A Monetary Business Cycle Model with Unemployment

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Abstract
To reproduce key features of the post-war U.S. data, most monetary business cycle models must assume there are high price markups and that agents have high labour supply elasticities. Unfortunately, microeconomic evidence indicates that markups and labour supply elasticities are generally low. This paper eliminates the need for these assumptions by introducing imperfectly observed effort into a limited participation model. In the model, detected shirkers forgo a bonus and households make their decisions about their level of monetary deposits for the period in advance of seeing the shocks to the economy. The estimated model is better able to capture the sluggish response of prices to a monetary policy shock than the standard model, and is consistent with recent evidence regarding the qualitative responses of the U.S. economy to technology shocks, fiscal policy shocks and monetary policy shocks. (JEL E32, E4)

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1. INTRODUCTION

To reproduce key features of the post-war U.S. data and the responses to monetary policy shocks, monetary business cycle models generally require the presence of high price markups and the assumption that agents have high labour supply elasticities.\(^1\) Microeconomic evidence, however, indicates that labour supply elasticities for workers and price markups are generally low.\(^2\) To reconcile these microeconomic estimates with the values needed in these models, papers like Christiano, Eichenbaum and Evans (1997) have recommended that future work focus on incorporating labour market frictions that mimic the effect of a high labour supply elasticity into limited participation and sticky price models.

Recently, Christiano, Eichenbaum and Evans (2003) have demonstrated that embedding frictions into a standard sticky price-sticky wage model helps improve its performance.\(^3\) In this paper, I follow a different approach. Specifically, I investigate how the performance of a standard limited participation model is altered when imperfectly observable work effort is added. My results suggest that introducing this microfounded labour market friction into the limited participation framework improves its ability to: (i) reproduce the estimated responses of the economy to a monetary policy shock, including the sluggish response of prices, (ii) account for the observed variation in employment and real wages over the business cycle without relying on high markups, and (iii) replicate the responses to fiscal policy shocks and technology shocks.\(^4\) Finally, the resulting model is better able to account for the

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\(^1\) See e.g., Christiano, Eichenbaum and Evans (1997), and Rotemberg and Woodford (1996).

\(^2\) See Basu and Fernald (1997,1999) for some evidence on price markups, and Pencavel’s (1986) survey paper for evidence that the labour supply elasticity of males is near zero in the U.S.

\(^3\) See Ambler, Guay and Phaneuf (2003) for a similar exercise that instead focuses on sticky nominal wages, costly labour adjustment and imperfectly competitive households.

\(^4\) Although Christiano, Eichenbaum and Evans (2003) examine a sticky price model, Christiano, Eichen-
Dunlop-Tarshis observation.

Unlike the model in Christiano, Eichenbaum and Evans (2003), where prices and nominal wages are assumed to be sticky, the friction embedded in the limited participation model endogenously causes wages and prices to be relatively unresponsive to monetary shocks. As a result, the model with imperfectly observed effort outperforms the benchmark limited participation model. The new model has five main characteristics: (i) households make nominal savings decisions before observing the period’s shocks, (ii) firms are monopolistically competitive, (iii) workers’ effort levels are imperfectly observed by firms, (iv) shirkers who are detected by their firm forgo a bonus, and (v) households provide unemployment insurance to their members. The first two features are common in limited participation models. Features 3 and 4 are associated with the shirking literature that originated with Shapiro and Stiglitz (1984), and feature 5 is introduced because there will be positive levels of unemployment in equilibrium. Two types of unemployment insurance schemes are examined - full income insurance (where non-shirkers and the unemployed received the same income) and partial income insurance (where detected shirkers and the unemployed receive the same income). These two scenarios are of interest since they correspond to cases where the workers without jobs are voluntarily and involuntarily unemployed.

In the limited participation model with imperfectly observed effort (herein referred to as

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6 Here the term voluntarily unemployed refers to the case where the utility of the unemployed agent exceeds that of a working agent so that unemployed workers would prefer not to find employment. In contrast, involuntarily unemployed refers to the case where the utility of the unemployed agent is less than that of a working agent so that unemployed workers would like to find jobs if possible.
the shirking model), wages affect a worker’s productivity.\textsuperscript{7} Similar to Shapiro and Stiglitz’s (1984) model, the link between wages and productivity emerges because firms only imperfectly observe workers’ effort levels on the job.\textsuperscript{8} Consequently, firms take their workers’ incentive compatibility constraints into account when they make their decisions and the model generates a positive level of equilibrium unemployment for reasonable parameter values. In this case, employment and wages are determined by the intersection of the labour demand curve and the individuals’ incentive compatibility constraints - not by the intersection of the labour demand and labour supply curves. This implies that, unlike the standard model, the shirking model’s predictions will not depend on the elasticity of the labour supply curve. However, as the results from a number of past shirking models have shown, this feature alone is not sufficient to guarantee an improvement in the model’s performance. For example, Gomme (1999) shows that when firms fire detected shirkers, the wage and employment volatilities seen in the U.S. data cannot be replicated by the model. The predicted wages are too volatile while employment is not volatile enough. This occurs because of the punishment associated with being detected shirking (being fired). The smaller the pool of unemployed, the faster unemployed find jobs. For this reason when unemployment is low (and output high), firms must raise wages significantly to prevent workers from shirking. This produces a strong comovement between wages and output, and causes the model’s predictions to be similar to those obtained from a standard business cycle model with a low labour supply elasticity.

In my model, individuals found shirking on the job forgo a portion of their possible pay

\textsuperscript{7} This feature is shared with the efficiency wage models discussed in Katz (1986).

\textsuperscript{8} See Danthine and Kurmann (2004) for a monetary efficiency wage model where the relationship between wages and productivity instead emerge because of issues related to fairness and wage norms.
for the period, (i.e., a bonus or a raise), instead of being dismissed. I use this “monetary punishment” instead of the Shapiro-Stiglitz dismissal punishment for two reasons. First, there is evidence which suggests that firms more commonly rely on this type of “monetary punishment” to discipline workers than on outright dismissal. Second, unlike the Shapiro-Stiglitz model, this type of model is better able to reproduce the behavior of wages and employment since firms can punish the workers even when there is full employment.

The analysis reveals that both the standard limited participation model and the shirking model are qualitatively consistent with some recent empirical evidence about the U.S. economy’s responses to monetary policy shocks, fiscal policy shocks and technology shocks. However, their quantitative responses differ substantially. Small shocks generate much larger output and employment responses in the shirking model than in the standard limited participation model, and these results do not depend on either high levels of markups or large labour supply elasticities. For example, with a markup of about 15%, following a monetary policy shock, the shirking model with full income insurance produces employment and output responses approximately twice as large as the responses generated in the standard limited participation model while the shirking model with partial income insurance produces em-

9 While this environment is similar to the ones seen in Alexopoulos (2004), and Burnside, Eichenbaum and Fisher (2000), these other papers do not investigate the economy’s response to monetary policy shocks.

10 For example, evidence in papers such as Agell and Lundborg (1995), Hall (1993), Bewley (1999), Weiss (1990), and Malcomson (1998) indicate that: (1) the majority of firms report that they initially reprimand detected shirkers instead of firing them, and (2) disciplined workers are not as likely to receive bonuses, raises or promotions. Moreover, MacLeod and Parent (1999) find that there are a significant number of firms offering workers contracts with bonuses and that the incidence of these contracts tend to increase during periods of low unemployment.

11 This occurs since the punishment affects the slope of the incentive compatibility curve.

ployment and output responses over three times larger. This occurs because, in the shirking model, the response of wages to the shock is muted. Consequently, firms hire significantly more labour in the shirking model and supply more goods to the market in response to the shock. This large output response, in turn, helps the shirking model deliver a more sluggish price response to the shock than the one produced in the standard model. In fact, the larger increase in supply enables the shirking model with partial income insurance to generate a price response over 40% smaller than in the standard model in the period of the shock and the full income insurance shirking model to generate a 17% smaller response. In addition, the price level in the model with frictions remains significantly lower than the price level in the standard model for over one year following the shock.\footnote{The response is over 1/3 lower in the partial income insurance shirking model and 15% lower in the full income insurance case one year later.} Finally, the differential wage response also helps the shirking model deliver significantly larger output and employment responses than the standard model following technology and fiscal policy shocks. This helps explains why the shirking model is more consistent with the Dunlop-Tarshis observation that the correlation between real wages and output is low.

The remainder of the paper is organized as follows. Section 2 presents the limited participation model with imperfectly observed effort. Section 3 then presents the results for the estimated versions of the model and the empirical implications for the model’s second moment properties. It also outlines the estimated model’s responses to technology, monetary and fiscal policy shocks, and compares these responses to the empirical evidence about the U.S. economy’s responses to these shocks, as well as to the results obtained from a standard limited participation model. Finally, Section 4 provides a summary of the results and
suggests areas for future research.

2. THE MODEL

The model economy has 6 sectors: the monetary authority, the financial intermediaries, the government, the families, the intermediate goods firms and the final good firms. Each agent’s problem is discussed in detail below.

2.1. The Monetary Authority

The monetary authority controls the period by period monetary injection, $X_t$, received by the financial intermediaries. The growth rate of money, $x_t$, is defined by $x_t = \frac{X_t}{M_t} = \frac{M_{t+1} - M_t}{M_t}$ where $M_t$ is the nominal stock of money at the beginning of period $t$. For the purpose of this investigation, I use the M2 measure of the money stock and assume that $x_t$ is the realization of an AR(1) process:

$$x_t - x^{ss} = \rho_x (x_{t-1} - x^{ss}) + \varepsilon_{xt}$$

where $-1 < \rho_x < 1$, $x^{ss}$ is the mean growth rate of money and $\varepsilon_{xt}$ is a serially uncorrelated process with mean zero and standard deviation $\sigma_x$.\(^{14,15}\)

2.2. Financial Intermediaries

At the beginning of time $t$, the continuum of perfectly competitive financial intermediaries receives nominal deposits, $D_t$, from families to invest for the duration of the period. After these funds are deposited, the financial intermediaries receive the lump sum cash injection, $X_t$, from the monetary authority.

