}_s', z_f', z_s') = \begin{cases} 
\sum_{h' = 1}^{3} \chi_j(h, h'; z_f, z_s)E_{\{z_f', z_s'/z_f, z_s\}}V_{j+1}(a', h', \overline{e}_f', \overline{e}_s', z_f', z_s') & \text{for } j < T, \\
\psi_T(z_s)(1 + n)^T E_{\{z_f', z_s'/z_f, z_s\}}V_1(a', 3, \overline{e}_s, 0, z_f', z_s') & \text{for } j = T,
\end{cases}$$
\(\chi_j(h, h'; z_f, z_s)\) is the probability that a household of age \(j\) and type \(h\) becomes type \(h'\) the next period given that the parent is of ability \(z_f\) and the children of ability \(z_s\). Notice that a household of age \(T\) faces two shocks. One is the life-span shock that affects the youngest members of the household, the other is the ability shock that affects the new generation of the dynasty. The youngest members will survive with probability \(\psi_T(z_s)\) and constitute \((1+n)^T\) new households; by construction these are type 3 households. The ability of the new generation of the dynasty is correlated with the ability of the parent; that is, \(z'_s\) is correlated with \(z_s\). The labor productivity of the (new) parent is denoted by \(z'_f\) and is correlated with the previous period realization \(z_s\) (the individuals was a ‘child’ in the previous household). Notice also that the new member of the household is born with zero average lifetime earnings.

### 2.5. Equilibrium

Stationary recursive competitive equilibrium: Given a fiscal policy \(\{G, B, \tau_\kappa, \tau_\ell\}\), a stationary recursive competitive equilibrium is a set of value functions \(\{V_j(x)\}_{j=1}^T\), households’ decision rules \(\{c_{s,j}(x), c_{f,j}(x), \ell_{s,j}(x), \ell_{f,j}(x), a_j(x)\}_{j=1}^T\), time-invariant measures of households \(\{X_j(x)\}_{j=1}^T\), with the state vector \(x = (a, h, \bar{e}_f, \bar{e}_s, z_f, z_s)\), relative prices of labor and capital \(\{\omega, \bar{r}\}\), a lump sum distribution of unintended bequests \(\xi\), a payroll tax \(\tau\), and a labor income tax \(\tau_\ell\), such that the following conditions are satisfied:

1. given fiscal policy, factor prices and lump-sum transfers, households’ decision rules solve households’ decision problems (2.4);
2. factor prices are competitive;
3. aggregation holds,
\[
\tilde{K} = \sum_{j,x} a_{j-1}(x)X_j(x)(1+n)^{1-j},
\]
\[
N = \sum_{j,x} [\phi_s(h)(1-\ell_{s,j}(x))\varepsilon_j(z') + \phi_f(h)(1-\ell_{f,j}(x))\varepsilon_{j+T}(z)]X_j(x)(1+n)^{1-j},
\]
\[
C = \sum_{j,x} [\phi_s(h)c_{s,j}(x) + \phi_f(h)c_{f,j}(x)]X_j(x)(1+n)^{1-j};
\]
4. the set of age-dependent measures of households satisfies
\[
X_{j+1}(a', h', \bar{e}_f', \bar{e}_s', z_f', z_s') = \sum_x X_j(x)\chi_j(h, h'; z_f, z_s) \Pr ob(z'_f, z'_s/z_f, z_s), \text{ for } j < T;
\]

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\(^{8}\)This transition probability matrix is a function of the age of the household and of the abilities of the parent and the child, and is given by

\[
[\chi_j(h, h'; z, z')]_{h, h' \in \{1, 2, 3\}} = \left[\begin{array}{ccc}
\psi_j(z) & 0 & 0 \\
0 & \psi_{j+T}(z) & 0 \\
\psi_j(z')(1-\psi_{j+T}(z)) & (1-\psi_j(z'))\psi_{j+T}(z) & \psi_j(z')\psi_{j+T}(z)
\end{array}\right].
\]
where $a'$, $\overline{c}_f$, and $\overline{c}_s$ are the next period optimal assets and average earnings given today’s state $x$; the invariant distribution of age-1 households is given by conditions

$$X_1(a', 3, \overline{c}_s, 0, z'_f, z'_s) = \sum_x X_T(x) \chi_T(h, 3; z_f, z_s) \Pr \{ z_f' = z_s' / z_s \}, \quad (2.6)$$

where $a'$, and $\overline{c}_s$ are the next period optimal assets and average earnings given today’s state $x$ ($\overline{c}_f = 0$ because the parent has died); and

$$X_1(0, 1, 0, 0, 0, z'_s) = \lambda(z'_s) - \sum_{a' \overline{c}_s, z'_f} X_1(a', 3, \overline{c}_s, 0, z'_f, z'_s), \quad (2.7)$$

that is, new dynasties, holding zero assets, substitute for the family lines broken during the last period, where $\lambda(z'_s)$ is the invariant measure of $z'_s$;

5. the lump-sum redistribution of unintended bequests aggregated over $\{j, a, h, \overline{c}_f, \overline{c}_s, z_f, z_s\}$ satisfies

$$(1+n)\xi \sum X_j(x)(1+n)^1-j = (1+r) \sum a_j(x)X_j(x) \left[ 1 - \sum_{h'=1}^{3} \chi_j(h, h'; z_f, z_s) \right] (1+n)^1-j,$$

6. the government’s budget is balanced

$$G = \tau_k r \left[ \tilde{K} - \frac{\xi}{1+r} \right] + \tau_k \omega N + \tau_c C;$$

7. the social security tax is such that the budget of the social security system is balanced

$$\sum_{j=R a, h, z_f, z_s} B_j(\overline{c}_f)X_j(x) = \tau_s \omega N;$$

8. the goods market clears

$$C + (1 + g)(1 + \delta) \tilde{K} - (1 - \delta) \tilde{K} + G = \tilde{K}^{\alpha} N^{1-\alpha}.$$

Since the purpose of this paper is to examine policies designed to eliminate the pay-as-you-go social security program, as our benchmark we start at a steady state where the average social security replacement rate is set to 44%. We then solve for a final steady state where the social security replacement rate is set to 0%. In order to solve for the transition path, we follow Auerbach and Kotlikoff (1987) and Huang, Imrohoroglu and Sargent (1997) and assume that the transition from the initial to the final steady state takes $S$ periods. The details of the computational approach are given in the Appendix.
3. Calibration of the Benchmark Economy

3.1. Demographics

We assume that individuals are born when they are 20 years old and live to be at most 90 years old. If they survive, they retire from the labor market at the age of 65. Also conditional on surviving, individuals’ fertile lifetimes conclude when they are 35 years old. At this time they have $m$ children. If individuals reach the age of 55, they form a household with their $m$ children. For computational reasons, a model period is five years. These assumptions imply the following parameter values for the model: $T = 7$ and $R = 10$. When children reach the model age 1 (real time age 20), the parent’s age is the model age of 8 (real time age 55) and this household starts making joint decisions. When the child is 3 periods old (real time age 30), the parent who is at the model age of 10 (real time age 65) retires.

Although the model period is five years, in what follows we express flow variables as rates per year. The population growth rate is constant and consistent with the average annual population growth rate of the U.S. economy, that is, 1.2%. This implies for the model that $n = 0.012$ and $m = 1.52$.

