UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL/MAY 2008 EXAMINATIONS 2008
ECO 325H1

Duration: 2 hours

NO AIDS ALLOWED

NOTE TO STUDENT: Enter your name and student number on each examination book that you use. There are three questions on this exam for a **TOTAL OF 200 POINTS**. Read each question carefully and note the number of points allocated to each part. Good Luck.

1. (Total 55 points) Consider a Ramsey-Cass-Koopmans economy that is on its balanced growth path. Suppose at some time, which we will call time t^*, the government switches to a policy of taxing investment income at rate τ_1 instead of taxing investment income at rate τ_0, where $\tau_1 < \tau_0$. Assume that this change in tax policy is unanticipated. Further assume that the government rebates the revenue that it collects to households.

   a. (4 points) How does the real interest rate that households face change when the taxes are altered? Justify your answer.

   b. (6 points) How does the change in taxes affect the $\dot{c}=0$ locus and the $\dot{k}=0$ locus?

   c. (8 points) What are the steady state values for k, c and $y,$ and the savings rate prior to time t^*?

   d. (8 points) What are the steady state values for k, c and y and the savings rate after to time t^*?

   e. (4 points) How are k^*, c^*, y^* and the savings rate effected by the tax change? Justify your answer.

   f. (9 points) What are the dynamics of the variables k, c and y after the tax change? Justify your answer.

   g. (7 points) How would your answer to part 1.e) change if the tax decrease was only temporary? Justify your answer.

   h. (9 points) Does the growth rates of capital per person, consumption per person and output per person depend on the tax rate in this economy? Justify your answer.

2. (Total 65 points) Assume that $Y(t) = ZK(t)^\alpha (A(t)L(t))^{1-\alpha}$, where $0 < \alpha < 1$, Z is a positive constant, $Y(t)$ is total output at time t, $K(t)$ is total capital at time t, $A(t)$ is the amount of knowledge at time t, and $L(t)$ is the amount of labour at time t. Use the Solow model to answer the following questions.

a. (9 points)Using the Solow model find the steady state values for k, y and c (i.e., k^*, y^* and c^*) as functions of the savings rate, s, the growth rate of labour, n, the rate of depreciation, δ, the growth rate of knowledge, g, Z, and α.

b. (3 points) What is the golden rule level of capital, k^G as a function of s, n, δ, g, Z, and α?

c. Assume that there is a permanent decrease in α. $\left(HINT : \frac{\partial z^x}{\partial x} = \ln(z)z^x \right)$

   i. (12 points) Using diagrams, show what happens to the steady state levels of k, c, and y, and the level of k^G. (Hint: There are multiple cases to consider here.)

   ii. (12 points) Find and determine the signs of:
   $$\frac{\partial k^*}{\partial \alpha}, \frac{\partial c^*}{\partial \alpha}, \frac{\partial y^*}{\partial \alpha}, \text{ and } \frac{\partial k^G}{\partial \alpha}$$

d. (10 points) If the elasticity of total output with respect to labour decreases from 0.66 to 0.5, and the population growth rate is 2%, depreciation is 2.5% and the growth rate of knowledge is 1%, how much time will it take for k and y to move half way to their new balanced growth path? (Note: You only need to provide the final equation. You do not need to evaluate it.)

e. Now assume the economy is initially on its balanced growth path and that there is a one time increase in the amount of both capital and labour in the economy in the economy. Answer the following questions.

   i. (6 points) At the time of the change, what happens to the level of output per unit of effective labour? Justify your answer.

   ii. (8 points) After the initial change (if any) in output per unit of effective labour, is there any further change in output per unit of effective labour? If so, does it rise or fall? Why?

   iii. (5 points) Once the economy has again reached its balanced growth path, is output per unit of effective labour higher, lower or the same as it was before the new workers appeared? Justify your answer.

3. (Total 80 points)Assume that there is no growth in the economy, there is no depreciation, and the household's expected lifetime utility is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t + G_t)^{1-\theta}}{1 - \theta} + b\ln(1 - l_t) \right] \frac{N}{H}$$

The household also has the following period by period budget constraint:

$$K_{t+1} - K_t + \frac{N}{H}c_t \leq (1 - \tau_t^K)r_t K_t + (1 - \tau_t^l)w_t l_t \frac{N}{H}$$

where $0 < \theta < 1$, K_t is the level of capital per household, c_t is consumption per person in period t, G_t is the amount of Government expenditures per person in period t, l_t is the individual's time devoted to labour, w_t is the wage rate in period t, r_t is the real rate of return on capital, τ_t^K is the tax on capital income and τ_t^l is the tax on labour income in the economy in period t, H is the number of households and N is the number of people in the economy. Further assume that output is produced according to the Cobb-Douglas Production function.

a. (10 points) Set up the Lagrangian for the household's problem. Find the first order necessary conditions.

b. (10 points) What effect does it have on the current level of consumption if the covariance of $(c_{t+1} + G_{t+1})$ and $r_{t+1}(1 - \tau_t^K)$ is positive instead of negative? Justify your answer.

c. (10 points) How is the household's choice of leisure in period t, affected by the wage in period t, the tax rate in period t, government expenditures in time period t and consumption in period t?

d. (5 points) From the individual's point of view, is their private consumption in time period t and government expenditures in time period t complements, substitutes or neither? Justify your answer.

e. (6 points) Write down the firms problem and find the first order necessary conditions.

f. (7 points) What equations describe the equilibrium for this economy?

g. (10 points) Would Ricardian Equivalence hold in this economy? Justify your answer.

h. (12 points) Write down the Social Planner's problem for this economy and find the corresponding first order necessary conditions. Will these conditions give the same allocations as the decentralized economy you examined in parts (a)-(f)? Justify your answer.

3.i. (10 points)Assume that the taxes described above are used to finance government expenditures and that the government runs a balanced budget each period. Show that in a competitive economy, if each household satisfies their period by period budget constraint, the economy's resource constraint is satisfied.