\(^{14}\) The findings of Christiano, Eichenbaum and Evans (1998) indicate that $x_t$ follows an AR(1) process if the money stock is measured by M2. Although M2 is used as the measure of money in this paper, simulations of the model using M1 as the measure and the MA(2) process for $x_t$ reported in Christiano, Eichenbaum and Evans (1998) suggest that the results are not dependent on which money stock is used.

\(^{15}\) The money growth rate, $x_t$, is assumed to be unaffected by fiscal and technology shocks. This assumption is consistent with the results in Eichenbaum, Edelberg and Fisher (1998), and Christiano, Eichenbaum and Vigfusson (2002), which suggest that the stock of money does not significantly respond to these shocks.
As in the standard limited participation model, it is assumed that intermediate goods firms require loans to finance their wage bills. The firms borrow these funds from the financial intermediaries for the duration of the period at a gross interest rate, $R_t$. This implies that the economy’s loan market clearing condition is:

$$ L_t = D_t + X_t $$

(1)

where $L_t$ is the total amount of funds demanded by firms in time period $t$ and $D_t + X_t$ is the total supply of available funds. At the end of the period, the intermediate goods firms repay their loans with interest, and the financial intermediaries distribute $R_tD_t$ to the households in return for their deposits and $R_tX_t$ in the form of profits.

2.3. The Government

The government finances its purchases of final goods, $G_t$, by levying lump sum taxes on families. Therefore, the government’s period $t$ budget constraint is:

$$ G_t \leq \Omega_t $$

where $\Omega_t$ is the amount of lump sum taxes collected. Since all families are identical in the model, each is assumed to face an equal tax burden. The value of $G_t$ is assumed to vary over time according to the following process:

$$ \ln(G_t) = \mu_g + \rho_g \ln(G_{t-1}) + \varepsilon_{g,t} $$

where $\mu_g$ is a constant, $-1 < \rho_g < 1$, and $\varepsilon_{g,t}$ is a serially uncorrelated process with mean zero and standard deviation $\sigma_g$.

2.4. Families and Individuals

In models with unemployment, when agents’ incomes are heterogeneous and agents are able to transfer wealth across periods, the individual’s savings decision becomes dependent
on his entire work history. To keep my model comparable to other standard representative agent models, and isolate the role of the imperfect observability of effort, I introduce a family construct to ensure that the workers’ problems will remain homogeneous.\textsuperscript{16}

Specifically, the economy is assumed to be populated by a large number of families, each of which contain a $[0, 1]$-continuum of infinitely lived individuals.\textsuperscript{17} The individual’s problem remains homogeneous since individual agents do not directly own assets in this model. Instead it is assumed that each individual’s family owns an equal portion of the capital stock, as well as equal amounts of shares in the intermediate goods firms and the financial intermediaries that entitle them to a percentage of the profits earned by the financial intermediaries and intermediate good firms. Families then use these funds for investments and to purchase some consumption goods for their members.

\textbf{2.4.1. A Representative Family}

The representative family owns all the assets in the economy, and makes all decisions regarding the amount of money dedicated to family-purchased consumption, $P_t c^f_t$, nominal deposits, $D_t$, capital investment, $P_t I_t$, and money holdings, $M_{t+1}$, where $P_t$ is the price of the final good in time $t$. Consistent with the limited participation constraint, the family chooses the level of nominal deposits before observing the values of the period’s shocks. After the shocks are revealed, the family pays taxes, $\Omega_t$, and decides how much money to devote to capital investment and family-purchased consumption.

\textsuperscript{16} Gomme (1999) uses an alternate approach to ensure that the agent’s problem remain homogeneous. In his model, there are two types of agents: entrepreneurs who can save and workers who cannot. Versions of my model using a similar structure yielded approximately the same results.

\textsuperscript{17} The number of families is assumed to be large enough so that no individual family assumes it can influence firms’ labour contracts. The role of the family is similar to the family’s role in Shi (1997), and is also seen in papers such as Alexopoulos (2004), Burnside, Eichenbaum and Fisher (2000), and Felices (2001).
The family finances its period $t$ family-purchased consumption, $c_t^f$, taxes, deposits, and capital investment using their beginning of period real money balances, $M_t/P_t$, and their return on capital, $r_tK_t$. Profits are not distributed to the family until the end of the period and therefore are unavailable for purchasing period $t$ goods. Under these assumptions, the family’s cash in advance constraint and money holdings are described by the following equations:

$$P_t c_t^f \leq M_t - D_t - P_t \Omega_t + P_t r_t K_t - P_t I_t$$  \hfill (#2)

$$M_{t+1} = [M_t - D_t - P_t c_t^f - P_t \Omega_t + P_t r_t K_t - P_t I_t] + R_t [D_t + X_t] + \pi_t$$  \hfill (#3)

where $\pi_t$ and $R_t X_t$ denote the profits from intermediate goods firms and financial intermediaries in period $t$ respectively. Since the family decides how to distribute $P_t c_t^f$ among the members before firms hire employees, each family member is given an equal share.

As in the standard limited participation model, families must be discouraged from removing large amounts of deposits from the financial intermediaries to generate persistent responses to monetary policy shocks (See e.g., Christiano and Eichenbaum (1992a)). Therefore, I assume that the family faces adjustment costs whenever they alter their stock of capital or their flow of funds to the goods market. The forms of the adjustment costs on the flow of funds to the goods market and capital are similar to those used in Christiano and Eichenbaum (1992a), and Christiano and Fisher (1998) respectively. Specifically, if the flow of funds to the goods market changes, I assume that the family members’ utility decreases because of the time that must be spent reorganizing the family’s purchases. In addition, the final good cannot be freely converted to the capital goods used in production by the intermediate goods sector. Here, the end of period capital stock, $K_{t+1}$, is determined by the
following technology:

\[ K_{t+1} = \left[ \gamma_1 I_t^v + \gamma_2 K_t^v \right]^{\frac{1}{2}} \]  \hspace{1cm} (#4)

where \( I_t \) is the amount of period \( t \) investment, \( v \leq 1 \) determines the cost of adjusting the capital stock, and \( \gamma_1 \) and \( \gamma_2 \) are positive constants.\(^{18}\) To ensure that the steady state values of the rental rate of capital and investment are invariant to the level of \( v \), \( \gamma_1 \) and \( \gamma_2 \) are set to the values \( \gamma_1 = \delta^{1-v} \) and \( \gamma_2 = (1 - \delta) \), where \( \delta \) represents the capital depreciation rate. In the benchmark case, I set \( v = 1 \) so that equation (#4) is the conventional linear capital accumulation equation.

### 2.4.2. Family Members

Although individual family members do not have direct access to financial or capital markets, they do receive some consumption financed by their family’s return on financial and capital investments through \( P_t c_f^t \). In addition to this family-purchased consumption, \( c_f^t \), family members may increase their consumption level by obtaining wage income from employment.

All intermediate goods firms are assumed to have identical production technologies. As a result, all firms will require the same effort levels from their workers and offer the same wage levels. Since effort is imperfectly observed by the firms, workers who accept employment must choose whether to provide the required effort level. Workers hired by the intermediate goods firms receive a one-period contract that specifies the number of hours an employee must work, \( f \), the level of effort a worker is required to provide, \( e_t \), and the wage rate, \( w_t \), that a worker can earn.\(^{19}\) All workers receive a fraction, \( s \), of their wages up front, but the

\(^{18}\) Here lower values of \( v \) correspond to higher adjustment costs.

\(^{19}\) To reduce the amount of notation in this subsection, the subscript that identifies which firm offers the contract is omitted here since all firms are identical and will choose the same values.
final payment of \((1 - s)w_t f\) is only paid to workers who are not disciplined for shirking.\(^{20}\)

In addition, workers understand that if they shirk, they will be detected by their firm’s exogenous monitoring technology with probability \(d < 1\).

In this model, the optimal wage chosen by intermediate goods firms can result in equilibrium unemployment.\(^{21}\) Unemployed family members who do not reject a job offer can increase their consumption level above \(c^f_t\) by purchasing extra consumption goods with the funds they receive from their family’s income insurance fund. This fully funded insurance is financed by working family members who each transfer a lump sum, \(F_t\), to a fund that is distributed among the unemployed during the period.\(^{22}\) This unemployment insurance ensures that an unemployment spell will not result in a counterfactually large drop in consumption for the individual. While the amount of the unemployment insurance is exogenous in this model, I examine two different risk sharing arrangements, partial income insurance and full income insurance, to determine how changes in the benefits affect the model’s predictions.\(^{23}\)

2.4.3. The Worker’s Problem

Workers are able to purchase consumption with their family’s consumption benefits, \(P_t c^f_t\),

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\(^{20}\) In this model, \(s\) is assumed to be an exogenous parameter. However, this model delivers the same results as a model where: (i) there is a restriction on the minimum value of \(s\), (e.g. a legal restriction or an industry norm), and (ii) \(s\) is chosen endogenously by firms, since firms would always set \(s\) to the lowest level possible to maximize the value of the punishment for shirking. The value of \(s\) in the model is determined by the US data and GMM procedure discussed later in the paper.

\(^{21}\) Although for some parameter values, the model will produce no equilibrium unemployment, in the remainder of the paper I focus on the more interesting case where the model predicts positive levels of unemployment in equilibrium.

\(^{22}\) The model’s predictions are unaltered if the government, instead of the family, runs the unemployment insurance by taxing each employed worker \(F_t\).