3.2. Preferences and Technology

The exogenous productivity growth rate is taken as $g = 1.4\%$, which is close to the postwar annual average in the U.S. Following Imrohoroglu, Imrohoroglu and Joines (1999), the income share of capital, $\alpha$, is taken as 0.31. The depreciation rate $\delta$ is given by

$$\delta = \frac{I/Y}{K/Y} - g - n - gn,$$

where we target an investment-output ratio equal to 21% and a capital-output ratio of 3, yielding $\delta = 0.044$. The subjective discount factor, $\beta$, is chosen so that the economy at the initial steady state produces a capital-output ratio of 3.0. This procedure yields a $\beta$ of 0.99.

The instantaneous utility function is assumed to be

$$u(c, \ell) = \left(\frac{c^{1-\nu} \ell^\nu}{1-\gamma}\right)^{1-\gamma} - 1.$$

We choose a value for the intensity of leisure in the utility function such that individuals work 33% of their discretionary time ($\nu = 0.63$). We assume $\gamma = 4$ which implies an elasticity of intertemporal substitution of consumption $(1 - (1-\nu)(1-\gamma))^{-1} = 0.474$ which is a value in the range of estimates (see Auerbach and Kotlikoff (1987)).

The Frisch elasticity of labor supply implied in this model is 1.29. Although this might seem too high there is a growing literature in labor economics that finds significant downward biases in the earlier estimates of the intertemporal elasticity of substitution in labor supply. MaCurdy (1981), Altonji (1986), and Browning, Deaton, and Irish (1985) use a

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9Note that the children are born when the parent was 35 years old, but the joint decision making only starts after the children reach the age of 20 and start working.
time-separable utility function which is also separable between consumption and leisure, find [under the joint assumptions of complete markets, exogenous wages, and using a data set consisting of prime working age males] a fairly low intertemporal elasticity of substitution of labor supply, between 0 and 0.5, with 0.2 a typical point estimate. Recently, Domeij and Floden (2001) argue that ignoring borrowing constraints biases this parameter downward, by as much as 50%, bringing the elasticity to between 0.3 and 0.56. Browning, Hansen, and Heckman (1999) estimate the Frisch elasticity to be 1.6. Aronson and French (2001) endogenize wages and take progressive taxation into account and argue that there could be a 10-20% downward bias due to ignoring these features. Ham and Reilly (2003) use an implicit contract model in their use of micro data and estimate the labor supply elasticity to be between 0.5 and 1.5. Kimball and Shapiro (2003) develop a theory of labor that imposes the restriction that the income and substitution effects cancel, takes into account the fixed cost of going to work, and the interactions of labor supply decisions within the household. Using survey data they find that the Frisch elasticity of labor supply is about 1. Chang and Kim (2003) study the mapping from individual to aggregate labor supply using a general equilibrium heterogeneous-agent model with incomplete markets. They calibrate the nature of heterogeneity among workers using wage data from the PSID. The gross worker flows between employment and non-employment and the cross-sectional earnings and wealth distributions in their model are comparable to those in the micro data. They find that the aggregate labor supply elasticity of such an economy is around 1. Finally, Imai and Keane (2004) develop a life cycle theory of labor that allows individuals to accumulate human capital and use a disutility of labor function similar to the earlier literature started by MaCurdy and Altonji. Using the NLYS79 data set, their estimate of the elasticity is about 3.8. Our implicit Frisch elasticity of labor is consistent with the recent literature but not as high as the Imai and Keane (2004) estimate, which would tend to reinforce our findings since an elastic labor supply makes reform easier.

3.3. Labor Productivity Shocks

We assume that the efficiency units of labor of an individual of age \( j \) depends on his ability \( z \) and the realization of the idiosyncratic shock \( u \). In particular, the individual’s labor productivity at age \( j \) is \( \varepsilon_j(z)e^{u_j(z)} \) where \( \varepsilon_j(z) \) denotes the mean efficiency units of labor of an age-\( j \)-individual of ability \( z \). We assume that shock \( u \) follows an AR(1) process and that this process is specific for the ability type of the individual, that is \( u_j(z) = \rho(z)u_{j-1}(z) + \eta_j(z) \), and \( \eta_j(z) \sim N(0,\sigma_{\eta_z}^2) \).

We calibrate the profiles of mean efficiency units of labor for high and low ability individuals, \( \varepsilon_j(z) \), to match the average profiles of efficiency units of labor of college and non-college graduate males, respectively. We construct these indices using data on earnings from the Bureau of the Census (1991).

We also have to calibrate the parameters \( \rho(z) \) and \( \sigma_{\eta_z}^2 \) which characterize the AR(1) processes \( u_j(z) \), for college and non-college graduate workers. We pick the values for these four parameters that match the estimates of Hubbard Skinner and Zeldes (1995) for the U.S. economy (\( \rho = 0.95 \) for both college and non-college graduates, \( \sigma_{\eta_z}^2 = 0.016 \) for college gradu-
ates and $\sigma_H^2=0.025$ for non-college graduates). Following Tauchen (1986) we approximate this process with a two state first order Markov chain.

The ability $z$ (college or non-college education) follows a first order Markov chain. We choose the values for the transition probabilities characterizing such process so that our benchmark economy matches two observations. First, the proportion of full-time male workers that were college graduates in 1991 was 28% (see Bureau of the Census (1991), pg. 145). Second, the correlation between the wages of parents and children is 0.4 according to the estimates by Zimmerman (1992) and Solon (1992). These observations imply for this model that $\pi_{HH} = 0.57$ and $\pi_{LL} = 0.83$.

Labor ability determines both the lifetime productivity of the individuals and the vector of conditional survival probabilities. We obtain these probabilities for college and non-college graduate males in the U.S. economy from Elo and Preston (1996) who document that lifetime expectancy at the real age of 20 is 5 years longer for a college graduate than for non-college graduate.

3.4. Social Security and Taxation

In the U.S. economy, retirement benefits depend on individuals’ average lifetime earnings via a concave, piecewise linear function. The marginal replacement rate decreases with average lifetime earnings indexed to productivity growth. It is equal to 0.9 for earnings lower than 20% of the economy’s average earnings. Above this limit and below 125% of the economy’s average earnings the marginal replacement rate decreases to 0.33. For income within 125% and 246% of the economy’s average earnings the marginal replacement rate is 0.15. Additional income above 246% of the economy’s average earnings does not provide any additional pension payment. In particular, the benefit function that we use is

$$B(M) = \begin{cases} 
0.9M & \text{for } M \leq 0.2M \\
0.9(0.2M) + 0.33(M - 0.2M) & \text{for } 0.2M \leq M \leq 1.25M \\
0.9(0.2M) + 0.33(1.25M - 0.2M) + 0.15(M - 1.25M) & \text{for } 1.25M \leq M \leq 2.47M \\
0.9(0.2M) + 0.33(1.25M - 0.2M) + 0.15(M - 1.25M) + 0.15(M - 2.47M) & \text{for } M > 2.47M 
\end{cases}$$

where $\overline{M}$ denotes the average earnings in the economy. This benefit formula implies that the average replacement rate (replacement rate of an individual that earns the average earnings of the economy) is 44%. We compute properties of two steady states, one in which the average replacement rate is 44% and another where it is set equal to zero.

