UNIVERSITY OF TORONTO

ECO 2061H1 S MA Midterm 2011
Duration: 1 hour 50 minutes
NO AIDS ALLOWED

NOTE TO STUDENT: Enter your name and student number on each examination book that you use. There are five questions on this test for a **TOTAL OF 200 MARKS**. Read each question carefully and note the number of points allocated to each part.

Good Luck.

Question 1

[Total 25 points] Using your knowledge of the Solow model discussed in class answer the following questions:

- **a.** (6 points) Find the elasticity of output per unit of effective labour on the balanced growth, y^*, with respect to the population growth rate n.
- **b.** (10 points) Assume that the economy is on a balanced growth path where δ initially equals 1%, $n = 2\%$, g=1% and the elasticity of output per unit of effective labor with respect to capital per unit of effective labor equals 1/2,, evaluate the elasticity computed in part 1. Now assume that there is a temporary shock that causes the amount of capital per unit of effective labour to fall below its steady state level. How long will it take for k and y to move half way to their new balanced growth path? (Note: You only need to provide the expression for the amount of time. You do not need to evaluate it.)
- **c.** (9 points) Assume that in addition to the assumptions made above, markets are perfectly competitive, the growth rate of capital per worker is 4%, and the growth rate of output per worker is 8%. What is the value of the Solow residual?

Question 2

[Total 55 points] Assume that households have the following lifetime utility function

$$U = \int_{t=0}^{\infty} e^{-\rho t}\left(\frac{C(t)^{1-\theta}}{1-\theta}\right)\frac{L(t)}{H}dt$$

where $0 < \theta < 1, \rho$ is the discount rate, C(t) is consumption per worker, L(t) is the number of workers and H is the number of households. Further assume that total output is produced according to the following production function, F(K(t),A(t)L(t)) which exhibits constant returns to scale in capital, K(t), and effective labour, A(t)L(t) where L(t)=L(0)e^{nt} and A(t)=A(0)e^{gt}. Finally, assume that $\rho - n - (1-\theta)g > 0$.

- **a.** (3 points) Express the household's lifetime utility in terms of consumption per unit of effective labour, c(t).
- **b.** (15 points) Using the Hamiltonian method set up the Social Planner's problem for this economy. Which variable is the control variable and which is the state variable? Use the Pontryagrin's

Maximum principle theorem to find the equations that characterize the equilibrium.

c. (8 points) What are the equations for $\dot{c}(t) = 0$ and $\dot{k}(t) = 0$. Justify your answer.

d. (8 points) Find the steady state values for k, c and y, (i.e., k^*, c^* and y^*) and the golden rule level of k, k^G, in terms of ρ, n, g, and θ.

e. (12 points) Find and determine the signs of:

$$\frac{\partial k^*}{\partial \rho}, \frac{\partial c^*}{\partial \rho}, \frac{\partial y^*}{\partial \rho}, \text{ and } \frac{\partial k^G}{\partial \rho}$$

f. (9 points) Show the effects of a permanent increase in ρ on k^*, c^* and k^G using the phase diagram.

Question 3

(Total 55 points) Assume that $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$, where $0 < \alpha < 1$, $Y(t)$ is total output at time t, $K(t)$ is total capital at time t, $A(t)$ is the amount of knowledge at time t, and $L(t)$ is the amount of labour at time t. Use the Solow model to answer the following questions.

a. (9 points)Using the Solow model find the steady state values for k, y and c (i.e., k^*, y^* and c^*) as functions of the savings rate, s, the growth rate of labour, n, the rate of depreciation, δ, the growth rate of knowledge, g, and α.

b. (3 points) What is the golden rule level of capital per unit of effective labour, k^G as a function of s, n, δ, g and α?

c. (3 points) Under what conditions will the economy converge to the golden rule level of capital per unit of effective labour? Justify your answer.

d. Assume that there is a permanent decrease in δ.

  i. (12 points) Show what happens to the steady state levels of k, c, and y, and the level of k^G using a diagram.

  ii. (12 points) Find and determine the signs of:

$$\frac{\partial k^*}{\partial \delta}, \frac{\partial c^*}{\partial \delta}, \frac{\partial y^*}{\partial \delta}, \text{ and } \frac{\partial k^G}{\partial \delta}$$

e. (16 points) Assume the economy is initially on its balanced growth path and that there is a one time decrease in the total amount of capital in the economy. What effect will this change have on the values of output per unit of effective labour, consumption per unit of effective labour, k^G and capital per unit of effective labor over time?

Question 4

[Total 15 points] Assume that the economy is initially on its balanced growth path where government expenditures are zero. Next, there is a temporary increase in the amount of government expenditures, such that the amount of government expenditures per unit of effective labour per unit of time increases to G, which is a positive constant, and that the date that government expenditures will return to zero is known with certainty. Draw the phase diagram showing the effect of the increase in government purchases on k and $c,$ by drawing the path the economy takes as a result of this shock. Discuss what happens to k, c and the real return on capital, $r,$ both immediately after the shock, and as the economy moves to its balanced growth path.

Question 5

[Total 50 points] Assume that the household's expected lifetime utility is given by:

$$E_0 \sum_{t=0}^{T} \left(\frac{1}{1+\rho} \right)^t U(C_t)$$

where ρ is a non-negative discount rate, and $U(C_t)$ is the utility individuals get from consuming C_t . Also assume that $U' > 0,\ U'' < 0,$ and the household has the following period by period budget constraint:

$$K_{t+1} - K_t + C_t \le r_t K_t + Y_t$$

where K_t is the level of capital in period $t,\ Y_t$ is the amount of income (given exogenously) that the household receives in period t and r_t is the real rate of return on capital.

a. (15 points) Show that the first order necessary conditions for this problem imply

$$U'(C_t) = \left(\frac{1}{1+\rho} \right) \{E_t(1 + r_{t+1}) E_t(U'(C_{t+1})) + Cov_t(1 + r_{t+1}, U'(C_{t+1}))\}$$

b. (15 points) What effect does it have on the current level of consumption if the covariance of (C_{t+1}) and r_{t+1} is positive instead of negative? Justify your answer.

c. (20 points) Under what conditions will consumption follow a random walk? Justify your answer.