\(^{23}\) As in the real models described in Alexopoulos (2004) and Woodford (1994), the full insurance case of this model has the same reduced form as a model with indivisible labour. However, as Alexopoulos (2004) demonstrates, the equivalence between the models does not hold when there is only partial income insurance.
and their after transfer wage income, while the unemployed purchase consumption with their family’s consumption benefits, \( P_t c_t^f \), and their transfer from the insurance fund. Here, \( c_t^{ns} \), \( c_t^s \) and \( c_t^u \) denote the consumption level enjoyed by all workers not detected shirking (i.e., non-shirking workers and non detected shirkers), the consumption level for a detected shirker and the unemployed’s consumption level respectively. As a result

\[
\begin{align*}
    c_t^{ns} &= c_t^f + w_t f - F_t \\
    c_t^s &= c_t^f + sw_t f - F_t \\
    c_t^u &= \begin{cases} 
        c_t^f + \frac{N_t}{1-N_t} F_t & \text{if the individual had no job offer} \\
        c_t^f & \text{if the individual rejects a job offer}
    \end{cases}
\end{align*}
\]

where \( N_t \) is the total number of family members employed in period \( t \). Individuals’ utility levels are then described by the function:

\[
u(c_i^t, \widehat{e}_t) = \ln (c_i^t) + \theta \ln (T - \vartheta (\widehat{e}_t > 0) (f\widehat{e}_t + \xi)) - \theta \ln (1 + H(Q_t, Q_{t-1})) \]

where \( c_i^t \) is the individual’s realized consumption level (where \( i = ns, s, \) or \( u \)), \( \widehat{e}_t \) is the level of effort provided by the individual, \( T \) is the individual’s time endowment, \( \vartheta(\cdot) \) is an indicator function taking on the value of 1 when effort is provided, and \( \xi \) is the disutility associated with providing any effort (which is unobservable to firms). The term \( -\theta \ln(1 + H(\cdot, \cdot)) \) is the adjustment cost on the flow of funds to the goods market that helps the model generate persistent responses to monetary policy shocks. One interpretation of the form of the adjustment costs used here would be that each individual spends a portion of his leisure time, \( \frac{H(\cdot, \cdot)}{1+H(\cdot, \cdot)} \), involved in reorganizing the household’s flow of funds to the goods market.\(^{24}\)

\(^{24}\) Under this interpretation unemployed family members have more time available and, hence, will spend more time reorganizing family finances than their working counterparts.
Below, I assume that:

\[ H(Q_t, Q_{t-1}) = a_0 \left\{ \exp \left[ a_1 \left( \frac{Q_t}{Q_{t-1}} - 1 - x^{ss} \right) \right] + \exp \left[ -a_1 \left( \frac{Q_t}{Q_{t-1}} - 1 - x^{ss} \right) \right] - 2 \right\} \]

where \( Q_t = M_t - D_t \). This formulation implies that, when there are no changes in the flow of funds to the goods market, no adjustment costs are realized.

Family members who are offered jobs must determine whether working for the firm is optimal and whether they will abide by the terms of the contract, if they accept employment. Given that individuals who turn down job offers are ineligible for unemployment insurance, workers will always accept employment. Therefore, an employed worker will choose to provide the required effort specified by the firm, \( e_t \), if his incentive compatibility (IC) constraint is satisfied, i.e., if:

\[ u(c_t^{ns}, e_t) \geq du(c_t^s, 0) + (1 - d)u(c_t^{ns}, 0) \text{ (IC)} \]

In other words, the value of a working member’s effort, \( \hat{e}_t \), can be described as:

\[ \hat{e}_t = \begin{cases} e_t & \text{if } e_t \leq \frac{T}{f} \left( 1 - \left( \frac{c_t^{ns}}{c_t^s} \right)^{\frac{d}{\theta}} \right) - \frac{\xi}{f} \\ 0 & \text{if } e_t \geq \frac{T}{f} \left( 1 - \left( \frac{c_t^{ns}}{c_t^s} \right)^{\frac{d}{\theta}} \right) - \frac{\xi}{f} \end{cases} \]

2.4.4. The Family’s Problem

During the period firms hire \( N_t \) family members. Of these workers, \( N_t^s \) are shirkers who exert no effort on the job, and \( N_t - N_t^s \) are non-shirkers who exert a positive level of effort, \( \hat{c}_t \). Using the previously specified utility function and adjustment costs, and the definitions
of \( c^u_t, c^s_t \) and \( c^{ns}_t \), the family’s problem can be written as:

\[
\max_{\{c^s_t, K_{t+1}, M_{t+1}, D_t\}} \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l}
(N_t - dN^s_t) \ln (c^{ns}_t) + dN^s_t \ln (c^s_t) + (1 - N_t) \ln (c^u_t) \\
+ (N_t - N^s_t) \theta \ln (T - f\tilde{c}_t - \xi) + (1 - (N_t - N^s_t)) \theta \ln (T) \\
- \theta \ln (1 + H(Q_t, Q_{t-1}))
\end{array} \right\}
\]

subject to equations (#2) through (#4) where:

\[
F_t = \begin{cases} 
(1 - N_t)sw_t f \text{ under partial income insurance} \\
(1 - N_t)w_t f \text{ under full income insurance}
\end{cases}
\]

In equilibrium, the cash in advance constraint holds with equality since the gross interest rate on deposits exceeds unity, and no one shirks on the job (i.e., \( N^s_t = 0 \)). Therefore, the family’s Euler equations for deposits and capital accumulation reduce to:\(^{25}\)

\[
E_{t-1} \left\{ \begin{array}{l}
\left( \frac{U_t}{P_t} \right) - \theta \frac{H_1(Q_t, Q_{t-1})}{1 + H(Q_t, Q_{t-1})} - \beta \theta \frac{H_2(Q_{t+1}, Q_t)}{1 + H(Q_t, Q_{t-1})} \\
- \beta R_t \left[ \left( \frac{U_{c^s_t}}{P_{c^s_t}} \right) - \theta \frac{H_1(Q_{t+1}, Q_t)}{1 + H(Q_t, Q_{t-1})} - \beta \theta \frac{H_2(Q_{t+2}, Q_{t+1})}{1 + H(Q_t, Q_{t-1})} \right]
\end{array} \right\} = 0
\]

\[
E_t \left\{ U_t \frac{\partial I_t}{\partial K_{t+1}} - \beta U_{c^s_t} (r_{t+1} - \frac{\partial I_{t+1}}{\partial K_{t+1}}) \right\} = 0
\]

where \( U_{c^s_t} = \frac{N_t}{c^{ns}_t} + \frac{1 - N_t}{c^u_t} \)

\[
H_1(Q_t, Q_{t-1}) = \frac{a_0 a_1}{Q_{t-1}} \begin{cases} 
\exp \left[ a_1 \left( \frac{Q_t}{Q_{t-1}} - 1 - x^s \right) \right] \\
- \exp \left[ -a_1 \left( \frac{Q_t}{Q_{t-1}} - 1 - x^s \right) \right]
\end{cases}
\]

\[
H_2(Q_t, Q_{t-1}) = -H_1(Q_t, Q_{t-1}) \frac{Q_t}{Q_{t-1}}
\]

\[
\frac{\partial I_t}{\partial K_{t+1}} = \delta^{\frac{v-1}{v}} (K^v_{t+1} - (1 - \delta)K^v_t) \frac{1}{v - 1} K^{v-1}_{t+1}
\]

and

\[
\frac{\partial I_{t+1}}{\partial K_{t+1}} = \delta^{\frac{v-1}{v}} (K^v_{t+2} - (1 - \delta)K^v_{t+1}) \frac{1}{v - 1} (\delta - 1) K^{v-1}_{t+1}
\]

\(^{25}\) For simplicity, it is assumed that families do not believe that their choices affect the employment probability of their members. Alexopoulos (2001) discusses a way to rationalize this assumption and shows the more complex environment leads to precisely the same allocations as in this model.
2.5. Final Good Firms

The perfectly competitive final good firms produce the final good, $Y_t$, by combining the output of the continuum of intermediate firms using the technology:

$$Y_t = \left[ \int_0^1 Y_{it} \frac{1}{\mu} di \right]^\mu$$

where $Y_{it}$ represents the input from the $i^{th}$ intermediate firm in period $t$, and $1 < \mu \leq \infty$. The parameter $\mu$ will determine the markup in the economy.

Given this technology, a representative final good firm faces the following profit maximization problem in period $t$:

$$\max_{Y_t, \{Y_{it}\}_0^1} P_t Y_t - \int_0^1 P_{it} Y_{it} di \text{ s.t. } Y_t = \left[ \int_0^1 Y_{it} \frac{1}{\mu} di \right]^\mu$$

where $P_t$ is the price of the final good and $P_{it}$ is the price of the $i^{th}$ intermediate good at time $t$.

The Euler equations from this problem define the demand functions for the intermediate goods firm’s output:

$$Y_{it} = Y_t \left( \frac{P_{it}}{P_t} \right)^{\frac{\mu}{1-\mu}}$$

Therefore, the demand for firm $i$’s product is both increasing in aggregate output, $Y_t$, and decreasing in the price of its intermediate good. Finally, using the demand equations and the zero profit condition, the price of the final good can be determined by:

$$P_t = \left[ \int_0^1 P_{it}^{\frac{\mu}{\mu-1}} di \right]^{(1-\mu)}$$

2.6. Intermediate Goods Firms

The economy’s intermediate goods are produced by a $[0, 1]$-continuum of monopolistic competitors. Entry and exit into the production of intermediate good $i$ are ruled out and
representative firm $i$ is assumed to have the following production function:$^{26}$

$$Y_{it} = A_tK_{it}^\alpha ((N_{it} - N_{it}^s)f\tilde{e}_{it})^{1-\alpha}$$

Here $0 < \alpha < 1$, and $A_t$ is the level of technology where $\ln A_t = (1 - \rho_A)\ln A + \rho_A\ln A_{t-1} + \varepsilon_{A,t}$, $-1 < \rho_A < 1$ and $\varepsilon_{A,t}$ is a serially uncorrelated process with mean zero and standard deviation $\sigma_A$. Further, $N_{it}, N_{it}^s$ and $K_{it}$ denote the number of workers hired, the number of shirkers hired, and the amount of capital rented in period $t$ by firm $i$ respectively. Each employee works a fixed shift, $f$, and provides an effective labour unit input of $f\tilde{e}_{it}$, if he chooses not to shirk on the job.

After the time $t$ shocks are observed, firms rent capital from a perfectly competitive market, and publicly advertise employment opportunities. The advertisement specifies the number of workers they want to hire, the real wage rates they will pay workers, and the effort level they require from each worker. All parties understand that, although workers providing the required effort level are guaranteed wages $w_{it}f$, $(1 - d)N_{it}^s$ shirkers will receive $w_{it}f$ while $dN_{it}^s$ shirking workers will receive compensation $sw_{it}f$.