In the benchmark economy, we set the government purchases of goods and services ($G$) equal to 22.5% of output and keep them constant across steady states. We assume a consumption tax rate of 5.5% and the capital income tax rate is taken to be 35%. The labor income tax is set such that the government budget balances which implies a tax rate equal

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10Since Hubbard et al. estimate an annual process, we compute the values of $\rho$ and $\sigma$ for a five year process which are consistent with the estimates of Hubbard et al. The parameter values for our five year process are $\sigma^2(H) = 0.0438$ and $\rho(H) = 0.8437$ for college individuals and $\sigma^2(L) = 0.06844$ and $\rho(L) = 0.8437$ for non-college individuals.
to 0.185 at the benchmark economy. The following table summarizes all the parameters used in the initial steady state.

Table 1: List of Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2T)</td>
<td>14</td>
<td>Maximum lifetime (90 years)</td>
</tr>
<tr>
<td>(R)</td>
<td>10</td>
<td>Retirement age (65 years)</td>
</tr>
<tr>
<td>(n)</td>
<td>0.012</td>
<td>Annual population growth rate.</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>4</td>
<td>Utility parameter</td>
</tr>
<tr>
<td>(\nu)</td>
<td>0.63</td>
<td>Utility parameter</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.99</td>
<td>Utility parameter</td>
</tr>
<tr>
<td>(g)</td>
<td>0.014</td>
<td>Annual rate of growth of technology</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.31</td>
<td>Capital share of GNP.</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.044</td>
<td>Annual depreciation rate.</td>
</tr>
<tr>
<td>(\lambda(H))</td>
<td>0.28</td>
<td>Measure of individuals with high ability.</td>
</tr>
<tr>
<td>(\pi_{LL})</td>
<td>0.83</td>
<td>Transition probability matrix of abilities.</td>
</tr>
<tr>
<td>(\pi_{HH})</td>
<td>0.57</td>
<td>Transition probability matrix of abilities.</td>
</tr>
<tr>
<td>(\rho_L)</td>
<td>0.95</td>
<td>Correlation coefficient in AR process for (u)</td>
</tr>
<tr>
<td>(\rho_H)</td>
<td>0.95</td>
<td>Correlation coefficient in AR process for (u)</td>
</tr>
<tr>
<td>(\sigma^2_{\eta}(L))</td>
<td>0.025</td>
<td>Variance of innovations in AR process for (u)</td>
</tr>
<tr>
<td>(\sigma^2_{\eta}(H))</td>
<td>0.016</td>
<td>Variance of innovations in AR process for (u)</td>
</tr>
</tbody>
</table>

| Fiscal Policy | \(\tau_k\) | 0.35 | Capital income tax rate |
|              | \(\tau_c\) | 0.055 | Consumption tax rate |
|              | \(G\)     | 0.65  | Government purchases |

4. Results

We start this section by discussing the properties of the steady-state representing the current U.S. social security system and compare them with the steady state properties of an economy where the social security program is eliminated. Next, we incorporate the equilibrium transition across steady-states and examine the effects of eliminating the social security system. All the reforms we consider are revenue neutral and start from the same steady state where the social security replacement rate is set equal to 44%, and end at a steady state with a 0% replacement rate.

4.1. Steady-State Results

Table 2 describes the properties of two steady states for this environment. In the initial steady state the economy has an unfunded social security system with an average replacement rate

\[\text{replacement rate} = 44\%\]
\( \theta = 0.44 \). At the ending steady state, the social security system is completely eliminated by setting the replacement rate to 0%.

While we have not tried to match the U.S. wealth distribution, this model generates a significant amount of wealth inequality with a wealth Gini of 0.75 at the initial steady state. The corresponding number for U.S. is 0.78 (see, for instance, Castañeda, Díaz-Giménez and Ríos-Rull (2003)).\(^1\)\(^2\) In this framework wealth becomes more concentrated with social security due to the increase in saving for bequests which is especially strong for the rich households. Consequently, eliminating social security decreases wealth inequality resulting in a Gini coefficient of 0.69.\(^1\)\(^3\)

<table>
<thead>
<tr>
<th>Pen/Y</th>
<th>( \theta )</th>
<th>( \tau )</th>
<th>( \tau_C )</th>
<th>( K )</th>
<th>( N )</th>
<th>( Y )</th>
<th>( K/Y )</th>
<th>( r(1-\tau_k) )</th>
<th>( C )</th>
<th>( C/Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15%</td>
<td>0.44</td>
<td>0.10</td>
<td>0.18</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>3.03</td>
<td>0.037</td>
<td>100.00</td>
<td>57.62</td>
</tr>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.16</td>
<td>110.80</td>
<td>105.23</td>
<td>106.95</td>
<td>3.14</td>
<td>0.035</td>
<td>108.14</td>
<td>58.27</td>
</tr>
</tbody>
</table>

A comparison of the two steady states reveals that the economy with a zero percent replacement rate generates 10.8% more capital, 5.2% more labor, 8.1% more consumption and 7% more output than an economy with a 44% replacement rate (average working hours increase from 33.6% to 35.8% of discretionary time due to the elimination of social security). Notice that taxation of labor income is considerably reduced when social security is eliminated since the combined payroll taxes for social security and personal income taxes decreases from 0.28 to 0.16.\(^1\)\(^4\) In this experiment where government revenues are held constant, \( G/Y \) decreases from 22% at the initial steady state to 20.4% at the new steady state. More important, consumption to output ratio in this economy increases from 57.6% to 58.3%.

As discussed before, households in this model differ in terms of their demographic composition and labor ability. Because of lifetime uncertainty households can be classified into three categories according to their demographic composition. A household in which only the children are alive is denoted as type 1. When the parent is the only member alive, the household is labelled as type 2. Households where both the parent and the children are alive are denoted as type 3. A very small fraction of the population is of type 2 and none of the newborns can be of this type (children live at least one period). At a given point in time, 29% of households are type 1, 2% of households are type 2, and 69% of households are type 3.\(^1\)\(^5\) Since individuals can be of high or low labor ability, type 3 households can be

\(^{1}\)While the model is successful in replicating the lower tail of the wealth distribution, it does not match the upper tail as it happens in most of the dynastic and life-cycle models. In order to match the U.S. wealth distribution Krusell and Smith (1998) and Erosa and Koreshkova (2003) introduce stochastic discount factors while Castañeda, Díaz-Giménez and Ríos-Rull (2003) calibrate their model to the Lorenz curves of U.S. earnings and wealth.

\(^{2}\)Fuster (1999) also finds this result. De Nardi (2003) and Laitner (2001) argue that intentional intergenerational transfers may explain the skewness of the empirical wealth distribution.

\(^{3}\)In the sensitivity analysis we will argue that there are significant welfare gains associated with the decrease in the labor income tax burden due to the elimination of social security.

\(^{4}\)There are three different measures for each type: percent of newborn households of a particular type; percent of (all ages) households of a particular type; and percent of individuals belonging to households of a particular type.
subdivided into four categories according to the abilities of the parent and his children. We thus denote by HH a type 3 household where both the parent and children are of high human capital. The remaining type 3 households are denoted as HL, LL, and LH, where the first letter indicates the ability of the parent and the second the ability of the children. In this model there is further heterogeneity in a given household type based on the labor income shocks that the parent and the child received at each year during their working years. For each given family type on the basis of the permanent generationally persistent shock H and L, we will distinguish between four types of families based on the individual income shock received by the parent and the child in their working years, $u_1$, and $u_2$, respectively. Since the individual income shock is persistent over the life cycle, this additional subdivision of the households will reveal the impact of social security reform on relatively poor and rich households within a given household ability type. In our classification, for example, $u_1u_2$ in an HL household indicates a high human capital parent who receives a high labor income shock of $u_1$ at age 55, and the low human capital child who receives a low labor income shock of $u_2$ in his first working year at age 20.

