UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER EXAMINATIONS 2009

ECO 325H1 F

Duration: 2 hours

NO AIDS ALLOWED

NOTE TO STUDENT: Enter your name and student number on each examination book that you use. There are **four questions** on this test for a **TOTAL OF 200 MARKS**. Count the exam pages to ensure you have all 4 pages. Read each question carefully and note the number of points allocated to each part. Good Luck.

1. (Total 40 points) Assume that $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$, where $0 < \alpha < 1$, $Y(t)$ is total output at time t, $K(t)$ is total capital at time t, $A(t)$ is the amount of knowledge at time t, and $L(t)$ is the amount of labour at time t. Use the Solow model to answer the following questions.
   - **a.** (8 points)Using the Solow model find the steady state values for k, y and c (i.e., k^*, y^* and c^*) as functions of the savings rate, s, the growth rate of labour, n, the rate of depreciation, δ, the growth rate of knowledge, g, and α. What is the golden rule level of capital, k^G as a function of s, n, δ, g and α?
   - **b.** (8 points) Prove that the elasticity of total output with respect to total capital is equal to the elasticity of output per unit of effective labour with respect to capital per unit of effective labour.
   - **c.** Assume that there is a permanent increase in s.
     - **i.** (8 points) Show what happens to the steady state levels of k, c, and y, and the level of k^G using a diagram.
     - **ii.** (8 points) Find and determine the signs of:

$$\frac{\partial k^*}{\partial s}, \frac{\partial c^*}{\partial s}, \frac{\partial y^*}{\partial s}, \text{ and } \frac{\partial k^G}{\partial s}$$

   - **d.** (8 points) If $s = 0.6$, δ decreases from 1% to 2%, a decreases from $\frac{1}{2}$ to $\frac{1}{3}$, $n = 1\%$ and $g = 1\%$, how much time will it take for k and y to move half way to their new balanced growth path? (Note: You only need to provide the final equation for each variable. You do not need to evaluate it.)

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2. (55 points) Using the Ramsey-Cass-Koopmans model discussed in class, where knowledge, $A(t)$ grows at rate $g,$ labour, $L(t)$ grows at rate $n,$ $Y(t) = F(K(t),A(t)L(t)),$ and $F(K(t),A(t)L(t))$ is constant returns to scale in $K(t),$ and $A(t)L(t),$ answer the following questions assuming that the rate of depreciation in the economy is positive.

a. (6 points) Write down the equations for $\dot{c}(t) = 0$ and $\dot{k}(t) = 0.$

b. (8 points) Find the steady state values for k, c and $y,$ (i.e., k^*, c^* and y^*) and the golden rule level of $k,$ $k^G,$ in terms of $\rho,$ $n, \delta, g,$ and $\theta.$

c. (12 points) Find and determine the signs of:

$$\frac{\partial k^*}{\partial \theta}, \frac{\partial c^*}{\partial \theta}, \frac{\partial y^*}{\partial \theta}, \text{ and } \frac{\partial k^G}{\partial \theta}$$

d. (10 points) Show the effects of a permanent increase in θ on k^*, c^* and k^G using the phase diagram.

e. (5 points) Assume that the economy is initially on its balanced growth path where government expenditures are zero. Next, assume that there is a permanent increase in the amount of government expenditures where the amount of government expenditures per unit of effective labour per unit of time is $G.$ Here, G is a positive constant.

Draw the phase diagram showing the affect of the increase in government purchases on steady state k and $c.$

f. (9 points) Assume that the economy is initially on its balanced growth path where government expenditures are zero. There is a temporary increase in the amount of government expenditures, such that the amount of government expenditures per unit of effective labour per unit of time is $G,$ which is a positive constant, and that the date that government expenditures will return to zero is known with certainty.

Draw the phase diagram showing the effect of the increase in government purchases on k and $c,$ by drawing the path the economy takes as a result of this shock.

Discuss what happens to k, c and the real return on capital, $r,$ both immediately after the shock, and as the economy moves to its balanced growth path.

g. (5 points) What is the growth rate of total output per person along the balanced growth path? Justify your answer.

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3. (Total 35 points) Assume that the representative household has no initial wealth and that it will be paid w_1 per hour in period 1 and w_2 per hour in period 2 with certainty for each hour worked. Further, assume that households know with certainty that they can borrow or lend at a real interest rate of r between periods 1 and 2, there is no growth in knowledge or population, and the Government funds its expenditures each period by taxing wage income each period at a constant rate τ. Answer the following questions using these assumptions, and the fact that the household has the following lifetime utility function:

$$\ln(c_1) + b\frac{(1-l_1)^{1-\theta}}{1-\theta} + \beta\left[\ln(c_2) + b\frac{(1-l_2)^{1-\theta}}{1-\theta}\right]$$

where $b > 0$, $0 < \theta < 1$, β is the household's discount factor, c_t denotes consumption in period t, and l_t denotes hours devoted to labour in period t

- **a.** (10 points) What is the household's budget constraint for period 1? For period 2? What is the implied lifetime budget constraint for the household?
- **b.** (5 points)What is the Government's lifetime budget constraint?
- **c.** (10 points) Write down the Household's problem and find the corresponding first order necessary conditions.
- **d.** (10 points) How does the relative demand for leisure in the two periods depend on the relative wage, the interest rate, and the tax rate on wages?

4. (Total 70 points)Assume that there is no growth in the economy, there is no depreciation, and the population of the economy is normalized to 1. Further assume that the household's expected lifetime utility is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t + G_t)^{\theta}}{\theta} + \gamma \ln(1 - L_t)\right]$$

where $0 < \theta < 1$, L_t is the fraction of time the household spends working for firms, C_t is the amount of the household consumption in time period t and G_t is the amount of Government expenditures per person in the economy. G_t is given exogenously for all dates and is financed by lump-sum taxes on the household. The household also has the following period by period budget constraint:

$$K_{t+1} - K_t + C_t \le r_t K_t + w_t L_t - Tax_t$$

where $\gamma > 0$, K_t is the level of capital, w_t is the wage rate in period t, Tax_t is the amount of the taxes paid by the household to the government and r_t is the real rate of return on capital.
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a. Question 4 continued.

b. (6 points) Describe the household's problem.

c. (8 points) Find the first order necessary conditions for this problem.

d. (8 points) What problem does the representative firm face in this economy if $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$ where $0 < \alpha < 1$ and what are the corresponding first order necessary conditions?

e. (10 points) Assume that $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$ where $0 < \alpha < 1.$ Show that in a competitive economy with H households and no growth, if each household satisfies their period by period budget constraint, the economy's resource constraint is satisfied.

f. (5 points) Does Ricardian Equivalence hold in this economy? Justify your answer.

g. (9 points) Assume that $G_t = G$ for all date t, where G is a positive constant. What effect does it have on the current level of consumption if the covariance of C_{t+1} and $(1 + r_{t+1})$ is negative instead of zero.

h. (12 points) Write down the Social Planner's Problem for this economy assuming $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$ where $0 < \alpha < 1$ and find the corresponding first order necessary conditions.

i. (6 points) Will the solution to the social planner's problem give the same allocations as the decentralized economy you examined in parts (a)-(e)? Justify your answer

j. (6 points) Is the first welfare theorm satisfied in this economy? Justify your answer.