Once the employees are hired, each firm borrows its nominal wage bill, $P_tw_{it}fN_{it} = W_{it}fN_{it}$, from the bank at the gross interest rate $R_t$. Although the firm possesses the funds to pay its entire wage bill before the workers begin production, they choose to withhold a fraction, $(1 - s)$, of the funds.$^{27}$ This shows their intent to pay their workers while making

$^{26}$ This representation assumes that non-shirking workers provide an effort level of $\tilde{e}_{it}$ while shirking workers provide no effort to firms.

$^{27}$ Alexopoulos (2001) presents an extension of this model where firms endogenously choose the value of $s$. In this richer environment: (i) shirkers are detected by an exogenous monitoring technology with probability $d$, and non-shirkers are labelled as shirkers with an exogenous probability $\varepsilon$, where $\varepsilon < d$, (ii) the employed worker’s time endowment may differ from an unemployed worker’s due to commuting time, and (iii) the family’s unemployment insurance takes the form $c_{it}^{nds} = bc_{it}$ where $c_{it}^{nds}$ is the consumption level of a worker not punished for shirking and $b$ is a constant. All workers that are detected incur the monetary punishment.
the punishment for shirking credible. In this case, each worker is paid a base salary of $sW_{it}$ at the beginning of the period and the additional amount of $W_{it}f(1-s)$ at the end of his contract period, provided he is not detected shirking. Although, the model only requires that $s \in [0, 1)$, data on base wages and bonuses suggest that $s$ is generally set to a value greater than $1/2$. Therefore, I allow this parameter to be determined by the data instead of setting $s = 0$ (the maximum punishment level) a priori.

After workers are hired, production takes place and the output is sold to the final good firms. The firm then pays for the rental of capital, $r_tK_t$, and repays the bank loan with interest. Any remaining proceeds from the sale are then distributed in the form of profits to the families at the end of the period.

Formally, a representative intermediate good firm, hiring identical workers, faces the following problem in period $t$:

$$\max_{\{P_{it}, w_{it}, N_{it}, K_{it}, e_{it}\}} \left\{ \frac{P_{it}}{P_t} \left( A_t K_t^\alpha \left( f e_{it} N_{it} \right)^{1-\alpha} \right) - R_t w_{it} f N_{it} - r_t K_{it} \right\}$$

subject to the period by period demand functions:

$$P_t^{\mu-t} \left( A_t K_t^\alpha \left( f e_{it} N_{it} \right)^{1-\alpha} \right) - P_t^{\mu-t} Y_t = 0$$

as well as the period by period incentive compatibility constraints (IC) and individual ratio-

---

28 See e.g., International Customer Service Association (2001) publication on bonuses, salaries and incentives.

29 The fraction of the wage forfeit by detected shirkers may be interpreted as a bond between firms and workers in the model. Katz (1986) discusses reasons why there may not be full bonding between workers and firms in the economy. These reasons may explain why $s \neq 0$ in practice.
nality constraints (IR):

\[ u(c_{it}^{ns}, c_{it}) \geq du(c_{it}^s, 0) + (1 - d)u(c_{it}^{ns}, 0) \] (IC)

\[ u(c_{it}^{ns}, c_{it}) \geq u(c_{it}^u, 0) \] (IR)

Here \( u(\cdot, \cdot) \) is the representative worker’s utility for the period, \( e_{it} \) is the effort level specified in the contract, and \{\( c_{it}^{ns}, c_{it}^s, c_{it}^u \)\} are given by equations (#5) through (#7) when wages are equal to \( w_{it} \).

In this problem, \( w_{it} \) denotes the real wage rate offered by the firm, and \( r_t \) is the real rate of return on capital. Since the form of the family’s unemployment insurance implies that the IR constraint does not bind, the IC constraint holds with equality in equilibrium. Using the definitions of \( c_{it}^{ns} \), and \( c_{it}^s \), effort can then be expressed as a function of the real wage, the price level, the intra-family transfers and the family’s non-deposited funds:

\[ e_{it}(w_{it}) = \frac{T_f}{f} \left( 1 - \left( \frac{c_{it}^{ns}}{c_{it}} \right)^{\frac{d}{\theta}} \right) - \frac{\xi}{f} \]

It follows that the Euler equations from an intermediate good firm’s problem can be re-arranged to obtain the following equilibrium conditions:

\[ \frac{e_{it}'(w_{it})w_{it}}{e_{it}(w_{it})} = 1 \] (The Solow Condition)

\[ \frac{Y_{it}}{N_{it}} \frac{(1 - \alpha)}{\mu} = \frac{P_t}{P_{it}} \frac{f w_{it} R_t}{Y_{it}} \]

\[ \frac{Y_{it}}{K_{it}} \frac{\alpha}{\mu} = \frac{P_t \rho}{P_{it}} \]

\[ P_{it}^{\gamma - \nu - \mu} Y_{it} = P_t^{\gamma - \nu} Y_t \]

The Solow Condition ensures that the firm’s choices will minimize the cost per unit of effort. Notice that since \( e'(w_{it}) = \frac{T_f}{f} \frac{d}{\theta} \left( \frac{c_{it}^{ns}}{c_{it}} \right)^{-\frac{d}{\theta} - 1} \left( \frac{c_{it}^s - s c_{it}^u}{(c_{it}^u)^2} \right) \) and \( w_{it} = \frac{c_{it}^{ns} - c_{it}^s}{1 - s} \), the equation
\[ \frac{e'(w_{it})w_{it}}{c_{1t}(w_{it})} = 1 \] can be written as a function of \( \frac{c_{it}}{c_{it}} \) and the parameters. As a result, it is optimal for firms to choose wages that keep the level of \( \frac{c_{it}}{c_{it}} \) (the punishment associated with shirking) and effort constant since:

\[ \frac{c_{it}}{c_{it}} = \chi ightarrow e_{it} = \frac{T}{f} \left( 1 - (\chi)^{-\frac{d}{f}} \right) - \frac{\xi}{f} = e \]

where \( \chi \) is a constant greater than 1. Furthermore, all of the intermediate goods firms require the same effort level and offer the same wages to workers since they have use identical technologies and workers.

These Euler equations also imply that the firm chooses to set its time \( t \) price equal to a constant markup over the period’s marginal costs:

\[ P_{it} = \mu MC_{it} \]

Given the representative firm’s problem for each good \( i \), it follows that the equilibrium demand for funds can be expressed as:

\[ L_t = \int_0^1 W_{it}fN_{it}di \]

Since firms borrow funds from the financial intermediaries to finance their wage bills, \( L_t = D_t + X_t \). No one shirks in equilibrium and the unemployment rate is \( 1 - \int_0^1 N_{it}di \).

3. THE EMPIRICAL RESULTS

The generalized method of moments (GMM) procedure discussed in Christiano and Eichenbaum (1992b), along with quarterly data from 1955 to 1992, are used to estimate the parameters and diagnose the performance of the model.\(^{30} \)

\(^{30}\) A detailed description of the data is found in Appendix A followed by an outline of the exactly identified GMM procedure, based on the Euler equations from the model with technological growth, in Appendix B.
grow over time in the data, I introduce growth to the model before estimating it by adding an exogenous labour augmenting technology to the production function:

\[
Y_{it} = A_t K_{it}^\alpha (\gamma f N_{it} f^\epsilon c_{it})^{1-\alpha}
\]

where \( \gamma \geq 1 \). In addition, government expenditures are assumed to evolve according to 
\[
G_t = \gamma^t \exp(g_t),
\]
where \( g_t = \mu_g + \rho_g g_{t-1} + \varepsilon_{g,t}, -1 < \rho_g < 1, \) and \( \varepsilon_{g,t} \) is a serially uncorrelated process with mean zero and standard deviation \( \sigma_g \). The data used to estimate the second moments, \( \{\sigma_c, \sigma_i, \sigma_y, \sigma_w, \sigma_n, \sigma_u, \sigma_y, \sigma_u\} \), are detrended using a Hodrick and Prescott (HP) filter to ensure that the estimates exist since the data displays marked time trends.\(^{31}\)

After the parameters and second moments are estimated, the model is tested by: (1) comparing the estimated second moments from the data to those computed from the model using a Wald test,\(^{32}\) and (2) examining if the model’s predictions regarding how the economy responds to shocks are qualitatively consistent with the estimated impulse responses reported in papers like Christiano, Eichenbaum and Evans (1997, 1998), Ramey and Shapiro (1997), Edelberg, Eichenbaum and Fisher (1998), Christiano, Eichenbaum and Vigfusson (2002).

Not all of the model’s parameters are estimated using the Euler equations. The values for \( \beta, T, f, \xi, \theta, v, a_1, \) and \( a_2 \), are chosen to coincide with values commonly seen in the literature. Specifically,\(^{33}\)

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( T )</th>
<th>( f )</th>
<th>( \xi )</th>
<th>( \theta )</th>
<th>( v )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \left( \frac{1}{1.03} \right)^{0.25} )</td>
<td>1369</td>
<td>1</td>
<td>30</td>
<td>0.72</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Here, since \( f \) only affects the scale of effort, it is normalized to 1, \( T \) is chosen to coincide

\(^{31}\) \( \sigma_j \) denotes the standard deviation of the log of variable \( j \) where \( j = c, i, y, w, n, \) and \( u \).

\(^{32}\) This Wald test formally explores the hypothesis that the two sets of estimates are the same in population \( \) (see Christiano and Eichenbaum (1992b)).