Table 3: Welfare of Newborns

<table>
<thead>
<tr>
<th>Type 3</th>
<th>HH</th>
<th>HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>$u_1u_1$</td>
<td>$u_2u_1$</td>
</tr>
<tr>
<td>0.10</td>
<td>-140.09</td>
<td>-134.98</td>
</tr>
<tr>
<td>0.0</td>
<td><strong>-137.75</strong></td>
<td><strong>-132.89</strong></td>
</tr>
<tr>
<td>Measure</td>
<td>3.7%</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

Table 3 cont.: Welfare of Newborns

<table>
<thead>
<tr>
<th>Type 3</th>
<th>LH</th>
<th>LL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>$u_1u_1$</td>
<td>$u_2u_1$</td>
</tr>
<tr>
<td>0.10</td>
<td>-162.83</td>
<td>-157.99</td>
</tr>
<tr>
<td>0.0</td>
<td><strong>-156.62</strong></td>
<td><strong>-152.31</strong></td>
</tr>
<tr>
<td>Measure</td>
<td>2.7%</td>
<td>2.6%</td>
</tr>
</tbody>
</table>

Table 3 cont.: Welfare of Newborns

<table>
<thead>
<tr>
<th>Type 1</th>
<th>H</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>$u_1$</td>
<td>$u_2$</td>
</tr>
<tr>
<td>0.10</td>
<td>-152.26</td>
<td>-145.68</td>
</tr>
<tr>
<td>0.0</td>
<td><strong>-139.97</strong></td>
<td><strong>-134.48</strong></td>
</tr>
<tr>
<td>Measure</td>
<td>1.28%</td>
<td>1.28%</td>
</tr>
</tbody>
</table>

Table 3 provides information on new-born household preferences over the two social security replacement rates.\textsuperscript{16} In the first panel, we provide the results of this experiment for

\textsuperscript{16}For Type 1 we present the measure of newborn households of each type. Given the steady state comparisons of welfare, this is the appropriate measure to consider.
type 3 households of HH and HL families. Within each family type there are four different types of households, $u_1u_1$, $u_1u_2$, $u_2u_1$, and $u_2u_2$. For simplicity, we provide information on only two sub-types ($u_1u_1$, $u_2u_1$) since the preferences of the remaining households on social security are not significantly different from the ones presented. Panel 2 of the table provides information for LH and LL families. In the last panel we have welfare results for type 1 households. Our results indicate that all households prefer to be born in an economy without social security. The welfare gains from the decrease of labor supply distortions (due to the elimination of the payroll tax for social security and the reduction in the personal income tax) and the increase in the aggregate capital stock more than compensate the welfare loses from losing the insurance roles provided by social security against life-span and earnings risks.

Previous social security analyses conducted in life-cycle frameworks also find that individuals would prefer to be born into an economy without social security. In that framework, the changes in labor supply due to the elimination of the social security tax do not play as important a role on the welfare effects as in our model. Indeed, in a life-cycle model, the long-run benefit of eliminating social security comes from a huge increase in the capital stock. For example, Auerbach and Kotlikoff (1987) find that a social security system with a 60% replacement rate reduces the steady state capital stock by 24%. İmrohoğlu, İmrohoğlu and Joines (1999) report that capital stock decreases by 26% with a 40% social security replacement rate. Storesletten, Telmer and Yaron (1999) report changes in the capital stock ranging between 10% to 25%. The change in the capital stock in those models is driven from an increase in the saving rate of the economy. Social security affects the saving rate because it redistributes income from individuals with high marginal propensities to save (young) to individuals with low marginal propensities to save (old). In our framework, however, old individuals do not necessarily have a low marginal propensity to save since they also save for a bequest motive and the aggregate saving rate does not increase with the elimination of social security (see $K/Y$ in Table 2). Thus, our findings are driven mostly by the welfare gains due to the decrease in labor supply distortions.

4.2. Transitions

The steady state results presented above confirm the earlier findings in the literature that agents would prefer to be born into an economy without social security. In this section we investigate the behavior of the economy and welfare of the individuals along alternative
transition paths that lead to elimination of the pay-as-you-go social security system. We consider several unanticipated elimination schemes and compute the compensating variation in consumption that would make each household indifferent between the initial steady-state with social security and going along the transition path toward the privatized system. The welfare effects of the elimination scheme depend on the fiscal policies that are considered during the transition to the new steady state. We start with an uncompensated elimination scheme where individuals who had paid into the system are not compensated at all. While this may be an unlikely scheme for the elimination of the unfunded social security system, it provides a useful benchmark because of the ease with which one can define the losses and the gains. We examine the behavior of consumption, leisure, and intervivos transfers in detail for this case. Later, we present several other elimination schemes where individuals who had paid into the social security system are fully compensated, and the compensation paid for by various tax and debt schemes.

Plan 1: Uncompensated elimination

This plan considers an uncompensated elimination scheme where the government sets the payroll tax and the benefits to zero from the initial period. Thus, in this case individuals who have already paid into the system are not compensated for their contributions and the retirees’ pensions are terminated. Figure 1 shows the evolution of capital stock and employment during the transition. Most of the convergence to the new steady state is completed in 40 years with this plan.

![Figure 1: Uncompensated elimination.](image)

Since employment increases immediately, the capital-labor ratio decreases first and then increases towards its higher long-run level. The evolution of the after tax interest rate,
displayed in Figure 2, is just the inverse of the evolution of the capital-labor ratio. The after
tax wage rate increases monotonically because both the social security tax and the labor
income tax decrease during the transition.

![Graph](image)

Figure 2: Uncompensated elimination.

Figures 3 and 4 display the compensating variation in consumption needed to equate
the expected discounted utility of a household in the benchmark steady-state and along the
transition to the no social security steady-state.\textsuperscript{18} If this value is greater than unity for a
household of a given age, then that household prefers to move along the transition to the
steady state with no social security program and the difference between this number and
unity is the consumption loss due to social security. The horizontal axis represents the age
of the child, which corresponds to the age of the household. At the time of the reform,
households of ages 20 to 50 are alive and they either have parents aged 55 to 85 (type 3 and
2), or their parents may have died sometime during their lifetime (type 1).

