UNIVERSITY OF TORONTO

ECO 2061H1 S Final 2008
Duration: 2 hours
NO AIDS ALLOWED

NOTE TO STUDENT: Enter your name and student number on each examination book that you use. There are five questions on this test for a **TOTAL OF 100 MARKS**. Read each question carefully and note the number of points allocated to each part.

Good Luck.

Question 1

[Total 17 points] Using your knowledge of the Solow model discussed in class answer the following questions:

1. **a.** (5 points) Find the elasticity of output per unit of effective labour on the balanced growth, y^*, with respect to the population growth rate n.

b. (8 points) Assume that the economy is on a balanced growth path where δ initially equals 1%, $n = 2\%$, g=1% and the elasticity of output per unit of effective labor with respect to n equals -1/2,, by how much does a fall in n from 2% to 1% raise y^*, and how long will it take for k and y to move half way to their new balanced growth path? (Note: You only need to provide the expression for the amount of time. You do not need to evaluate it.)

c. (4 points) Assume that in addition to the assumptions made above, markets are perfectly competitive, the growth rate of capital per worker is 4%, and the growth rate of output per worker is 8%. What is the value of the Solow residual?

Question 2

[Total 32 points] Consider the following model of a monetary economy. A representative household maximizes the criterion function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

where $0 < \beta < 1$, C_t is time t consumption and L_t is time t hours worked. The household begins the period with M_t units of money. The household faces the cash-in-advance constraint

$$P_t C_t \leq M_t - N_t + W_t L_t$$

where P_t is the time t dollar price of consumption, $U(C_t, L_t) = \ln\left[C_t - \frac{\psi_0}{1+\psi} L^{1+\psi}\right]$, N_t is dollars deposited with a financial intermediary at time t, and W_t is the time t nominal wage rate. The gross interest rate on deposits is denoted by R_t. The representative household owns the stock of capital, K_t. This capital can't be augmented nor does it depreciate. So the aggregate stock of capital is a just a constant K for all t. The household rents out the capital in a perfectly competitive rental market at the nominal rate r_t. The household's stock of money evolves according to

$$M_{t+1} = M_t - N_t + W_t L_t - P_t C_t + R_t N_t + r_t K_t + \pi_t^f + \pi_t^{fi}$$

Here π_t^f and π_t^{fi} denote time t lump sum profits received from firms and financial intermediaries.

A representative firm produces time t output using the technology

$$Y_t = \theta_t K_t^{1-\alpha} H_t^{\alpha}$$

where θ_t is the time t level of technology, H_t denotes time t hours worked and K_t is the firm's stock of capital. θ_t evolves according to

$$\theta_t = \theta + \varepsilon_t$$

where θ is a positive constant and ε_t is a mean zero i.i.d. shock. The firm must borrow the wage bill $W_t H_t$ from a financial intermediary. The interest rate on these loans is given by R_t. All loans are repaid at the end of the period.

The firm maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{C,t+1}}{P_{t+1}} \pi_t^f$$

where

$$\pi_t^f = P_t \theta_t K_t^{1-\alpha} H_t^{\alpha} - W_t H_t R_t - r_t K_t$$

and $U_{C,t+1}$ denotes the time $t+1$ marginal utility of the representative consumer.

A representative financial intermediary receives N_t from households at time t as well as a monetary injection X_t from the government. All funds received are lent out to firms at the rate R_t.

The aggregate stock of money evolves according to

$$M_{t+1}/M_t = (M_t + X_t)/M_t = 1 + x_t.$$

where x_t is an i.i.d. shock to the net growth rate of money. All markets and all agents in this economy (except for the government) are perfectly competitive. Suppose that N_t must be chosen by households before the current period realization of ε_t and x_t. All other time t variables under control of the household are chosen after the realization of ε_t and x_t.

1. **a.** (6 points) Formally state the problem of the representative household. Display the first order necessary conditions for the problem.

b. (6 points) Formally state the problem of the representative firm. Display the first order necessary conditions for the problem.

c. (4 points) Formally define the equilibrium for this economy.

d. (8 points) Prove that a positive shock to x_t reduces R_t and increases equilibrium time t employment.

e. (8 points) Show what happens to the equilibrium level of employment and R_t after a positive shock to ε_t. (Assume that the money supply doesn't change in response to the shock in ε_t).

Question 3

[Total 25 points] Consider the following economy where a representative household has the following lifetime utility

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

where $\beta < 1$ is the household's discount rate,

$$u(c_t, n_t) = \frac{\left((c_t)^{\mu}(1-n_t)^{1-\mu}\right)^{1-\sigma}}{1-\sigma}$$

and the values of μ and σ are such that the utility function is differentiable, increasing and concave in both arguments ($\mu \in (0,1), \sigma \in (0,1)$). Here c_t is the amount of consumption in period t and n_t is the amount of labor the household chooses to provide to the market during the period, $n_t \in [0,1]$. Let k_t and g_t denote the per capita amount of capital and the per capita amount of exogenous government expenditures in period t. The rate of depreciation is $\delta \in (0,1)$. Furthermore goods are produced according to the following production function

$$Y_t = K_t^{\alpha}(Ln_t)^{1-a}$$

where K_t is the economy wide capital stock, L is the number of households in the economy and $\alpha \in (0,1)$. Goods can used for consumption, investment or government expenditures.

1. **a.** (12 points) Write down the economy wide resource constraint in terms of the per capita variables and the social planners problem for this economy. Find the first order necessary conditions for this problem.

b. (13 points) Solve for the non-stochastic steady state. Let the variables without the time subscript denote steady state levels. How does $\frac{Y}{Ln}$ vary with g? Find the elasticity of the steady state level of n_t with respect to the steady state level of per person government expenditures g?

Question 4

[Total 16 points] Assume that households have the following lifetime utility function

1.

$$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta}}{1-\theta} \frac{L(t)}{H} dt$$

where $\theta > 0, \theta \neq 1, \rho$ is the discount rate, C(t) is consumption per worker, L(t) is the number of workers and H is the number of households.

a. (4 points) Express the household's lifetime utility in terms of consumption per unit of effective labour, c(t), using the fact that L(t)=L(0)e^{nt}, A(t)=A(0)e^{gt}, and $\rho - n - (1-\theta)g > 0$

b. (6 points) Given the fact that the household has the following lifetime budget constraint,

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t} dt \leq k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t} dt$$

Prove that the household's Euler equation has the form:

$$\frac{\dot{c}\ (t)}{c(t)} = \frac{r(t) - \rho - \theta g}{\theta}$$

(6 points) Let $w(t)$ be the wage per unit of effective labour, $r(t)$ be the rate of return on capital at time t, and $Y(t) = (K(t))^{\alpha} (A(t)L(t))^{1-\alpha}$. Write down the firm's maximization problem for time and prove that the first order necessary conditions from the firm's problem imply that:

$$w(t) = (1 - \alpha)k(t)^{\alpha}$$

$$r(t) = \alpha k(t)^{\alpha - 1}$$

Question 5

[Total 10 points] Using the Tobin's q model of investment discussed in class discuss what happens to q and K after a permanant increase in the interest rate, r. Justify any movements to the $\dot{q} = 0$ and $\dot{K} = 0$ locuses and show the movements (if any) on a phase diagram.