\(^{33}\) The main findings are robust to small changes in the parameters \( \xi, \theta, v, a_1 \) and \( a_2 \).
with a time endowment of 15 hours per day per quarter, ξ represents a fix cost of 10 minutes a day, v = 1 implies there are no capital adjustment costs, a1 and a2 are set equal to the values in Christiano, Eichenbaum and Evans (1997), and θ and β are chosen so that they fall in the range commonly seen in the literature. An additional assumption about the ratio \( \frac{c_{n\sigma}}{c_f} \) is made to help identify the ratio \( \frac{d}{\theta} \), and the parameter s, in the IC constraint.34

Here, this ratio is assumed to equal 1.2853 based on Gruber’s (1997) results that an unemployment spell results in a 22.2% drop in food consumption when there is no government run unemployment insurance.36 The GMM procedure and the data are used to estimate the remaining parameters

\[
\left\{ \frac{d}{\theta}, \delta, \tau_g, \mu_g, \rho_g, \sigma_g, \ln(A), \rho_A, \sigma_A, A_y, \ln(\gamma), \alpha, \ln(\frac{g}{y}), x^{ss}, \mu, \rho_x, \sigma_x, \sigma_u \right\}
\]

in the model.37 The resulting parameter estimates for each case of the limited participation model with imperfectly observed effort (the shirking model) are reported in Table 1 alongside estimates for a standard limited participation model with divisible labour for the cases where \( v = \{1, 0.97, 0.95\} \).38

Since the results of a standard limited participation model with

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34 Once \( \frac{d}{\theta} \) is estimated, the value of s can be determined from the Solow condition.

35 Similar results are obtained if instead of assuming a value for \( \frac{c_{n\sigma}}{c_f} \), the value of s is set within the middle of the range suggested by surveys on the size of bonuses and values of \( \frac{c_{n\sigma}}{c_f} \) and \( \frac{d}{\theta} \) are obtained from the estimation. See e.g, Alexopoulos (2003).

36 The model’s sensitivity was assessed by varying the value of \( \frac{c_{n\sigma}}{c_f} \) since this value is never observed in equilibrium. In general, the findings indicate that small movements in \( \frac{c_{n\sigma}}{c_f} \) have little effect on the model’s second moments and responses to shocks.

37 The parameter \( \tau_g \) is included in the equation for \( g_t \) to capture the time trend in government purchases observed in the data, \( A_y \) is estimated using the condition: \( E \left( \ln(Y_t) - A_y - t \times \ln(\gamma) \right) = 0 \), and \( \frac{d}{\theta} \) is estimated using the condition: \( E \left( \ln(G_t) - \ln(Y_t) - \ln \left( \frac{g}{y} \right) \right) = 0 \).

38 The standard limited participation model with divisible labour, based on Christiano, Eichenbaum and Evans (1997), is described in Appendix C. The parameters estimated in this model are: \( \left\{ \theta, \delta, \tau_g, \mu_g, \rho_g, \sigma_g, \ln(A), \rho_A, \sigma_A, A_y, \ln(\gamma), \alpha, \ln(\frac{g}{y}), x^{ss}, \mu, \rho_x, \sigma_x \right\} \).
indivisible labour are identical to those from the shirking model with full income insurance, they are not reported separately.\footnote{See Appendix D for a proof of the observational equivalence of the models.}

An examination of these estimates reveals that the predicted values for both models are similar to those commonly seen in the literature. Further, the estimated parameters for the shirking model are virtually identical to those in the standard limited participation model, with the exception of the values of the coefficient on leisure in the individual’s utility function, $\theta$, and $\frac{d}{\theta}$. For example, the value of $\alpha$ falls in the range $[0.25,0.43]$ reported in Greenwood, Rogerson, and Wright (1995), the depreciation rate is around 2\% per quarter and the values for $\rho_x$ and $a_{ss}$ are similar to the values estimated in Christiano, Eichenbaum and Evans (1998). The value of $s$ implied by the estimation is approximately 80\% which is within the range reported in the International Customer Service Association’s (2001) publication on bonuses, salaries and incentives.

The estimates also reveal that the results for the shirking model are not attributable to a high markup value. Here, the estimated markup, $\mu$, is 1.15. Although this value is generally higher than the markups reported in Basu and Fernald (1993), it is lower than the value of 1.2 reported by Hornstein (1993) and much lower than the values Christiano, Eichenbaum and Evans (1997) report are necessary to obtain realistic price movements in the standard limited participation model (i.e., $\mu$ between 1.4 and 2).

Next, I solve the models using the estimated parameter values and the linearization technique described in Christiano (1998), and I compute the models’ impulse response functions for fiscal policy shocks, technology shocks and monetary policy shocks. I begin by presenting the second moments of the various models and then turn to a discussion of the models’
impulse response functions. Given the parameter estimates, it is clear that the differences between the models’ predictions discussed below are primarily attributable to the different assumptions about the labour market.

3.1. The J-Test

Table 2 presents the Wald tests that formally explore the hypothesis that the second moments from the estimated models are the same in population as the second moments estimated from the data. The results clearly indicate that the limited participation model with imperfectly observed effort is better able to produce low wage variation and high employment variation. The lowest real wage variation and highest employment variation is obtained from the shirking model with partial income insurance. Moreover, the model with partial income insurance is better able to reproduce the variation of unemployment.

This table demonstrates that with the low markup of $\mu = 1.15$ the standard limited participation model produces too little employment variation and too much variation in wages. In contrast, the shirking model tends to overshoot the point estimate of employment variation, $\sigma^2_{n}$. Increasing the cost of adjustment on capital decreases the employment variation and increases the wage variation in both models. Consequently, as the different panels in Table 2 show, small increases in the costs of adjustment $v$ tend to improve the fit of the shirking model, but decrease the performance of the standard limited participation model further. This occurs since large costs of capital adjustment cause investment to become less responsive to shocks and consumption to become more responsive. Since the volatility of wages in the shirking model are linked to the volatility of family purchase consumption, $c^f_t$,

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40 These results are also found if instead of using the actual unemployment series to estimate the models, I use the total employment hours series and convert this series to a full time equivalent employment rate.
and $F_t$, a decrease in $\nu$ increases the variation in real wages and decrease the variation in employment.

In addition to producing low real wage variation and high employment variation, the shirking model is also able to produce a much lower correlation between hourly wages and employment hours than the limited participation model. For the benchmark cases where $\nu = 1.0$ the shirking model with partial income insurance has a correlation of approximately -0.13; the full income insurance case has a correlation of approximately 0.55 and the standard limited participation model has a correlation of about 0.70. Therefore, the shirking model is more consistent than the standard model with the Dunlop-Tarshis observation that real hourly wages and employment hours are not highly correlated (and may in fact be negatively correlated in the data).\textsuperscript{41}

3.2. Monetary Policy Shocks

Figures 1 and 2 display the shirking model’s responses to an expansionary monetary policy shock alongside the response’s predicted by the limited participation model. They suggest that consumption, investment, output, employment, and profits increase, real wages generally rise and nominal interest rates fall. Despite the fact that all the models are consistent with the qualitative results outlined in papers like Christiano, Eichenbaum and Evans (1997), they differ with respect to their quantitative responses. With the relatively small markups ($\mu = 1.15$) the figures confirm the previous findings for the standard limited participation model - it falls to deliver large employment and output responses alongside smalls price and wage responses. However, it appears that adding the shirking friction to the model

\textsuperscript{41} The correlation between hours worked and the hourly wage rate in the data for the time period examined is approximately 0.0085. This estimate was insignificantly different from 0.
vastly improves upon its predictions. In particular, the figures illustrate that the shirking model is better able than the standard limited participation model to generate sticky real wages, a sluggish price response, and large output and employment responses.

To understand why the shirking model is able to deliver smaller wage and price responses alongside large employment and output gains in response to a monetary policy shock, it is necessary to first examine the shock’s impact on the marginal cost of labour. As in the standard model, the nominal interest rate declines in response to the shock which places downward pressure on the firms’ labour costs. However, the difference between the response of the marginal cost of labour in standard model and the shirking model is not due to this channel - it is due to the differential wage responses.

In the shirking model the response of real wages is determined by the effect the shock has on the punishment associated with shirking. Specifically, given that firms optimally want to keep effort constant, when the shock causes the punishment for shirking, \( \frac{c^s_t}{c_1} \), to deviate from its equilibrium level, \( \chi \), firms must alter the wage to prevent workers from shirking. Using the firm’s Euler equations and the definitions of \( c^{ns}_t \) and \( c^s_t \), it is clear that the response of the real wages paid by a firm depends on the magnitude of the change in \( (c^f_t - F_t) \) since their workers’ wages can be written as \( w_t = \frac{\chi - 1}{1 - \gamma} (c^f_t - F_t) \). The response in the value of family-purchased consumption, \( c^f_t \), depends on how the shock alters the family’s purchasing power, \( \frac{M_t - D_t}{P_t} + r_t K_t - G_t \), and the family’s investment decision, while the response of the insurance premium, \( F_t \), depends on the change in the unemployment rate and the aggregate wage.

The monetary policy shock affects the family’s purchasing power, \( \frac{M_t - D_t}{P_t} + r_t K_t - G_t \), through two channels. The first channel increases \( r_t K_t \) since the influx of loanable funds
decreases the interest rate which, in turn, increases labour demand and the return on capital. The second channel affects the families’ purchasing power through the price level since:\(^{42}\)

\[ P_t = \frac{1}{\left(1 - \frac{\alpha}{\nu}\right)} \frac{(M_t + X_t)}{Y_t} \]

This equation implies that, if the output response to the shock is small, an unexpected increase in \(X_t\) will cause prices to rise and erode the families’ purchasing power, while if the output response is large, prices will fall and increase the amount of goods families can buy with \(\frac{M_t - D_t}{P_t}\). Given that the two channels may work in opposite directions, the effects of an unexpected increase in the money supply on \(\frac{M_t - D_t}{P_t} + r_tK_t - G_t\) will depend on which channel dominates. This is clearly illustrated in Figures 1 and 2.

Even if the family’s purchasing power increases in response to a positive monetary policy shock, the response of \(c^f_t\) depends on how much the household chooses to invest. Given the shock is relatively persistent, the family finds it optimal to use the majority of their resources to purchase investment and decrease the amount of family purchase consumption given to their members.\(^{43}\) As a result, the movement in the punishment associated with shirking depends on the magnitude of the fall in \(F_t\).

In the partial income insurance case with no adjustment costs on capital \((v = 1)\), the punishment associated with shirking rises only modestly in the first period since the shock causes \(F_t\) to decline slightly more than \(c^f_t\). Firms respond to this change by mildly increasing

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\(^{42}\) This goods market clearing condition is obtained by: (i) substituting the firm’s Euler equation for capital, and the accounting equation \(Y_t = I_t + C_t + G_t\) into the family’s cash in advance constrain, and (ii) using the fact that \(C_t = c^f_t + w_tN_t\) and \(w_tN_t = \frac{D_t + X_t}{P_t}\).