The first panel in Figure 3 displays the welfare of type 1 households. None of the house-
holds of this type are against this plan. These are young households whose parents have
died, thus they do not have parents whose welfare they need to consider. As they get closer
to the retirement age of 65 their support for the reform diminishes because they lose more
social security claims while they enjoy the higher wage for a shorter period of time. However,
for the ages over which this household is defined, there is overall support for the elimination
of social security.\textsuperscript{19}

\textsuperscript{18}All the welfare graphs are displayed for the average household with respect to the labor income shocks.
\textsuperscript{19}If this individual survives to age 55, then a child will join him to transit this household from a type 1 to
a type 3 household.
Among individuals belonging to type 1 households, those with low ability are the ones that benefit the most from the elimination of social security (LL and HL) even though the social security benefit formula is progressive. There are two reasons why low ability individuals benefit from the elimination more than high ability ones. First, low ability individuals have a shorter life expectancy and, thus, they care less about the annuity insurance provided by social security than high ability ones. Second, low ability individuals are more likely to be borrowing constrained, and the elimination of the social security payroll tax relaxes these constraints.\footnote{In our framework, 23\% of the individuals are borrowing constrained (have zero assets) at the initial steady state. Jappelli, Pischke and Souleles (1998) discuss the difficulty in identifying the liquidity constrained households in the data. They consider several measures of liquidity constrains and report that, 14.4\% of the households ‘have been turned down for a loan’, 23.6\% of households have ‘no credit card or a line of credit’, and 65\% of households ‘have low assets’ in the Survey of Consumer Finances.}

Figure 3: Welfare Effects of Uncompensated Elimination

The second panel in Figure 3 displays the welfare of type 2 households where the child had died sometime during the lifetime of the parent. Thus for this type, the horizontal axis represents the age of the parent. These individuals are hurt the most by the sudden elimination scheme since they are all either retired or very close to retirement age. In addition, they have no children whose welfare they might care about or from whom they may receive transfers. In fact, the welfare losses for these individuals are extremely high. All of these households, who make up 2\% of the population, are against this reform.\footnote{Later, when we introduce elimination schemes that at least partially compensate the losses of these individuals, we observe a big decline in their welfare losses.} Among individuals belonging to type 2, the ones that lose the least with the elimination of social security are those with high ability (HH and HL). These households are wealthier and thus rely less on pension income for their consumption than individuals with low ability.
Type 3 households that constitute the majority of households in this economy have different preferences about this elimination depending on their age and the abilities of their members, as can be seen from Figure 4. In general, the welfare gains display a non-monotonic path since welfare gains are decreasing with the age of the household from ages 20 to 30 and then increasing with the age of the household from ages 30 to 50. The household that loses the most is the age-30 household because the parent is about to retire. This parent (age 65) has contributed to the system until his retirement and loses all the benefits. Younger households lose less than the age-30 household because their members have contributed less to the system. Older households lose less than the age 30-household because the parent has received some benefits already.

Figure 4: Welfare Effects of Uncompensated Elimination.

In addition to differences in welfare due to age, there are also significant differences among different ability types. For example, a household of ability LH is in favor of the elimination regardless of its age. This household gets a low return on social security taxes because it pays high taxes due to the fact that the child has high ability and receives a relatively low pension since the parent has low ability. On the contrary, the household HL pays low taxes and receives a relatively high pension which explains why it is the one that benefits the least from the elimination of social security. In general, households where the parent is low ability benefit more from the elimination than households where the parent has high ability. Although households with low ability parent are poorer and, thus, rely more on pensions to finance their retirement, they are in favor of the elimination because they receive family transfers as we will see in the next section. Moreover, they have a shorter life expectancy and care less about the annuity insurance provided by social security than households with high ability parents.

We have also examined how household’s wealth affects the compensating variation in consumption associated with the elimination of social security. As displayed in Figure 2,
the interest rate decreases and the wage rate increases significantly during this transition. This particular change in factor prices benefits households whose main income comes from labor and hurts the asset-rich households. At the same time, since the asset-rich households are in a better position to buffer the sudden elimination of social security, they require a smaller compensation amount for the reform. These opposing effects give rise to a rich set of results in examining the relationship between wealth and changes in welfare due to reform. Our findings indicate that in general the required consumption compensation decreases with wealth; that is, a wealthier household is less likely to agree to the reform. For example, HH households of age 25 and older, who are in the 60th or higher wealth decile actually are against the elimination of social security. For LL individuals, on the other hand, their willingness to go through reform increases as they get wealthier.

Overall, 54.87% of the individuals in this economy are in favor of this elimination scheme. Households that are against the elimination of the social security system for this case are of abilities HL and ages 25-45, HH of ages 25-50, and LL of age 25-40. Welfare losses for these individuals are in the range of 3% or less. Conesa and Krueger (1999) who study a life-cycle model with an uncompensated elimination scheme report that support for such a reform in their model ranges between 40% to 21%. In one of their cases all the agents of age 37 or younger vote for the elimination, and everybody older votes against it. Welfare losses for the older generation range between 20% to 60% (in equivalent consumption). In Kotlikoff (1996), an uncompensated elimination scheme causes the oldest members of the economy to suffer a reduction in welfare that is equivalent to a 26% decrease in life-time consumption and leisure. Our results indicate that in this framework with two sided altruism the results of uncompensated elimination look significantly different for a majority of households compared to a pure life-cycle model. There is more support for this elimination scheme, and except for type 2 households who constitute a very small fraction of the population, the welfare losses are much smaller in this model than in a life-cycle framework. In the following section we examine the role of intervivos transfers in allowing families to share the burden of the transition through changes in transfers between the parent and the child.

**Additional Properties** In this section we analyze some of the properties of the economy under this plan in order to gain more insight into the preferences of different households towards the elimination of social security. In particular, we examine the intervivos transfers and consumption profiles of different households to assess their attitudes towards eliminating social security.22 Figure 5 displays net intervivos transfers as a fraction of income per effective labor at the initial steady state between the parent and the child for a household of type 3, whose child is born at the time of the reform. In the following panels, positive numbers indicate a transfer from the parent to the child and negative numbers indicate transfers from the child to the parent. The dashed line in each panel indicates the net transfers at the steady state with social security and the solid line indicates the transfers during the transition. For some of these households there is no significant difference between the transfers in the steady state versus during the transition. For example in the HL household where the parent has

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22 A description of the computation of intervivos transfers, which are transfers made between the parent and the child while they are all alive, can be found in Fuster, Imrohoroglu and Imrohoroglu (2003).
a higher income than the child there are only transfers from the parent to the child. We observe a small decrease in these transfers when social security is eliminated. For the HH household most of the intervivos transfers are also from the parent to the child. Even though they both have the same ability level, when the child is born the parent attains a high income due to his higher position on the age-efficiency profile relative to the child and is in a better position to support his child. If we examine the intervivos transfers for the LH household, one in which the parent has low ability and the child has high ability, we observe that the steady state with social security implies transfers from the child to the parent after the child is 25 years old (parent is 60 years old). When social security is eliminated the transfers from the child to the parent increase, perhaps compensating some of the loss the parent experiences. A similar pattern is detectable for the LL household. Notice that among all these households LH and LL households are in a better position to support their parents. In fact, these are the households who support the elimination of the social security system in Figure 4. In particular, LH households of all ages benefit the most from the transition to the new system. In this household, the child enjoys the elimination of the social security tax, increase in the wage rate and is able compensate his parent who suffers due to the abrupt elimination of social security benefits.

Using data from The Survey of Consumer Finances for 1983-85, Gale and Scholz (1994) find that in the U.S. about 75% of transfers involve parents giving to children. In our model with social security we find that 84% of intervivos transfers are from parents to children. Our results also indicate that intervivos transfers can play an important role in case of a change in policy. In the final steady state of this experiment, when social security is eliminated, transfers to children decrease to 65.8% of total transfers.

![Figure 5: Intervivos transfers.](image-url)
In the following graph we display the consumption profiles for the child, born at the time of the reform, of a type-3 household to further examine what takes place during the transition. Notice that the consumption profiles of all the households are higher during the transition compared to the steady state. These are the young members of the household who now are working for a higher wage rate. Indeed, the leisure profiles of these individuals reveal the fact that they all work more hours during the transition.

