ECO2061 MA Macroeconomics
February 25, 2010
Midterm Duration: 2 hours
NO AIDS ALLOWED

NOTE TO STUDENT: Enter your name and student number on each examination book that you use. There are **four questions** on this test for a **TOTAL OF 220 MARKS**. Read each question carefully and note the number of points allocated to each part. Good Luck.

1. (Total 60 points) Assume that $Y(t) = ZK(t)^{\alpha}(A(t)L(t))^{\gamma} + BA(t)L(t),$ where $Y(t)$ is total output at time t, $K(t)$ is total capital at time $t, A(t)$ is the amount of knowledge at time t, and $L(t)$ is the amount of labour at time t.
   a. (12 points) What restrictions on the parameter values Z, B, α and γ are necessary to make the production function consistent with constant returns to scale? Are any further restrictions necessary to make the intensive form of the production function consistent with a positive marginal product of capital per unit of effective labour, diminishing marginal returns on capital per unit of effective labour and the inada conditions?
   b. (8 points)Using the production function and the restrictions derived in part a find the steady state values for k, y and c (i.e., k^{*}, y^{*} and c^{*}) and the golden rule level of capital per unit of effective labour in the Solow.?
   c. Assume that there is a shock that hits the economy that temporarily increases. s for exact10 periods (and it is know for certainty s will be higher for only 10 periods)
      i. (12 points) What paths do k, c, and y follow after the shock? Justify your answer
      ii. (8 points) Show what happens to the steady state levels of $k, c,$ and $y,$ and the level of k^{G} using a diagram.
      iii. (12 points) Find and determine the signs of:

$$\frac{\partial k^{*}}{\partial s}, \frac{\partial c^{*}}{\partial s}, \frac{\partial y^{*}}{\partial s}, \text{ and } \frac{\partial k^{G}}{\partial s}$$

   d. (8 points) If $s = 0.6,$ δ decreases from 1% to 2%, a decreases from $\frac{1}{2}$ to $\frac{1}{3}, n = 1\%$ and $g = 1\%,$ how much time will it take for k and y to move half way to their new balanced growth path? (Note: You only need to provide the final equation for each variable. You do not need to evaluate it.)
2. (Total 96 points) Assume that households have the following lifetime utility function

$$U = \int_{t=0}^{\infty} e^{-\rho t}(\frac{C(t)^{1-\theta}}{1-\theta})\frac{L(t)}{H}dt$$

   where $0 < \theta < 1, \rho$ is the discount rate, C(t) is consumption per worker, L(t) is the number of workers and H is the number of households.
   a. (4 points) Express the household's lifetime utility in terms of consumption per unit of effective labour, c(t), using the fact that L(t)=L(0)e^{nt}, A(t)=A(0)e^{gt}.
   b. (6 points) Are there any restrictions we should impose on the parameter values $\rho, n, \theta,$ or g? Why or why not?
   c. (20 points) Using the Hamiltonian method set up the Social Planner's problem for this

economy. Which variable is the control variable and which is the state variable? Use the Pontryagrin's Maximum principle theorem to find the equations that characterize the equilibrium.

d. (10 points) Assume that the household has the following flow budget constraint:

$$\frac{\dot{K}(t)}{H} + C(t)\frac{L(t)}{H} \leq \frac{r(t)K(t)}{H} + \frac{w(t)A(t)L(t)}{H}$$

where K(t) is the aggregate amount of capital in the economy at time t. Express this equation in terms of the per unit of effective labor variables.

e. (20 points) Use the information in part d to set up the current value Hamiltonian for the Household' problem. Which variable is the control variable and which is the state variable? Use the Pontryagrin's Maximum principle theorem to find the equations that characterize the solution to the household problem.

f. (10 points) Let $w(t)$ be the wage per unit of effective labour, $r(t)$ be the rate of return on capital at time t, and $Y(t) = F(K(t), A(t)L(t))$ Write down the firm's maximization problem for time t. Prove that when Y(t) exhibits constant returns to scale the first order necessary conditions from the firm's problem imply that:

$$w(t) = f(k(t)) - k(t)f'(k(t))$$

$$r(t) = f'(k(t))$$

g. (6 points) Write down the equations for $\dot{c}(t) = 0$ and $\dot{k}(t) = 0$.

h. (8 points) Find the steady state values for k, c and y, (i.e., k^*, c^* and y^*) and the golden rule level of k, k^G, in terms of ρ, n, g, and θ.

i. (12 points) Find and determine the signs of:

$$\frac{\partial k^*}{\partial \theta}, \frac{\partial c^*}{\partial \theta}, \frac{\partial y^*}{\partial \theta}, \text{ and } \frac{\partial k^G}{\partial \theta}$$

j. (10 points) Show the effects of a permanent increase in θ on k^*, c^* and k^G using the phase diagram.

3. (Total 24 points) Using the Tobin's q model of investment discussed in class discuss what happens to q and K after a permanent increase in the interest rate, r. Justify any movements to the $\dot{q} = 0$ and $\dot{K} = 0$ locuses and show the movements (if any) on a phase diagram.

4. (Total 40 points) Assume that the household's expected lifetime utility is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[C_t - \frac{a(C_t)^2}{2}) \right]$$

where $0 < \beta < 1$ and $a > 0$. The household also has the following period by period budget constraint:

$$K_{t+1} - K_t + C_t \leq r_t K_t + Y_t$$

where K_t is the level of capital in period t, Y_t is the amount of income (given exogenously) that the household receives in period t and r_t is the real rate of return on capital.

a. (15 points) Show that the first order necessary conditions for this problem imply

$$u'(C_t) = \beta\{E_t(1+r_{t+1})E_t(u'(C_{t+1})) - aCov_t(1+r_{t+1}, C_{t+1})\}$$

b. (10 points) What effect does it have on the current level of consumption if the covariance of (C_{t+1}) and r_{t+1} is positive instead of negative? Justify your answer.

c. (15 points) Now assume that individuals only live for T periods, and β =1=r_t for all dates. Show consumption follows a random walk in this case.