\(^{43}\) Although the standard limited participation model does not have a family structure, it is still possible to examine the responses of non-wage funds, \(\frac{M_t - D_t}{P_t} + r_tK_t - G_t\), and the response of \(\frac{M_t - D_t}{P_t} + r_tK_t - G_t - I_t\) in the model. The response of these values are reported in the figures for the purpose of comparison, and to illustrate that \(\frac{M_t - D_t}{P_t} + r_tK_t - G_t - I_t\) decreases in the standard model as well.
wages to the point where \( \frac{w^u}{c^u} \) again equals \( \chi \) since they require a higher wage to obtain the optimal effort level, \( e \), from workers. Since the interest rate declines significantly while the wage increase is small, there is a large decrease in the marginal cost of labour which induces sizable increases in both employment and output alongside a relatively moderate increase in prices in the first period. Since \( c^u_t = \frac{\chi(1-s)w_t}{(\chi-1)} \) and \( c^s_t = c^u_t = \frac{w^u_t}{\chi} \), the consumption of the non-shirkers and the unemployed, like wages, are virtually unresponsive to the shock in period one. Aggregate consumption, however, rises since there are now more individuals who are employed and consuming the higher consumption level, \( c^u_t \).

The same forces are in effect in the second period. However, without capital adjustment costs, the further increase in investment causes the value of \( c^f_t - F_t \) to decline in period two. Consequently, firms respond by reducing wages to restore the value of \( \frac{w^u_t}{c^u_t} \) back to \( \chi \). This decrease in \( w_t \) alongside the further decline in \( R_t \) leads firms to expand employment and output even more than they had in the first period. The size of the output affect then causes prices in the second period to mildly decrease in period two. Families find their large second period investment optimal even though it causes a small decline in total consumption, \( N_t c^u_t + (1 - N_t) c^u_t \), because it translates into large, and persistent gains in future consumption (i.e., two and half years after the shock, total consumption in the partial insurance model is over 80% larger than total consumption in the full income insurance model).

Despite the fact that a positive monetary policy shock can cause real wage to fall in the shirking model, this does not always occur. In particular, as Figures 1 and 2 demonstrate, the response of real wages is sensitive to the degree of income insurance and the size of the capital adjustment costs. For example, when there is full income insurance or mild capital adjustment costs, the shock’s affect on investment is muted. As a result, \( c^f_t - F_t \) rises more
than in the case with partial income insurance and this forces firms to mildly increase real wages in response to the unexpected increase in the money supply.

The empirical evidence in Christiano, Eichenbaum and Evans (1997) suggest that in response to a positive monetary policy shock output, employment, aggregate consumption and investment increase, interest rates decrease, and prices and real wages mildly rise. The partial income insurance model with mild capital adjustment costs appears to be the most consistent with their results. Specifically, in the partial income insurance model with small adjustment costs, real wages remain relatively unresponsive for approximately one year following the shock. As a result, the wage response for this model is 1/5 the size of the response seen in the standard limited participation model on impact and about 1/10 the size of the response of the standard limited participation model one year later. Consequently, the shirking model is better able to reproduce large employment and output responses. Moreover, the larger increase in supply enables the shirking model with partial income insurance to generate a more sluggish price response. In fact, the partial income insurance model’s price response is over 40% smaller when $v = 1$ than in the standard model in the period of the shock and remains over 1/3 lower for the following year. The difference is diminished when the adjustment costs of capital increase, however, even when $v = 0.95$ the partial income insurance model’s price response is over 30% smaller in the period of the shock and remains about 30% lower for the first year.45

44 The responses reported in Christiano, Eichenbaum and Evans (1997) suggest that, real wages could initially fall in response to a positive monetary policy shock in some sectors. However, for the vast majority of cases, the response of real wages became mildly positive within a couple of quarters.

45 The full income insurance case also produces a more sluggish price response. When $v = 1$, it produces a 17% smaller price response than the standard model in the period of the shock, and a 13% smaller response when $v = 0.95$. Furthermore, in both cases, the price response remains over 10% lower than the standard models’ response during the first year.
Next I examine the models’ responses to fiscal policy and technology shocks. These responses are reported for completeness and to determine how the models with limited participation frictions respond to real shocks. The responses to real shocks in these models may differ from the responses seen in non-monetary models since the shocks will affect the wealth of families through their impact on the price level and family purchasing power.46 47

3.3. Fiscal Policy Shocks

The shirking and limited participation models’ responses to an exogenous increase in government expenditures are depicted in Figures 3 and 4. These figures demonstrate that both models share the same qualitative responses to a positive government expenditure shock. Consistent with the results reported in Edelberg, Eichenbaum and Fisher (1998) and Ramey and Shapiro (1997), in response to the shock output, employment, investment and interest rates all increase, while consumption and real wages decrease.

In the shirking model, an exogenous increase in government expenditures raises taxes. Holding everything else constant, this reduces the purchasing power of the family and increases the punishment associated with shirking (i.e., \( \frac{\alpha}{c^t} \) increases). Firms respond to this change by lowering workers’ real wages back to the point where \( \frac{\alpha}{c^t} = \chi \) to keep effort constant. At this lower wage rate, the marginal product of labour exceeds the marginal cost so firms increase the size of their labour force. Consequently, employment and output rise.

The increases in output and employment have three effects. First, the unemployment

46 The families’ purchasing power is affected since \( M_t \) and \( D_t \) are determined prior to the shock being realized and \( P_t \) responds to the shock.

47 The responses will also differ from those seen in Alexopoulos (2004) since firms borrow money to finance their wage bill in this environment.
insurance premium, $F_t$, decreases. Second, the current and expected return on capital increase. Third, the price of the final good declines to clear the final good market. Both the increase in the return on capital and the effect of the price decrease on nominal balances, affects the families purchasing power. Families invest more in capital goods because of the increase in the expected future return on capital. Despite the fact that the decline in prices and increase in $r_t$ work to increase family purchases, in practice, $c^f_t$ decreases because of the rise in taxes and the large increase in investment. $F_t$ falls, however, the decrease in $c^f_t$ is larger which causes $c^f_t - F_t$ to fall in the end. Consequently, in response to the shock, real wages and consumption decline while $I_t$, $N_t$ and $Y_t$ increase. Prices then fall due to the rise in $Y_t$. In contrast, interest rates are initially unresponsive to the shock because the level of deposits is determined prior to the shock and there is no increase in the money stock.

A comparison of Figures 3 and 4 again demonstrates that: (1) the shirking model produces larger responses than the standard limited participation model in all variables except aggregate consumption, (2) the form of the intra-family transfer affects the magnitude of the shirking model’s responses, and (3) increasing the capital adjustment cost decreases the shock’s effect on employment, real wages and output by lowering the response of investment. The figures also show that neither model can account for the observation in Edelberg, Eichenbaum and Fisher (1998) that prices increase following an exogenous increase in government expenditure. However, given the models’ responses to a monetary policy shock seen in Figures 1 and 2, if the monetary authority expands the money supply in response to the shock, prices could rise in response to a fiscal policy shock and nominal interest rates would immediately decline.

3.4. Technology Shocks
Figures 5 and 6 display the limited participation and shirking models’ responses to a shock that increases the level of technology by 1%. The findings suggest that, in both models, a positive technology shock causes prices to decrease and all other variables to increase. These responses are consistent with the findings of Christiano, Eichenbaum and Vigfusson (2002) and Alexopoulos (2006).48

A positive technology shock increases both output and the marginal product of labour for firms in the shirking model. The increase in current output again increases the families’ purchasing power through two channels. First, holding all else equal, it decreases the price of the final goods, which causes an increase in $\frac{M_t-D_t}{P_t}$.49 Second, the increase in $A_t$ causes the real return on capital to rise. Although both of these forces allow the family to purchase more consumption and investment goods, as in the real business cycle model presented in Alexopoulos (2004), the resulting wages and employment responses depend on how much $c_t^f$ and $F_t$ are affected.

In addition to increasing the families’ purchasing power, the expected future return on capital rises since the increase in technology is very persistent. This induces the family to allocate the majority of the increase in their purchasing power towards investment which causes $c_t^f$ to fall in response to the shock. Even though the decrease in $c_t^f$ works to raise the punishment for shirking, the increase in employment causes a significant decline in $F_t$ which offsets the fall in $c_t^f$. Overall, $c_t^f - F_t$ only mildly increases so firms only need to increase the real wage modestly to dissuade workers from shirking. This increases the marginal cost

48 The results in other papers, such as Basu, Fernald and Kimball (1999) and Gali (1999), have suggested that technology improvements may, in fact, be contractionary in the short run and expansionary only in the long run.

49 This affect is not present in Alexopoulos’ (2004) shirking model and help to explain why real wages are slightly more responsive in the monetary shirking model.
of labour, however, the rise in the marginal product of labour is larger. As a result, output, employment, consumption, investment and wages increase in response to the technology shock, while prices decrease. Finally, the models predict that the interest rate is initially unaffected because the money stock is unaffected by the shock and the level of deposits is initially fixed.

Similar to the findings for monetary policy and fiscal policy shocks, the form of the intra-family transfer and the adjustment cost on capital again affect the predicted responses. Specifically, the figures imply that: (1) limiting the amount of income insurance available to agents will again increase the employment and output responses, and (2) increasing the costs of adjustment on capital dampens the shock’s impact on output, employment and investment. Moreover, it appears that compared with the standard limited participation, the shirking model is able to produce larger price decreases and smaller real wage increases alongside more substantial increases in employment, output, investment and consumption.

4. CONCLUSIONS

This paper develops and analyzes a monetary business cycle model where: (1) individuals make nominal savings decisions before observing the period’s shocks, (2) a worker’s effort level is only imperfectly observed by firms, (3) detected shirkers forgo an increase in their compensation, and (4) unemployed workers receive income insurance through their families. Two types of unemployment insurance are examined - full income insurance (where non-shirkers and the unemployed receive the same income) and partial income insurance (where unemployed workers receive the same income as a detected shirker). These two schemes are explored because they correspond to cases where the unemployed workers are voluntarily
and involuntarily unemployed respectively.