![Graph](image)

Figure 6: Consumption of Son.

Figure 7 displays the consumption profile of the parent in the same type 3 household who is age 55 at the time of the reform. The profiles in this figure confirm the conjecture that intervivos transfers allow for the parents to maintain their consumption levels during the transition.
Plan 2: Full Compensation-Labor Income Tax Finance  In this scheme individuals who have paid into the social security system are fully compensated. The government announces that individuals, from the reform date onwards, will not accumulate any more social security claims, and that retired individuals and others who have paid into the system will receive a pension corresponding to the social security claims that they have accumulated in the past. Initially these pensions are financed by a labor earnings tax only. This tax is eliminated at the year 60 of the transition. Overall, our results indicate that there is very small support for this elimination plan with 93% of individuals against it.

Figure 8 displays the decline in aggregate pensions as a fraction of GDP along the transition where the horizontal axis represents the years.
Capital and employment converge to their long-run level more slowly than in Plan 1. This is because there are pensions during 60 years while in Plan 1 pensions are eliminated in the first period of the reform. The capital stock and employment decrease at the initial periods of the transition and then increase slowly towards their new long run levels (see Figure 9).\(^\text{23}\) At the initial periods of the transition labor supply decreases because the labor income tax that is being used to pay for the transition is more distorting that the payroll tax that is being eliminated since there is a link between social security payroll taxes and retirement benefits in the initial steady state. The decrease in employment induces a decrease in saving and, as a result, a decrease in output.\(^\text{24}\)

\(^{23}\)The first value represented in these figures corresponds to the value at the initial steady state.

\(^{24}\)There is an additional increase in the labor income tax that is required to keep revenues constant, which adds to the distortions created in this economy.
The capital-labor ratio increases at the first period of the transition and then decreases during the next 45 years and then increases towards its higher long-run level. The decrease of the capital-labor ratio is due to the fact that employment increases before the capital stock increases. Figure 10 shows the evolution of the after tax prices during this transition.
An interesting outcome of this transition is its effect on the intervivos transfers. Figure 11 displays these transfers for type 3 households. In the uncompensated elimination scheme that was explained previously we had observed significant changes in the pattern of intervivos transfers during the transition. Since in this elimination scheme parents are fully compensated by the government, there is no need for the children to compensate their parents. Consequently, the pattern of intervivos transfers between the steady state and the transitions are now very similar.

![Figure 11: Intervivos Transfers](image)

Overall, our results indicate that there is only minimal support for this elimination plan with 7% of individuals in favor of it. In this social security reform, all type 3 households are hurt by the elimination of social security with the exception of the HL household with a child of age 30. This is a household where the parent is 65 years old and receives the full pension. In general, the welfare gains of this elimination scheme are higher if the child has low ability than otherwise (that is, LL and HL benefit more than LH and HH do at ages 30-40). The households where the child has low ability like less the annuity insurance of social security than the households with high ability children. Moreover, the welfare changes for different household types are in the range of $-0.9$ to $0.02$ percent of consumption at the benchmark.
steady state (for Type 3). In absolute value these welfare changes are much smaller than the ones implied by the uncompensated elimination (see Figures 3 and 4).

Next, we extend our analysis of this case to incorporate government debt where we use debt and labor income tax to finance the existing social security claims during the transition. The social security claims are honored in the same way as the previous experiment. In this case, the pensions are financed by an earnings tax and public debt. The maximum debt is 11% of GDP and the debt is eliminated by the year 60 of the transition. There is more support for this reform compared to the case without debt, but even in this case only 14.77% of individuals are in favor of the reform. Our results indicate that the compensated elimination of social security that is financed by labor income taxes (with or without debt) does not generate much support from the households in this economy. Indeed, even if we increase the time period in which debt is repaid, we can not generate significant support for this reform.

Plan 3: Full Compensation-Consumption Tax Finance and Debt  Similar to plan 2, this scheme fully compensates the individuals who have paid into the social security system. The government announces that individuals, from the reform date onwards, will not accumulate any more social security claims, and that retired individuals and others who have paid into the system will receive a pension corresponding to the social security claims that they had accumulated in the past. The existing social security claims are financed by a new consumption tax and public debt. In this economy, 60.41% of individuals are found to be in favor of this reform. In order to understand the reasons behind this support it is necessary to examine the changes that take place during the transition in more detail.

Figure 12 shows the ratio of aggregate pensions to GDP during the transition. The pensions are positive during 60 years. This means that individuals of ages 35-85 at the moment of the reform will receive a pension corresponding to their social security contributions. Individuals of age 20-30 will not receive pensions since they had not accumulated any contributions.\footnote{Notice that, at the benchmark economy, individuals start accumulating social security claims at age 30 because pensions depend on the earnings during the last 35 working years.}
Figure 13 shows the ratio of public debt to GDP during the transition. We assume that the government eliminates the debt after 60 years. The maximum stock of debt amounts to 11 percent of GDP. The consumption tax used to financed the pensions is kept constant during 60 year. In particular, the consumption tax increases from 5.5% to 13.7% at period 1 and decreases to 5.5% at year 60 of the transition.
The payroll tax is eliminated in the first year of the transition. As a consequence, the after-tax wage increases at the moment of the elimination of the tax as Figure 14 shows. While the after-tax wage increases monotonically during the transition, the after-tax interest rate decreases up to the year 60 when it jumps up and then decreases monotonically to the new steady state value. The jump in the interest rate in the year 60 is due to the jump in employment in that year.

Figure 15 shows that at year 60 employment jumps up considerably which is due to the fact that consumption tax is reduced which induces individuals to consume more and decrease their leisure. At that time, output in the economy also increases considerably due to the increase in the labor input. Notice that there is a significant difference between the transition path of employment that takes place in this plan as opposed to plan 2. In plan 2 the transition is financed by the labor income tax that is quite distortionary. This leads to a decline in employment initially. In the current plan, the transition is financed by an additional consumption tax which does not result in an initial decline in employment. This feature of the transition is crucial in generating positive gains that result in the overall support observed for this plan.\textsuperscript{26}

\textsuperscript{26}Comparing our findings for Plan 3 and Plan 2, we conclude that the transition financed with a consumption tax has a lower negative impact on employment. In understanding this finding, it is important to consider the overall distortion on labor supply generated by the taxation of both labor income and consumption. To this end, we divide both sides of the
Welfare. Figures 16 and 17 display the compensating variation in consumption needed to equate the expected discounted utility of a household in the benchmark steady-state and along the transition to the no social security steady-state. As before if this value is greater than unity for a household of a given age, then that household prefers to move along the transition to the steady state with no social security program. The horizontal axis represents the age of the child, which we take to be the age of the household. At the time of the reform, households of ages 20 to 50 are alive and they either have parents aged 55 to 85 (type 3 and 2), or their parents may have died sometime during their lifetime (type 1).

