

# UNIVERSITY OF TORONTO

## ECO 2061H1 S Final 2010

Duration: 2 hours

**NO AIDS ALLOWED**

**NOTE TO STUDENT:** Enter your name and student number on each examination book that you use. There are five questions on this test for a **TOTAL OF 200 MARKS**. Read each question carefully and note the number of points allocated to each part.

Good Luck.

### Question 1

[Total 18 points] Using your knowledge of the Solow model discussed in class answer the following questions:

- (6 points) Find the elasticity of output per unit of effective labour on the balanced growth,  $y^*$ , with respect to the population growth rate  $n$ .
- (6 points) Assume that the economy is on a balanced growth path where  $\delta$  initially equals 1%,  $n = 2\%$ ,  $g=1\%$  and the elasticity of output per unit of effective labor with respect to  $n$  equals  $-1/2$ , by how much does a fall in  $n$  from 2% to 1% raise  $y^*$ , and how long will it take for  $k$  and  $y$  to move half way to their new balanced growth path? (Note: You only need to provide the expression for the amount of time. You do not need to evaluate it.)
- (6 points) Assume that in addition to the assumptions made above, markets are perfectly competitive, the growth rate of capital per worker is 4%, and the growth rate of output per worker is 8%. What is the value of the Solow residual?

### Question 2

[Total 82 points] Assume that households have the following lifetime utility function

$$U = \int_{t=0}^{\infty} e^{-\rho t} \left( \frac{C(t)^{1-\theta}}{1-\theta} \right) \frac{L(t)}{H} dt$$

where  $0 < \theta < 1$ ,  $\rho$  is the discount rate,  $C(t)$  is consumption per worker,  $L(t)$  is the number of workers and  $H$  is the number of households.

- (4 points) Express the household's lifetime utility in terms of consumption per unit of effective labour,  $c(t)$ , using the fact that  $L(t)=L(0)e^{nt}$ ,  $A(t)=A(0)e^{gt}$ .
- (4 points) Are there any restrictions we should impose on the parameter values  $\rho, n, \theta$ , or  $g$ ? Why or why not?
- (12 points) Using the Hamiltonian method set up the Social Planner's problem for this economy. Which variable is the control variable and which is the state variable? Use the Pontryagin's

Maximum principle theorem to find the equations that characterize the equilibrium.

- d. (7 points) Assume that the household has the following flow budget constraint:

$$\frac{\dot{K}(t)}{H} + C(t) \frac{L(t)}{H} \leq \frac{r(t)K(t)}{H} + \frac{w(t)A(t)L(t)}{H}$$

where  $K(t)$  is the aggregate amount of capital in the economy at time  $t$ . Express this equation in terms of the per unit of effective labor variables.

- e. (12 points) Use the information in part d to set up the current value Hamiltonian for the Household' problem. Which variable is the control variable and which is the state variable? Use the Pontryagin's Maximum principle theorem to find the equations that characterize the solution to the household problem.
- f. (10 points) Let  $w(t)$  be the wage per unit of effective labour,  $r(t)$  be the rate of return on capital at time  $t$ , and  $Y(t) = F(K(t), A(t)L(t))$ . Write down the firm's maximization problem for time  $t$ . Prove that when  $Y(t)$  exhibits constant returns to scale the first order necessary conditions from the firm's problem imply that:

$$w(t) = f(k(t)) - k(t)f'(k(t))$$

$$r(t) = f'(k(t))$$

- g. (4 points) Write down the equations for  $\dot{c}(t) = 0$  and  $\dot{k}(t) = 0$ .
- h. (8 points) Find the steady state values for  $k, c$  and  $y$ , (i.e.,  $k^*, c^*$  and  $y^*$ ) and the golden rule level of  $k, k^G$ , in terms of  $\rho, n, g$ , and  $\theta$ .
- i. (12 points) Find and determine the signs of:

$$\frac{\partial k^*}{\partial \theta}, \frac{\partial c^*}{\partial \theta}, \frac{\partial y^*}{\partial \theta}, \text{ and } \frac{\partial k^G}{\partial \theta}$$

- j. (9 points) Show the effects of a permanent increase in  $\theta$  on  $k^*, c^*$  and  $k^G$  using the phase diagram.