The results demonstrate that introducing imperfectly observed effort into a standard limited participation model helps improve its performance. First, the shirking model is better able than the standard model to reproduce the sluggish response of prices and wages following a monetary policy shock - especially in the model with partial income insurance. Second, the shirking model’s ability to reproduce low wage variation alongside high employment variation does not depend on high labour supply elasticities or high levels of markups. Third, the shirking model is better able to account for the Dunlop-Tarshis observation.

The shirking model’s impulse responses to monetary and fiscal policy shocks and technology shocks are examined. The findings reveal that the magnitude of the responses depend on both the type of unemployment insurance and the level of capital adjust costs. However, the qualitative responses are similar and generally in line with the empirical evidence on how the U.S. economy responses to these shocks. Specifically, the shirking model predicts that, in response to a positive monetary policy shock, real wages, output, employment, investment and prices increase, while the gross interest rate decreases. In response to a positive fiscal policy shock, employment, output, and investment increases, the gross interest rate weakly increases, while real wages and consumption decrease. Finally, in response to an unexpected exogenous increase in the level of technology, real wages increase, employment increases, output increases, investment increases, gross interest rates increase, and the price level decreases.

A comparison of the implied impulse responses to fiscal policy, technology and monetary policy shocks for the standard limited participation and shirking models demonstrates that the models’ responses are similar qualitatively but differ in magnitude. For example, the
shirking model produces much larger increases in employment, output, and investment in response to all the shocks considered. The responses of wages, prices and consumption, however, depend on the type of shock. In particular, the estimated shirking model predicts larger movements in real wages and smaller movements in consumption in response to a fiscal policy shock while technology and monetary policy shocks predicted smaller wage movements alongside larger movements in aggregate consumption. Moreover, prices in the shirking model respond less to a monetary policy shock, and more to fiscal policy and technology shocks, in comparison to those in the standard limited participation model.

Overall, the results in the paper support the hypothesis that introducing efficiency wage considerations can help improve the performance of the standard limited participation model. Future work should concentrate on: (i) quantifying the importance of this type of friction, (ii) eliminating the need for adjustment costs on the flow of funds to the goods market, and (iii) introducing labour adjustment costs.

APPENDIX A: The Data

The models were estimated using quarterly data from the time period 1955-1992. This data is available from: the Citibase/DRI Database, the Federal Reserve Bank and the Bureau of Economic Analysis. The official capital stock was obtained from the Bureau’s Survey of Current Business-Fixed Reproducible Tangible Wealth in the U.S. Using these statistics, the capital stock, $K_t$, was defined as the sum of the net stocks of consumer durables, producer structures, and equipment and private residential capital plus the government non-residential capital.

Private consumption, $C_t$, was then defined as the sum of private-sector expenditures on
non-durable goods and services plus the imputed service flow from the stock of consumer durable goods. Moreover, the private sector data on non-durables and services was obtained from Citibase, while the imputed service flow from the stock of durable goods was created using the method described in Brayton and Mauskopf (1985).

Next, gross investment, \( I_t \), was measured as the sum of consumer expenditures on durable good, gross private non-residential (structures and equipment) and residential investment, as well as the change in the gross stock of government capital (computed using the data from the Survey of Current Business). Government expenditures, \( G_t \), was computed using the statistics reported in Citibase for federal, state and local expenditures on goods and services minus real government investment (measured by the change in the gross stock of government capital). The measure of output at time \( t \) was then defined as real GDP as found in Citibase. Specifically, this measure includes \( C_t + I_t + G_t \) plus net exports and time \( t \) inventory adjustments.

The GDP deflator with base year 1987, obtained from Citibase, was used to convert data variables between their nominal and real levels. The interest rate was measured using data available from the Federal Reserve Bank on the prime lending rate on loans. Finally, the monetary aggregate represented in the model was measured by M2, and was obtained by combining the numbers available from Citibase with the earlier estimates from Rasche (2001).

Two additional variables were needed to estimate the model: wages and employment/hours. The employment data was defined by using Citibase’s unemployment rate, while the wage series was created by combining the Citibase data on wages and other labour income. Although the employment rate is directly used in the shirking models, the standard divisible
and indivisible labour models generally are estimated using the number of hours worked normalized by the number of leisure/labour hours available to individuals over the period.

To keep the dataset as consistent as possible across the different models, all the data was converted to per-capita terms by dividing by the size of the labour force obtained from Citibase. This normalization then allowed for a computation of an implied hourly employment series by taking a stand on the number of hours an individual worked per week. For the purposes of this model, individuals were assumed to work 40 hours per week. The series for quarterly hours worked was then created using the formula: \( \frac{40 \times 52}{4}(1 - u_t) \) where \( u_t \) is the unemployment rate. This series was used in the divisible and indivisible labour models after it was normalized by the number of leisure/labour hours available to individuals during the quarters.

**APPENDIX B: The Estimated Equations**

The shirking model’s parameters,

\[
\left\{ \frac{d}{\partial}, \delta, \tau, k, \mu, \rho, \sigma, \ln(A), \rho_A, \sigma_A, A_y, \ln(g_y), \alpha, \ln(g), x^{ss}, \mu, \rho, \sigma \right\}
\]
are simultaneously estimated using the following restrictions:

\[
E\left( \frac{(1 - \alpha)}{\mu} - \frac{R_t w_t N_t}{Y_t} \right) = 0
\]

\[
E \left( \ln(A_t) - \ln A - \rho_A \ln(A_{t-1}) \right) = 0
\]

\[
E \left( \ln(A_t) - \ln A - \rho_A \ln(A_{t-1}) \times \ln(A_{t-1}) \right) = 0
\]

\[
E \left( \ln(A_t) - \ln A - \rho_A \ln(A_{t-1}) \right)^2 - \sigma_A^2 = 0
\]

\[
E \left( \ln(Y_t) - A_y - t \times \ln(\gamma) \right) = 0
\]

\[
E \left( \ln(Y_t) - A_y - t \ln(\gamma) \times \frac{t}{149} \right) = 0
\]

\[
E \left( \ln(G_t) - \ln(Y_t) - \ln \left( \frac{g}{\gamma} \right) \right) = 0
\]

\[
E \left( \ln \left( \frac{G_t}{\gamma^t} \right) - \mu_g - t \times \tau_g \right) = 0
\]

\[
E \left( \left( \ln \left( \frac{G_t}{\gamma^t} \right) - \mu_g - t \times \tau_g \right) \times \frac{t}{149} \right) = 0
\]

\[
E \left( (1 - \rho_g \alpha) \ln \left( \frac{G_t}{\gamma^t} \right) \times \ln \left( \frac{G_{t+1}}{\gamma^{t+1}} \right) - \mu_g - t \times \tau_g \times (t - 1) \right) = 0
\]

\[
E \left( \left( (1 - \rho_g \alpha) \ln \left( \frac{G_t}{\gamma^t} \right) - \mu_g - t \times \tau_g \right)^2 - \sigma_g^2 \right) = 0
\]

\[
E \left( x_t - (1 - \rho_x) x^{ss} - x_{t-1} \right) = 0
\]

\[
E \left( (x_t - (1 - \rho_x) x^{ss} - x_{t-1}) \times x_{t-1} \right) = 0
\]

\[
E \left( (x_t - (1 - \rho_x) x^{ss} - x_{t-1})^2 - \sigma_x^2 \right) = 0
\]

where \( muc_t \) is the marginal utility of \( c_t^f \) for the family. The standard limited participation
model’s parameters are estimated using the same identification scheme with this model’s marginal utility of consumption data, $muc_t$, hourly employment data, $N_t$, and the its expression for the steady state value of employment, $N^{ss}$. In this case, the equation equating the employment hours and the steady state value of employment hours identifies the parameter value $\theta$, instead of the variable $\frac{d}{\theta}$ as in the shirking model.

### The Identifying Restrictions for the J-test:

To test the models’ predictions for $\left\{ \frac{\sigma_{c}}{\sigma_{y}}, \frac{\sigma_{i}}{\sigma_{y}}, \frac{\sigma_{g}}{\sigma_{y}}, \sigma_{w}, \sigma_{n}, \sigma_{y} \right\}$ the HP filtered data, $\left\{ c_{t}^{hp}, i_{t}^{hp}, y_{t}^{hp}, y_{t}, n_{t}^{hp}, w_{t}^{hp}, u_{t}^{hp} \right\}$ are used along with the follow equations:\(^{50}\)

\[
E \left( \left( \frac{\sigma_{j}}{\sigma_{y}} \right)^{2} - \left( \sigma_{j}^{hp} \right)^{2} \right) = 0 \quad \text{for} \quad j = g, i, \text{ and } c
\]

\[
E \left( \left( \sigma_{j}^{hp} \right)^{2} - (\sigma_{j})^{2} \right) = 0 \quad \text{for} \quad j = n, y, w, \text{ and } u
\]

These moments were simultaneously estimated with the models’ parameters.

### APPENDIX C: The Standard Limited Participation Model

The standard limited participation model with divisible labour is based on Christiano, Eichenbaum and Evans (1997), and has the same six sectors as the shirking model presented in the paper. The problems facing the monetary authority, the final goods firms and the government are identical to the ones described in the shirking model. However, the individual’s problem, the intermediate goods firms’ problems and the loan market clearing condition differ slightly due to the observability of effort and the divisibility of labour, and are stated below.