The first panel in Figure 16 displays the welfare of type 1 households. None of the households of this type are against this plan. These are young households whose parent had died, thus they do not have parents whose welfare they need to consider. As they get closer budget constraint by $(1 + \tau_c)$ and find that the "effective marginal tax rate on labor income" is given by $(\tau_\ell + \tau_c)/(1 + \tau_c)$. This "effective" labor income tax rate is 6 percentage points lower if the transition is financed with a consumption tax (plan 3) relative to the transition financed with a labor income tax (plan 2). If the transition is financed with a consumption tax, there is an implicit tax of the initial capital holdings which allows for a lower effective tax rate on labor income (see Erosa and Gervais (2002)). Moreover, a tax on consumption also raises revenue from retired individuals, which further allows for a reduction on the effective tax on labor income.
to the retirement age of 65 their support for the reform gets diminished because they lose more social security claims while they enjoy the higher wage for a shorter period of time. Among individuals belonging to type 1 households, those with low ability are the ones that benefit the most from elimination of social security (LL and HL) even though the social security benefit formula is progressive. Similar to the discussions before this result is due to two features of the model. First, low ability individuals have a shorter life expectancy and, thus, they care less about the annuity insurance provided by social security than high ability ones. Second, low ability individuals are more likely to be borrowing constrained, and the elimination of the social security payroll tax relaxes these constraints. All individuals gain with the elimination with the exception of abilities HH and HL of ages 40-50. Since the parent of these households was of high ability, these households are the richest. The increase in the after tax wage is less important for them because labor earnings are not the main source of income for these households.

The second panel displays the welfare results for individuals where the child had died sometime during the life of the parent. For this type, the horizontal axis represents the age of the parent. All of these individuals are against the elimination of social security even though they are fully compensated. This is due to the fact that in this elimination scheme the consumption tax increases from 5.5% to 13.7% hurting all these individuals.

Type 3 households, who constitute the majority of households, have different preferences about this elimination depending on their age and the abilities of their members, as can be seen from Figure 17. In general, the welfare gains display a non-monotonic path since welfare gains are decreasing with the age of the household from ages 20 to 30 and then increasing with the age of the household from ages 30 to 50. Consumption compensation is flatter than for the case of plan 1 since claims of parents in type 3 households are honored in this case. Similar to plan 1, the household that benefits the most is the LH household because it is the
one with lowest return of social security (pay high taxes and get low pension) and, moreover, the parent has low life expectation. The household that loses the most is HL because they pay low taxes, get high pension and the parent has high life expectancy. In general, 60.41\% of individuals benefit from this reform.

Overall, these experiments indicate the importance of the policies that are used in the elimination of the existing social security system. While payroll taxes that are used to finance a pay-as-you-go system are distortionary, the links that exist between social security benefits and the payroll taxes reduce the severity of these distortions. In this environment, the benefits of eliminating social security are mainly due to the effects of reform on labor supply. Consequently, to generate welfare gains, the reform needs to focus on tools that minimize distortion on labor supply. Our findings indicate that consumption taxes are less distortionary than labor income taxes, resulting in a larger welfare gain and larger overall support for reform.

5. Sensitivity Analysis

In the results presented so far, it is fairly clear that the gains from removing social security are mainly due to the increase in labor supply that is generated along the transition. The presence of two-sided altruism allows for these gains to be distributed across generations. In order to further investigate the major factors behind our results we conduct several sensitivity analyses. We use the uncompensated elimination scheme that was investigated in Plan 1 for the sensitivity analysis for its simplicity. In this scheme, payroll taxes and benefits are eliminated as soon as the reform is announced. In the following analysis, we conduct three experiments and compare their results with the findings that are described in Section 4.
In the first experiment, we decrease the link between the social security benefits and the payroll taxes that exists in the benchmark economy by assuming that an individual’s pension is a function of the average earnings of his ability group, instead of his own past labor income history. This is often an assumption that is made due to its simplicity. However, a major consequence of this assumption is its effect on the link between benefits and payroll taxes. Under this scheme, the payroll tax is more distorting than it actually is in the U.S. economy. Comparing the results in the first and second rows of Table 4 allows us to see the importance of this assumption. Notice that if the link between payroll taxes and benefits is not properly modeled then eliminating payroll taxes increases labor supply by 7.6%. This is higher than the increase that was obtained in the benchmark case, since more distortions are eliminated from the economy with this reform in this case as opposed to the benchmark. Consequently, capital and output all increase further in this case resulting in the 74% of the individuals supporting this case. In other words, elimination has more support if the economy before the reform has more distortions.\footnote{We have also checked the sensitivity of our results to the existence of idiosyncratic income risk. Removing that from the model turns out not to matter significantly for the quantitative results. For example, for the no link case support goes down to 73.5% compared to 73.88%. However, the economy with idiosyncratic risk is able to match the U.S. wealth distribution better.}

<table>
<thead>
<tr>
<th>% in favor</th>
<th>ΔK</th>
<th>ΔN</th>
<th>ΔY</th>
<th>ΔC</th>
<th>↓τ+τε</th>
</tr>
</thead>
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<tr>
<td>Benchmark</td>
<td>54.87</td>
<td>10.8</td>
<td>5.2</td>
<td>7.0</td>
<td>8.1</td>
</tr>
<tr>
<td>No Link</td>
<td>73.88</td>
<td>13.0</td>
<td>7.6</td>
<td>9.1</td>
<td>11.5</td>
</tr>
<tr>
<td>Inelastic Labor</td>
<td>18.46</td>
<td>6.9</td>
<td>0.0</td>
<td>2.1</td>
<td>1.2</td>
</tr>
<tr>
<td>G/Y constant</td>
<td>18.71</td>
<td>10.4</td>
<td>5.1</td>
<td>6.7</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Another way to examine the importance of the role of labor supply is to examine the impact of a reform in an environment with inelastic labor supply. The results for this case are displayed in row 3 of Table 4. By design there is no increase in the labor supply in this case. The capital stock response is smaller (6.9%) as is the increase in output (2.1%). The fraction of individuals in support of this system is very small (18.46%). These are mainly borrowing constrained young type 1 individuals (those without a parent) who enjoy higher consumption due to the elimination of the payroll tax. When labor supply is elastic, the benefits of eliminating social security are substantially higher for several reasons: 1) the elimination of the social security tax reduces labor distortions; 2) the increase in labor supply due to the elimination of the social security increases individual’s earnings inducing a further increase in capital in the long run; 3) the resulting increase in output increases government’s revenues, allowing a further but small reduction in the personal income tax.

In all the experiments that are carried out so far, we conduct revenue neutral elimination schemes. There are important implications of this assumption. For example, since the elimination of social security leads to an increase in output in these experiments, it also leads to an increase in the level of government revenues that are rebated back in the form of lower labor income tax rate. While a revenue neutral experiment is the natural one to
conducted, we examine the sensitivity of our results to this assumption in the last row of Table 4. The only difference in this experiment compared to the benchmark is due to keeping G/Y constant instead of G. The labor income tax in this case is not reduced since it is used to finance higher government purchases. The labor supply increases due to the decrease in the payroll tax resulting in a 6.7% increase in output. However, the increase in consumption between the two steady states in this case is smaller (5.7%) due to the increase in government purchases. Only 18.7% of the individuals are in favor of this reform. Again these are the young type-1 individuals.

6. Conclusions

In this paper we study the welfare effects of eliminating social security in a dynastic framework where social security provides insurance against life-span and individual income risks. Retirement benefits are financed with a payroll tax that distorts labor supply decisions and may also hurt borrowing constrained individuals. Social security also affects saving for retirement and for bequests since our framework nests the life-cycle and altruistic models.

