

ECO325S: Mid-Term Examination

Duration: 2 hours

NO AIDS ALLOWED

Note to student: Enter your name and student number on each examination book that you use. There are four questions on this test for a total of 120 marks. Read each question carefully and note the number of points allocated to each part. Good Luck.

1. (Total 30 points) Assume that final output is produced by the following production function:

$$\ln(Y(t)) = \alpha_0 \ln(K(t)) + \alpha_2 \ln(A(t)L(t) + \alpha_1) + \alpha_3 A(t)L(t)$$

- (6 points) What restrictions on $\alpha_0, \alpha_1, \alpha_2$ and α_3 are necessary to make this function exhibit constant returns to scale in capital and effective labour.
 - (6 points) What is the intensive form of the Cobb-Douglas production function?
 - Is the intensive form of the production function consistent with:
 - (2 points) A positive marginal product of capital per unit of effective labour?
 - (2 points) Diminishing marginal returns on capital per unit of effective labour?
 - (4 points) The Inada conditions
 - (10 points) What is the economic interpretation of: (1) a positive marginal product of capital per unit of effective labour, (2) diminishing returns on capital per unit of effective labour, and (3) the Inada conditions.
2. (Total 20 points) Assume that the final output of the economy is defined by

$$Y(t) = (K(t))^\alpha (A(t)L(t))^{1-\alpha}$$

- (6 points) Derive the growth rate of Total output in terms of the growth rates of $K(t), A(t)$ and $L(t)$. (Note: just writing down the final equation will only give you 3 points)
For the remaining parts of this question let α_K and α_L represent the elasticity of output with respect to capital and the elasticity of output with respect to labour respectively, and assume markets are perfectly competitive.
 - (4 points) Prove that $\alpha_K = \alpha$
 - (4 points) If $\alpha = 0.4$, what is the labour's share of income in the economy? Justify your answer.
 - (4 points) Assume that in addition to the assumptions made above, the growth rate of capital per worker is 5%, and the growth rate of output per worker is 7%. What is the value of the Solow residual?
 - (2 points) What does the Solow residual tell us?
3. (Total 30 points) Assume that households have the following lifetime utility function

$$U = \int_{t=0}^{\infty} e^{-\rho t} ((L(t)C(t))^{\alpha} + 1) \frac{L(t)^{1-\alpha}}{H} dt$$

where $1 > \alpha > 0$, ρ is the discount rate, $C(t)$ is consumption per worker, $L(t)$ is the number of workers and H is the number of households.

- (3 points) Express the household's lifetime utility in terms of consumption per unit of effective labour, $c(t)$, using the fact that $L(t)=L(0)e^{nt}$, $A(t)=A(0)e^{gt}$.
- (4 points) Are there any constraints on the values of ρ, n, α and g ? If so what are they? Justify your answer.
- (4 points) Given the fact that the household has the following lifetime budget constraint

$$\int_{t=0}^{\infty} e^{-R(t)} C(t) \frac{L(t)}{H} dt \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} w(t) A(t) \frac{L(t)}{H} dt$$

where $R(t) = \int_0^t r(\tau) d\tau$, $k(0)$ is capital per unit of effective labour at time 0, and $w(t)$ is the wage per unit of effective labour. Find the household's Euler equation $\frac{\dot{c}(t)}{c(t)}$, where $c(t)$ is the amount of consumption per unit of effective labour.

- (4 points) Describe in words what the household's budget constraint says.
- (3 points) Let $w(t)$ be the wage per unit of effective labour, $r(t)$ be the rate of return on capital at time t , and $Y(t) = F(K(t), A(t)L(t))$ Write down the firm's maximization problem for time t .
- (6 points) Prove that when $Y(t)$ exhibits constant returns to scale the first order necessary conditions from the firm's problem imply that:

$$w(t) = f(k(t)) - k(t)f'(k(t))$$

$$r(t) = f'(k(t))$$

- (6 points) What is the economic interpretation of the firm's first order necessary conditions?
- (Total 40 points) Let c and k be consumption per unit of effective labour and capital per unit of effective labour. Using the models studied in class complete the following.
 - (3 points) Draw the Phase diagram for the Ramsey-Cass and Koopmans model and label the point that corresponds to the economy's balanced growth path.
 - Assume that the Ramsey-Cass and Koopmans model's economy is initially on its balance growth path. Suppose that there is a permanent **rise** in n .
 - (4 points) How, if at all does this affect the $\dot{k}=0$ and $\dot{c}=0$ curves?
 - (3 points) What happens to c at the time of this change.
 - (3 points) Draw the diagram that shows break-even and actual investment per unit of effective labour in the Solow model. Label the steady state level of capital per unit of effective labour
 - Assume that the Solow model's economy is initially on its balanced growth path. Suppose that there is a permanent **rise** in n .
 - (2 points) On the graph of break-even and actual investment, how, if at all does this change affect the steady state level of capital per unit of effect labour, k^* .
 - (2 points) Derive the expression for $\frac{\partial k^*}{\partial n}$ and determine whether this expression is positive, negative or equal to zero.

- iii. (3 points) What happens to c at the time of the change?
- e. Now assume that there is a one time unexpected decrease in the number of people in the economy.
 - i. (8 points) Explain what affect this shock has on the balanced growth paths in the Ramsey-Cass-Koopmans model and in the Solow model? Explain what impact this has on the golden rule of capital per unit of effective labour in each of the models.
 - ii. (8 points) Explain what happens to c , k and y after the shock, in each of the models?
- f. (4 points) Find the growth rate of consumption per person along the balanced growth rate in each model.