ECO325S: Mid-Term Examination

February 2010

Duration: 2 hours

NO AIDS ALLOWED

Note to student: Enter your name and student number on each examination book that you use. There are five questions on this test for a total of 110 marks. Read each question carefully and note the number of points allocated to each part. Good Luck.

1. (Total 18 points) Assume that final output is produced by the following production function:

$$Y(t) = \left[K^{\alpha_1}(A(t)L(t))^{1-\alpha_1}\right] - \alpha_2 A(t)^{\alpha_3} L(t)$$

   a. (6 points) What restrictions on $\alpha_1, \alpha_2,$ and α_3 are necessary to make this function exhibit constant returns to scale in $A(t)L(t)$, and $K(t)$.
   b. (4 points) What is the intensive form of this production function when $\alpha_1 = 0.34, \alpha_2 = 2$, and $\alpha_3 = 1$?
   c. Is the intensive form of the production function consistent with:
      i. (2 points) A positive marginal product of capital per unit of effective labour?
      ii. (2 points) Diminishing Returns on capital per unit of effective labour?
      iii. (4 points) The inada conditions

2. (Total 18 points) Assume that the final output of the economy is defined by

$$Y(t) = (K(t))^{\alpha}\,(A(t)L(t))^{\gamma}$$

   Further, assume that $\alpha_A = 0.55$ where α_A is the elasticity of output, Y(t), with respect to the labour augmenting technology A(t).
   a. (4 points) Using this production function derive the growth rate of total output in terms of the growth rates of $K(t), A(t)$ and $L(t)$.
   b. (4 points) What is the value of the elasticity of output, Y(t), with respect to labour, L(t)? Justify your answer.
   c. (6 points) Under what condition(s) will:

$$\alpha_K + \alpha_L = 1$$

      where α_K is the elasticity of total output with respect to total capital and α_L is the elasticity of total output with respect to labour. Give an economic interpretation of the mathematical condition(s).
   d. (4 points) Assume that the we know $\alpha_K + \alpha_L = 1$, and are given that the growth rate of total capital is 3%, the growth rate of labour is 2% and the growth rate of total output per worker is 3%. What is the value of the Solow residual?

3. (Total 18 points) Assume that households have the following lifetime budget constraint:

$$\int_{t=0}^{\infty} e^{-R(t)} C(t)\frac{L(t)}{H}dt \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} w(t)A(t)\frac{L(t)}{H}dt$$

   where $R(t) = \int_o^t r(\tau)d\tau$, C(t) is consumption per worker, L(t) is the number of workers, K(0) is the intial capital stock and H is the number of households.
   a. (4 points) Express the household's lifetime budget constraint in terms of the per unit of effective labour variables using the fact that $L(t)$ and $A(t)$ grow exponentially at rates n and g respectively.
   b. (6 points) Assume that the household has the following lifetime utility function in terms of the

consumption per unit of effective labour, $c(t)$:

$$U = B\int_{t=0}^{\infty} e^{-\beta t}\left(\frac{c(t)^{\Delta}-1}{\Delta}\right)dt$$

where B is a constant greater that zero and $\beta = \rho - n - \Delta g > 0,\ 0 < \Delta < 1$ and ρ is the discount rate. Write down the Lagrangian for the Household's problem and use the first order necessary conditions to find the household's Euler equation $\frac{\dot{c}(t)}{c(t)}$.

c. (4 points) Let $w(t)$ be the wage per unit of effective labour, $r(t)$ be the rate of return on capital at time t, and $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$ Write down the firm's maximization problem for time t and prove that when $Y(t)$ exhibits constant returns to scale the problem's first order necessary conditions imply that:

$$w(t) = f(k(t)) - k(t)f'(k(t))$$

$$r(t) = f'(k(t))$$

d. (4 points) Can the steady state value of capital per unit of effective labour ever exceed the golden rule level of capital per unit of effective labour in this model? Justify your answer.

4. (Total 24 points) Let c and k be consumption per unit of effective labour and capital per unit of effective labour respectively. Using the Ramsey Cass Koopman model answer the following questions.

a. (3 points) Draw the Phase diagram for the model and label the point that corresponds to the economy's balanced growth path and the golden rule level of capital per person.

b. Assume that the economy is initially on its balance growth path. Suppose that the depreciation rate increases from 0 to δ.

i. (4 points) What are the affects of this change on the $\dot{k}$=0 and $\dot{c}$=0 curves if any? Justify your answer.

ii. (3 points) What happens to c after δ increases?

c. (8 points) How would your answer to b.i change if the increase in the depreciation rate was temporary? What would the change have on the balanced growth path be? Justify your answer.

d. (4 points) Now assume that there is a one time unexpected decrease in the level of total capital in this economy. What happens to k and y after the shock? How does this shock effect k^* and k^G where k^* is the steady state level of capital per unit of effective labour and k^G is the golden rule of capital per unit of effective labour?

5. (Total 32 points) Assume that $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$, where $0 < \alpha < 1$, $Y(t)$ is total output at time t, $K(t)$ is total capital at time t, $A(t)$ is the amount of knowledge at time t, and $L(t)$ is the amount of labour at time t. Use the Solow model to answer the following questions.

a. (6 points) Using the Solow model find the steady state values for k, y and c (i.e., k^*, y^* and c^*) as functions of the savings rate, s, the growth rate of labour, n, the rate of depreciation, δ, the growth rate of knowledge, g, and α.

b. (2 points) What is the golden rule level of capital, k^G as a function of s, n, δ, g and α?

c. Assume that there is a permanent decrease in δ.

i. (8 points) Show what happens to the steady state levels of k, c, and y, and the level of k^G using a diagram.

ii. (12 points) Find and determine the signs of:

$$\frac{\partial k^*}{\partial \delta}, \frac{\partial c^*}{\partial \delta}, \frac{\partial y^*}{\partial \delta}, \text{ and } \frac{\partial k^G}{\partial \delta}$$

d. (4 points) If δ decreases from 2% to 1%, and $\alpha = \frac{1}{3}, n = 1\%$ and $g = 1\%$, how much time will it take for k and y to move half way to their new balanced growth path? (Note: You only need to provide the final equation. You do not need to evaluate it.)