---

\(^{50}\) $\lambda = 1600$ when the data was filtered.
The individual’s problem in the standard limited participation model is:

\[
\max_{\{C_t, K_{t+1}, M_{t+1}, D_t\}} \sum_{t=0}^{\infty} \beta^t \left\{ \ln (C_t) + \theta \ln (1 - N_t) - \theta \ln (1 + H(Q_t, Q_{t-1})) \right\}
\]

subject to \(P_tC_t \leq M_t - D_t - P_t\Omega_t + P_tr_tK_t - P_tI_t + W_tN_t\) and

\[
M_{t+1} = M_t - D_t - P_tC_t - P_t\Omega_t + P_tr_tK_t + W_tN_t - P_tI_t + R_t [D_t + X_t] + \pi_t
\]

where \(H(Q_t, Q_{t-1}) = a_0 \left\{ \exp \left[ a_1 \left( \frac{Q_t}{Q_{t-1}} - 1 - x^* \right) \right] + \exp \left[ -a_1 \left( \frac{Q_t}{Q_{t-1}} - 1 - x^* \right) \right] - 2 \right\}
\]

and \(Q_t = M_t - D_t\)

where \(C_t\) is the individual’s consumption, \(N_t\) is the percent of time the individual spends working for firms, \(r_t\) is the real return on capital, \(K_t\) is the amount of capital available during time \(t\), \(P_t\) is the price level, \(M_t\) is the beginning of period stock of money, \(I_t\) is investment, \(W_t\) is the nominal wage, \(D_t\) is the amount of nominal deposits chosen in advance of the shocks, \(R_t\) is the nominal interest rate, \(\Omega_t\) is the amount of taxes owned at time period \(t\) and \(R_tX_t\) and \(\pi_t\) are the profits received from financial intermediaries and intermediate goods firms respectively.

In the limited participation model, firms do not require effort from workers for production. The resulting intermediate good firm’s period \(t\) problem in this case is:

\[
\max_{\{P_{it}, N_{it}, K_{it}\}} \left\{ P_{it} \left( A_tK_{it}^\alpha \left( N_{it} \right)^{1-\alpha} \right) - R_tP_tw_{it}N_{it} - P_tr_tK_{it} \right\}
\]

subject to the period by period demand functions:

\[
P_{it}^\frac{-\mu}{1-\mu} \left( A_tK_{it}^\alpha \left( N_{it} \right)^{1-\alpha} \right) - P_t^\frac{-\mu}{1-\mu} Y_t = 0
\]

Finally, using the fact that the intermediate goods firms borrow their wage bill from the
financial intermediaries, the loan market clearing condition becomes:

\[ \int_0^1 W_{it} N_{it} di = D_t + X_t \]

where \( W_{it} N_{it} \) is firm \( i \)'s wage bill in period \( t \), \( D_t \) are the deposits and \( X_t \) is the amount of the monetary injection.

**APPENDIX D: The Observational Equivalence**

To prove the reduce form equivalence between the full income insurance shirking model and the limited participation model with indivisible labour, first notice that in the full income insurance shirking model, \( c^{uu}_t = c^u_t = C_t \) so:

\[
C_t = \chi(s w_t f - (1 - N_t) w_t f + c^f_t) \\
= (s - 1) \chi w_t f + \chi C_t \\
\Rightarrow w_t f = \frac{C_t (\chi - 1)}{(1 - s) \chi}
\]

Combining this with the firms’ Euler equation for \( N_t \) in the shirking model yields the equation:

\[
\frac{Y_{it}(1 - \alpha)}{N_{it}(1 - \mu R_t)} = C_t \Psi
\]

where \( \Psi = \frac{(\chi - 1)}{(1 - s) \chi} \). The equilibrium allocations can then be described by the following conditions:

\[
\frac{Y_{it}(1 - \alpha)}{N_{it}(1 - \mu R_t)} = C_t \Psi \\
\widetilde{\Lambda}_t K_{it}^\alpha (N_{it})^{1-\alpha} = Y_{it} = Y_t \\
\frac{Y_{it} \alpha}{K_{it} \mu} = r_t \\
Y_t - C_t - I_t - G_t = 0
\]
where $\tilde{A}_t = A_t (f e)^{1-\alpha}$ and

$$E_{t-1} \left\{ \left( \frac{1}{C_t} \frac{1}{P_t} \right) - \theta \frac{H_1(Q_t, Q_{t-1})}{1+H(Q_t, Q_{t-1})} - \beta \theta \frac{H_2(Q_{t+1}, Q_t)}{1+H(Q_{t+1}, Q_t)} \right\} = 0$$

$$-\beta R_t \left[ \left( \frac{1}{C_{t+1}} \frac{1}{P_{t+1}} \right) - \theta \frac{H_1(Q_{t+1}, Q_t)}{1+H(Q_{t+1}, Q_t)} - \beta \theta \frac{H_2(Q_{t+2}, Q_{t+1})}{1+H(Q_{t+2}, Q_{t+1})} \right]$$

$$E_t \left\{ \frac{1}{C_t} \frac{\partial I_t}{\partial K_{t+1}} - \beta \frac{1}{C_{t+1}} \left( r_{t+1} - \frac{\partial I_{t+1}}{\partial K_{t+1}} \right) \right\} = 0$$

$$P_tC_t = M_t - D_t - P_t G_t + P_t r_t K_t - P_t I_t + P_t w_t N_t$$

$$M_{t+1} = R_t [D_t + X_t] + \pi_t$$

In the limited participation model with indivisible labor model (with hours normalized to 1), the individual’s period $t$ utility function is \{ln ($C_t$) + $N_t \Lambda - \theta \ln (1 + H(Q_t, Q_{t-1}))$\} where $\Lambda$ is a constant. In this case, the Euler equations for capital and deposits reduce to:

$$E_{t-1} \left\{ \left( \frac{1}{C_t} \frac{1}{P_t} \right) - \theta \frac{H_1(Q_t, Q_{t-1})}{1+H(Q_t, Q_{t-1})} - \beta \theta \frac{H_2(Q_{t+1}, Q_t)}{1+H(Q_{t+1}, Q_t)} \right\} = 0$$

$$-\beta R_t \left[ \left( \frac{1}{C_{t+1}} \frac{1}{P_{t+1}} \right) - \theta \frac{H_1(Q_{t+1}, Q_t)}{1+H(Q_{t+1}, Q_t)} - \beta \theta \frac{H_2(Q_{t+2}, Q_{t+1})}{1+H(Q_{t+2}, Q_{t+1})} \right]$$

$$E_t \left\{ \frac{1}{C_t} \frac{\partial I_t}{\partial K_{t+1}} - \beta \frac{1}{C_{t+1}} \left( r_{t+1} - \frac{\partial I_{t+1}}{\partial K_{t+1}} \right) \right\} = 0$$

the first order condition for employment can be written as:

$$\Lambda C_t = w_t$$

and the money evolution equation and the cash-in-advance constraint become:

$$P_tC_t = M_t - D_t - P_t G_t + P_t r_t K_t - P_t I_t + P_t w_t N_t$$

$$M_{t+1} = R_t [D_t + X_t] + \pi_t$$
In addition, the firms first order conditions are described by the following equations:

\[
\frac{Y_{it} (1 - \alpha)}{N_{it} \mu} = w_{it} R_t \\
\frac{Y_{it} \alpha}{K_{it} \mu} = r_t \\
Y_{it} = Y_t = A_t K_{it}^\alpha (N_{it})^{1-\alpha}
\]

Notice that combining the condition \( \Lambda C_t = w_t \) with the labour demand equation implies that \( \frac{Y_{it} (1 - \alpha)}{N_{it} \mu R_t} = C_t \Lambda \). It follows that if the same data set is used to estimate the reduced forms for the full income insurance shirking model and the indivisible labour limited participation model, they should have the same empirical predictions about the variables’ second moments and impulse response functions since the equations that describe the equilibrium are identical.

References


Figure 1. Models’ Responses to a Positive Monetary Policy Shock ($\nu = 1.0$)

Partial Income Insurance Shirking Model: - - -
Full Income Insurance Shirking Model : -x-x-
Standard Limited Participation Model : - - -

Notes:
(1) The impulse responses of ($c^f - F$), $c^{ux}$, $c^u$ & $c^e$ are identical to the real wage response in the Shirking Models
(2) Although there is no family purchased consumption in the standard limited participation model, I report the value of $rK-G+(M-D)/P-I$ for comparative purposes
Figure 2. Models’ Responses to a Positive Monetary Policy Shock ($\nu = 0.95$)

Notes:

(1) The impulse responses of ($c^F - F$), $c^{ut}$, $c^u$ & $c^s$ are identical to the real wage response in the Shirking Models

(2) Although there is no family purchased consumption in the standard limited participation model, I report the value of $rK-G+(M-D)/P-I$ for comparative purposes.
Figure 3. Models’ Responses to a Positive Fiscal Policy Shock ($\nu = 1.0$)

Partial Income Insurance Shirking Model: -o-o-
Full Income Insurance Shirking Model : -x-x-
Standard Limited Participation Model : - - -

Notes:
(1) The impulse responses of $(c^f - F), c^u, c^u & c^s$ are identical to the real wage response in the Shirking Models.
(2) Although there is no family purchased consumption in the standard limited participation model, I report the value of $rK-G+(M-D)/P-I$ for comparative purposes.
Figure 4. Models’ Responses to a Positive Fiscal Policy Shock ($\nu = 0.95$)

Partial Income Insurance Shirking Model: -ooo-
Full Income Insurance Shirking Model : -x-x-
Standard Limited Participation Model : - - -

Notes:
(1) The impulse responses of $(c^f - F), c^{ut}, c^u$ & $c^a$ are identical to the real wage response in the Shirking Models.
(2) Although there is no family purchased consumption in the standard limited participation model, I report the value of $rK-G+(M-D)/P-I$ for comparative purposes.
Figure 5. Models’ Responses to a Positive Technology Shock ($\nu = 1.0$)

Notes:

(1) The impulse responses of $(c_f - F), c^u, c^u \& c^u$ are identical to the real wage response in the Shirking Models.

(2) Although there is no family purchased consumption in the standard limited participation model, I report the value of $rK-G+(M-D)/P-I$ for comparative purposes.
Figure 6. Models’ Responses to a Positive Technology Shock ($\nu = 0.95$)

Partial Income Insurance Shirking Model: -o-o-
Full Income Insurance Shirking Model : -x-x-
Standard Limited Participation Model : - - -

Notes:

(1) The impulse responses of $(c_f - F)$, $c^{ua}$, $c^u$ & $c^s$ are identical to the real wage response in the Shirking Models

(2) Although there is no family purchased consumption in the standard limited participation model, I report the value of $rK-G+(M-D)/P-I$ for comparative purposes
### TABLE 1.
PARAMETER ESTIMATES FOR THE MODELS

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<td>Full Income Insurance Case</td>
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Notes: Standard errors in parentheses.
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