We evaluate alternative schemes for eliminating the U.S. social security system that differ in the compensation of past social security claims and on the fiscal policy used to finance such compensation. Overall, our findings indicate the importance of the policies that are used in the elimination of the existing social security system. While payroll taxes that are used to finance a pay-as-you-go system are distortionary, the links that exist between social security benefits and the payroll taxes reduce the severity of these distortions. In this environment, the benefits of eliminating social security are mainly due to the effects of reform on labor supply. Consequently, to generate welfare gains, the reform needs to focus on tools that minimize distortion on labor supply. Our finding indicate that consumption taxes are far less distortionary than labor income taxes, resulting in a larger welfare gain and larger overall support for reform.

Most analyses about the elimination of social security have been conducted in a pure life-cycle framework. In this paper, we consider an environment with two-sided altruism. A contribution of our paper is to show that this assumption is both qualitatively and quantitatively important. It would be interesting to study the sensitivity of our results to the degree of altruism of individuals, that is, the discount rate of the utility of descendents and predecessors. Our model does not allow us to conduct this sensitivity analysis because it assumes that individuals do not discount the utility of their relatives. Such assumption implies that parents and children have the same objective function during the periods when their lifetime overlaps and, therefore, they pool their resources and jointly solve a maximization problem. Relaxing this assumption would imply that parents and children behave strategically. Modeling this behavior is not a trivial task and we leave it for future work.

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28 By increasing such discount rate the model will get closer to a pure life cycle framework.
7. Appendix: Computational Method

In this appendix we present the algorithms used to compute the steady states and the transition paths. In what follows, $k$ denotes the capital stock per effective units of labor, that is $k_t = K_t/(A_t N_t)$.

7.1. Algorithm to compute a steady state equilibrium

1. Guess $k, N, \tau_{\ell}, \tau$ and $\xi$.
2. Compute prices $r = \alpha k^{\alpha - 1} - \delta$ and $\omega = (1 - \alpha) k^{\alpha}$.
3. Compute bend points for pension formula $\{0, 0.2\bar{M}, 1.24\bar{M}, 2.47\bar{M}\}$, where $\bar{M}$ is the average earning in the economy (given by $N$ and $\omega$).
4. Compute after tax prices $r(1 - \tau_k)$ and $\omega(1 - \tau_{\ell} - \tau)$.
5. Solve individuals’ problem
6. Use optimal policies to simulate the outcomes of 200,000 dynasties during 1,500 model periods.
7. Compute averages (across 200,000 families) at period 1,500 of next period assets, labor supply, aggregate pension payments, accidental bequests.
8. Use market clearing conditions and government’s balance budget conditions to update the guesses of $k, N, \tau_{\ell}, \tau$ and $\xi$, if needed.

7.2. Algorithm to compute an equilibrium transition path

The algorithm used to compute the transition depends on the fiscal policy used to finance the social security claims. In what follows we describe the algorithm used to solve for the equilibrium when the elimination is uncompensated (plan 1).

We assume that the transition from the initial to the final steady state takes $S$ periods

1. Guess the paths for $k_t, \tau_{\ell,t}$, and $\xi_t$.
2. Compute prices $r_t = \alpha k_t^{\alpha - 1} - \delta$ and $\omega_t = (1 - \alpha) k_t^{\alpha}$.
3. Compute after tax prices $r_t(1 - \tau_k)$ and $\omega_t(1 - \tau_{\ell_t})$.
4. Solve individuals’ problem backwards starting at period $S - 1$ of the transition (using the value functions of the final steady state).
5. Use optimal policies at the initial steady state to simulate the outcomes of 200,000 dynasties over 1,500 model periods.
6. Assume that the period 1500 the government eliminates social security. Use the policies
computed at (5) to simulate the outcomes of each of the families during the $S$ periods
of the transition.

7. Compute averages (across 200,000 families) at each of the $S$ periods of the transition
of next period assets, labor supply, consumption, and accidental bequests.

8. Use market clearing conditions and government’s balance budget conditions to update
the guesses of the paths for $k_t$, $\tau_{\ell,t}$, and $\xi_t$.

7.3. Algorithm to solve the households’ problem.

1. Guess the value function of a household of age $1$ $V_1(a, 3, s_f, 0, z_f, z_s)$.

2. Solve the individuals’ problem by backward induction. At any age $i$ solve the problem
in two steps:

   1. Given an initial and final asset levels, $(a, a')$, solve for the optimal level of con-
sumption and leisure of the members of the household using the first order con-
ditions.

   2. Given the initial level of assets $a$, find the asset level $a'$ using the Euler condition.

3. Update the guess of $V_1(a, 3, s_f, 0, z_f, z_s)$ and iterate until convergence.

   **Step 2.1: Solving the household’s problem for a given final assets $a'$**

   Individuals’ average lifetime earnings are a state variable in the problem because pensions
are linked to contributions. We use a grid of 4 values for the average lifetime earnings
which are given by the bend points in the U.S. benefit formula. In particular the grid is
\{0, 0.2M, 1.24M, 2.47M\} where $M$ denotes the average lifetime earnings in the economy.
We use linear interpolation between grid points of average earnings to compute the optimal
labor supply using the first order conditions.

   **Step 2.2: Computing the optimal final assets $a'$**

   We apply bisection method to the Euler equation using linear interpolation between the
grid points of assets. The dimension of the grid for assets is 100 but the grid is finer at
low levels of assets. First, we find the grid point that maximizes the utility of the household.
Second, we apply the bisection method to solve for the optimal asset level around the grid
point that maximizes the utility of the household.

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29 At ages 1 and 2 we solve simultaneously for the labor supply of the father and the son and, then, we
need to carry on both the average earnings of the father and the one of the son. In order to reduce the
dimensionality of the household’s problem, we assume that the son’s earnings at ages 1 and 2 do not affect
his future pension. In other words, we assume that the pension depends on the earnings of the last 7 periods
(35 years) of an individual career. We think that this assumption is reasonable because in the U.S. economy,
pensions depend on the average earnings of the best 35 years which are likely to be the last 35 years of an
individual’s career.

30 This procedure is similar to the one used by Huggett and Ventura (1999).

31 Before solving this equation, we check if the borrowing constraint is binding.
7.4. Simulation

We keep the state-space coarse but allow the decision space to be near-continuous. As a result we can no longer compute the analytical distributions $X_j(a, h, v_f, v_s, z_f, z_s)$. Instead, we simulate the histories of 200,000 dynasties which we follow for 1,500 periods. Each of these dynasties has zero initial assets and zero initial average earnings. The initial household (in the dynasty) is of type 1 (individual of age 1 without a father). The individual makes a draw of the ability (college or noncollege) from the invariant distribution. The individual also makes a draw from the invariant distribution of labor productivity conditional on his ability type. We discretize the Markov process of labor productivity to only two values (following Tauchen (1986)) where the maximum grid point is 0.35 times the standard deviation of the shock.

Given the initial assets, the ability, and the labor productivity of the household, we compute the optimal consumption, leisure, final assets, and next period average lifetime earnings. At the end of the period, the individual makes a draw from the distribution of the mortality shock. If the individual survives, he makes a new draw of the labor productivity at the beginning of the next age. If the individual dies, he leaves an accidental bequest and a new dynasty starts as we described. At the end of age 7, the individual makes a draw of the ability of his children which is correlated with his own ability.

We compute averages of consumption, leisure, final assets and pensions using the outcomes of each of the dynasties from the period 1494 to 1500 (7 periods). When we aggregate decision rules across households, we use the corresponding population weights (taking into account population growth).

References