### Question 3

[Total 60 points] Consider the following model of a monetary economy. A representative household maximizes the criterion function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

where  $0 < \beta < 1$ ,  $C_t$  is time  $t$  consumption and  $L_t$  is time  $t$  hours worked. The household begins the period with  $M_t$  units of money. The household faces the cash-in-advance constraint

$$P_t C_t \leq M_t - N_t + W_t L_t$$

where  $P_t$  is the time  $t$  dollar price of consumption,  $U(C_t, L_t) = \ln \left[ C_t - \frac{\psi_0}{1+\psi} L_t^{1+\psi} \right]$ ,  $N_t$  is

dollars deposited with a financial intermediary at time  $t$ , and  $W_t$  is the time  $t$  nominal wage rate. The gross interest rate on deposits is denoted by  $R_t$ . The representative household owns the stock of capital,  $K_t$ . This capital can't be augmented nor does it depreciate. So the aggregate stock of capital is a just a constant  $K$  for all  $t$ . The household rents out the capital in a perfectly competitive rental market at the nominal rate  $r_t$ . The household's stock of money evolves according to

$$M_{t+1} = M_t - N_t + W_t L_t - P_t C_t + R_t N_t + r_t K_t + \pi_t^f + \pi_t^{fi}$$

Here  $\pi_t^f$  and  $\pi_t^{fi}$  denote time  $t$  lump sum profits received from firms and financial intermediaries.

A representative firm produces time  $t$  output using the technology

$$Y_t = \theta_t K_t^{1-\alpha} H_t^\alpha$$

where  $\theta_t$  is the time  $t$  level of technology,  $H_t$  denotes time  $t$  hours worked and  $K_t$  is the firm's stock of capital.  $\theta_t$  evolves according to

$$\theta_t = \theta + \varepsilon_t$$

where  $\theta$  is a positive constant and  $\varepsilon_t$  is a mean zero i.i.d. shock. The firm must borrow the wage bill  $W_t H_t$  from a financial intermediary. The interest rate on these loans is given by  $R_t$ . All loans are repaid at the end of the period.

The firm maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{C,t+1}}{P_{t+1}} \pi_t^f$$

where

$$\pi_t^f = P_t \theta_t K_t^{1-\alpha} H_t^\alpha - W_t H_t R_t - r_t K_t$$

and  $U_{C,t+1}$  denotes the time  $t + 1$  marginal utility of the representative consumer.

A representative financial intermediary receives  $N_t$  from households at time  $t$  as well as a monetary injection  $X_t$  from the government. All funds received are lent out to firms at the rate  $R_t$ .

The aggregate stock of money evolves according to

$$M_{t+1}/M_t = (M_t + X_t)/M_t = 1 + x_t.$$

where  $x_t$  is an i.i.d. shock to the net growth rate of money. All markets and all agents in this economy (except for the government) are perfectly competitive. Suppose that  $N_t$  must be chosen by households before the current period realization of  $\varepsilon_t$  and  $x_t$ . All other time  $t$  variables under control of the household are chosen after the realization of  $\varepsilon_t$  and  $x_t$ .

- a. (16 points) Formally state the problem of the representative household. Display the first order necessary conditions for the problem.
- b. (16 points) Formally state the problem of the representative firm. Display the first order necessary conditions for the problem.
- c. (14 points) Formally define the equilibrium for this economy.
- d. (14 points) Prove that a positive shock to  $x_t$  reduces  $R_t$  and increases equilibrium time  $t$  employment.

### Question 4

[Total 40 points] Assume that the relationship between a worker's effort,  $e$ , and his wage,  $w$ , is given by the following function:

$$e = \begin{cases} \left(\frac{w-\chi}{\chi}\right)^\beta & \text{if } w > \chi \\ 0 & \text{otherwise} \end{cases}$$

where  $\chi = (1 - bu)w_a$ ,  $0 < \beta < 1$ ,  $b > 0$  and  $w_a$  is the wage paid to workers at other firms in the economy

Assume that firms maximize profits and that their production function is given by  $F(eL)$ .

- a. (10 points) Set up the representative firm's maximization problem and find the first order necessary conditions.
- b. (10 points) Show that the first order necessary conditions found in part a. imply that  $\frac{\partial e}{\partial w}w = e$ . What is the interpretation of this condition?
- c. (10 points) Find the equilibrium wage rate assuming all firms are identical.
- d. (10 points) Find the equilibrium rate of unemployment for this economy.