

ECO325F: Mid-Term Examination

October 2010

Duration: 2 hours

NO AIDS ALLOWED

Note to student: Enter your name and student number on each examination book that you use. There are five questions on this test for a total of 100 marks. Read each question carefully and note the number of points allocated to each part. Good Luck.

1. (Total 12 points) What conditions are put on the intensive form of the production for the Solow model and what is the economic interpretation of these conditions?
2. (Total 8 points) Which of the following functions are constant returns to scale in total capital, $K(t)$, and total effective labour, $A(t)L(t)$? Justify your answer.
 - a. (4 points) $F(K(t), A(t)L(t)) = Z(K(t))^\alpha (A(t)L(t))^{1-\alpha}$, where Z is a positive constant
 - b. (4 points) $F(K(t), A(t)L(t)) = K(t) + A(t)L(t)$
3. (Total 14 points)
 - a. (2 points) What is the expression that describes the path of convergence of capital per unit of effective labour in the Solow model?
 - b. (2 points) What is the expression that describes the path of convergence of output per unit of effective labour in the Solow model?
 - c. (4 points) If the elasticity of total output with respect to total capital is equal to 0.5 , $\frac{A(t)}{A(t)} = g = 1\%$, $\delta = 3\%$ and the growth rate of population is 2% , how long does it take for the economy to move half way from an initial point $y(0)$ to the steady state level y^* ? Just give the final expression.
 - d. (6 points) What is the growth rate of capital per worker along the balanced growth path? Is this rate higher or lower than the growth rate when capital per unit of effective labour is below its steady state value? Justify your answers?
4. (Total 24 points)
 - a. (3 points) Find the intensive form of the production function defined in 2a)
Using the intensive form of the production function in part a), and assuming depreciation in this economy is δ , the population grows at rate n , knowledge, $A(t)$, grows at a constant rate g , and the households in this economy save a constant fraction of output, s , each period, answer the following:

- b.** (5 points) Draw the diagram that shows break-even and actual investment per unit of effective labour, and output. Label the steady state levels of capital per unit of effective labour, consumption per unit of effective labour, and output per unit of effective labour. In addition, label the golden rule level of capital per unit of effective labour.
- c.** Assume that the economy is initially on its balanced growth path. Suppose that there is a permanent **rise** in Z .
- (4 points) On the graph of break-even and actual investment, show how this change affects the steady state level of capital per unit of effective labour, k^* .
 - (4 points) Derive the expression for $\frac{\partial k^*}{\partial Z}$ and determine whether this expression is positive, negative or equal to zero.
 - (4 points) What happens to the golden rule level of capital per unit of effective labour? Justify your answer.
 - (4 points) What happens to consumption per unit of effective labour, c , at the time of the change and in the long run? Justify your answer.
- 5.** (Total 42 points) Assume that households have the following lifetime budget constraint:

$$\int_{t=0}^{\infty} e^{-R(t)} C(t) \frac{L(t)}{H} dt \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} w(t) A(t) \frac{L(t)}{H} dt$$

where $R(t) = \int_0^t r(\tau) d\tau$, $C(t)$ is consumption per worker, $L(t)$ is the number of workers and H is the number of households.

- (4 points) Express the household's lifetime budget constraint in terms of the per unit of effective labour variables using the fact that $L(t)$ and $A(t)$ grow exponentially at rates n and g respectively.
- (4 points) What is the value of the household's capital stock at time s ?
- (6 points) Assume that the household has the following lifetime utility function in terms of the consumption per unit of effective labour, $c(t)$:

$$U = B \int_{t=0}^{\infty} e^{-\beta t} \frac{c(t)^{1-\theta}}{1-\theta} dt$$

where $\theta > 0, \theta \neq 1$, B is a constant greater than zero and $\beta = \rho - n - (1 - \theta)g > 0$, and ρ is the discount rate. Write down the Lagrangian for the Household's problem and use the first order necessary conditions to find the household's Euler equation $\frac{c'(t)}{c(t)}$.

- d.** (4 points) Let $w(t)$ be the wage per unit of effective labour, $r(t)$ be the rate of return on capital at time t , and $Y(t) = ZK(t)^\alpha(A(t)L(t))^{1-\alpha}$. Write down the firm's maximization problem and the problem's first order necessary conditions.
- e.** (7 points) Prove that the golden rule level of capital per unit of effective labour is always greater than the steady state level of capital per unit of effective labour.
- f.** (3 points) Draw the Phase diagram for this version of the Ramsey-Cass-Koopmans model and label the point that corresponds to the economy's balanced growth path.
- g.** Assume that the economy is initially on its balanced growth path. Suppose that there is a permanent rise in Z .
- i.** (4 points) How, if at all does this affect the $\dot{k}=0$ and $\dot{c}=0$ curves?
 - ii.** (4 points) What happens to c at the time of this change? In the long run? Justify your answer.
 - iii.** (2 points) What happens to the golden rule level of k ? Justify your answer.
 - iv.** (4 points) What happens to output per unit of effective labour at the time of the change? In the long run? Justify your answer.