UNIVERSITY OF TORONTO

ECO 2061H1 S MFE Midterm 2011
Duration: 1 hour 50 minutes
NO AIDS ALLOWED

NOTE TO STUDENT: Enter your name and student number on each examination book that you use. There are four questions on this test for a **TOTAL OF 200 MARKS**. Read each question carefully and note the number of points allocated to each part.

Good Luck.

Question 1

[Total 30 points] Assume that final output is produced by the following production function:

$$Y(t) = \alpha_3[K(t)^{\alpha_1}(A(t)L(t))^{\alpha_2}] + \alpha_4$$

- **a.** (8 points) What restrictions on α_1, α_2, α_3 and α_4 are necessary to make this function exhibit constant returns to scale in $K(t)$ and $A(t)L(t)$?
- **b.** (6 points) Assume that final output is now produced by the following production function:

$$Y(t) = BK(t) + K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$$

  where $0 < \alpha < 1$ and $B \geq 0$. What is the intensive form of this production function?
- **c.** Under what conditions are the intensive form of the production function in b consistent with:
  - **i.** (4 points) A positive marginal product of capital per unit of effective labour?
  - **ii.** (4 points) Diminishing marginal returns on capital per unit of effective labour?
  - **iii.** (8 points) The Inada conditions?

Question 2

[Total 65 points] Assume that households have the following lifetime utility function

$$U = \int_{t=0}^{\infty} e^{-\rho t}\left(\frac{C(t)^{1-\theta}}{1-\theta}\right)\frac{L(t)}{H}dt$$

where $0 < \theta < 1, \rho$ is the discount rate, C(t) is consumption per worker, L(t) is the number of workers and H is the number of households. Further assume that total output is produced according to the following production function, F(K(t),A(t)L(t)) which exhibits constant returns to scale in capital, K(t), and effective labour, A(t)L(t).

- **a.** (4 points) Express the household's lifetime utility in terms of consumption per unit of effective labour, c(t), using the fact that L(t)=L(0)e^{nt}, A(t)=A(0)e^{gt}.

b. (4 points) Are there any restrictions we should impose on the parameter values $\rho, n, \theta,$ or g? Why or why not?

c. (12 points) What is the lifetime budget constraint for households assuming that w(t) is the wage paid per unit of effective labour and r(t) is the return on capital paid by firms. Find the first order necessary conditions for the household's problem. Derive the equation for $\frac{\dot{c}(t)}{c(t)}$.

d. (4 points) What is the equation for $\dot{k}(t) = 0$. What is the interpretation of this equation.

e. (8 points) Find the steady state values for k, c and y, (i.e., k^*, c^* and y^*) and the golden rule level of k, k^G, in terms of ρ, $n, g,$ and θ.

f. (12 points) Find and determine the signs of:

$$\frac{\partial k^*}{\partial n}, \frac{\partial c^*}{\partial n}, \frac{\partial y^*}{\partial n}, \text{ and } \frac{\partial k^G}{\partial n}$$

g. (11 points) Show the effects of a permanent increase in n on k^*, c^* and k^G using the phase diagram

h. (10 points) Let $w(t)$ be the wage per unit of effective labour, $r(t)$ be the rate of return on capital at time t, and $Y(t) = F(K(t), A(t)L(t))$ Write down the firm's maximization problem for time t. Prove that when Y(t) exhibits constant returns to scale the first order necessary conditions from the firm's problem imply that:

$$w(t) = f(k(t)) - k(t)f'(k(t))$$

$$r(t) = f'(k(t))$$

Question 3

1. [Total 55 points] Assume that $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$, where $0 < \alpha < 1$, $Y(t)$ is total output at time t, $K(t)$ is total capital at time t, $A(t)$ is the amount of knowledge at time t, and $L(t)$ is the amount of labour at time t. Use the Solow model to answer the following questions.

a. (9 points)Using the Solow model find the steady state values for k, y and c (i.e., k^*, y^* and c^*) as functions of the savings rate, s, the growth rate of labour, n, the rate of depreciation, δ, the growth rate of knowledge, g, and α.

b. (3 points) What is the golden rule level of capital per unit of effective labour, k^G as a function of s, n, δ, g and α?

c. (3 points) Under what conditions will the economy converge to the golden rule level of capital per unit of effective labour? Justify your answer.

d. Assume that there is a permanent decrease in n.

i. (12 points) Show what happens to the steady state levels of $k, c,$ and y, and the level of k^G using a diagram.

ii. (12 points) Find and determine the signs of:

$$\frac{\partial k^*}{\partial \delta}, \frac{\partial c^*}{\partial \delta}, \frac{\partial y^*}{\partial \delta}, \text{ and } \frac{\partial k^G}{\partial \delta}$$

e. (16 points) Assume the economy is initially on its balanced growth path and that there is a one time decrease in the total amount of workers in the economy. What effect will this change have on the values of output per unit of effective labour, consumption per unit of effective labour, k^G and capital per unit of effective labor over time?

Question 4

[Total 50 points] Assume that the household's expected lifetime utility is given by:

$$E_0 \sum_{t=0}^{T} \beta^t \left[C_t - \frac{a(C_t)^2}{2}) \right]$$

where $0<\beta < 1$ and $a > 0$. The household also has the following period by period budget constraint:

$$K_{t+1} - K_t + C_t \leq r_t K_t + Y_t$$

where K_t is the level of capital in period t, Y_t is the amount of income (given exogenously) that the household receives in period t and r_t is the real rate of return on capital.

a. (18 points) Show that the first order necessary conditions for this problem imply

$$u'(C_t) = \beta\{E_t(1 + r_{t+1})E_t(u'(C_{t+1})) - aCov_t(1 + r_{t+1}, C_{t+1})\}$$

b. (15 points) What effect does it have on the current level of consumption if the covariance of (C_{t+1}) and r_{t+1} is positive instead of negative? Justify your answer.

c. (17 points) Under what conditions will consumption follow a random walk? Justify your answer.