

UNIVERSITY OF TORONTO
ECO 2061H1 S Midterm 2008
Duration: 2 hours
NO AIDS ALLOWED

NOTE TO STUDENT: Enter your name and student number on each examination book that you use. There are three questions on this test for a **TOTAL OF 200 MARKS**. Read each question carefully and note the number of points allocated to each part. Good Luck.

1. (Total 60 points) Assume that households have the following lifetime utility function

$$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta}}{1-\theta} \frac{L(t)}{H} dt$$

where $\theta > 0, \theta \neq 1, \rho$ is the discount rate, $C(t)$ is consumption per worker, $L(t)$ is the number of workers and H is the number of households.

- a. (2 points) Express the household's lifetime utility in terms of consumption per unit of effective labour, $c(t)$, using the fact that $L(t)=L(0)e^{nt}$, $A(t)=A(0)e^{gt}$, and $\rho - n - (1 - \theta)g > 0$
- b. (5 points) Given the fact that the household has the following lifetime budget constraint,

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t} dt \leq k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t} dt$$

Prove that the household's Euler equation has the form: $\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho - \theta g}{\theta}$

- c. Assume this economy is initially on a balance growth path where there are no taxes. However, at some time, t^* , the government unexpectedly increases the tax rate on investment income to $\tau < 1$. Further assume that the government uses all revenue that it collects to purchase goods.
- i. (2 points) What is the real interest rate households face before the tax change? Justify your answer.
 - ii. (2 points) What is the real interest rate that households face just after the policy change? Justify your answer.
 - iii. (8 points) How does the change in taxes affect the $\dot{c} = 0$ locus and the $\dot{k} = 0$ locus? Justify your answer.
 - iv. (8 points) What are the steady state values for k, c and y , and the golden rule of k, k^g , prior to time t^* ?
 - v. (8 points) What are the steady state values for k, c and y and the golden rule of k, k^g , after time t^* ?
 - vi. (12 points) What are the dynamics of the variables k, c and y after the tax shock? Justify your answer.
 - vii. (9 points) Do the growth rates of capital per person, consumption per person and output per person depend on the tax rate? Justify your answer.
 - viii. (4 points) How do the properties of the balanced growth path in this model differ from the properties of the balanced growth path in the Solow Model?

2. (Total 70 points) Assume that in discrete time output is determined by a Cobb-Douglas Production Function: $Y_t = ZK_t^\alpha(A_tL_t)^{1-\alpha}$, where Z is a positive constant, and $0 < \alpha < 1$. Also let knowledge, A_t and population L_t evolve according to: $\ln(A_t) = \bar{A} + gt$ and $\ln(L_t) = \bar{L}$. Finally, we are given that $K_{t+1} = K_t + sY_t - \delta K_t$ and that $Y_t = C_t + I_t$. All lower case variables are in terms of units of effective labour.
- a. (10 points) Find, and graph the expression for k_{t+1} as a function of k_t . Label the steady state value of k . How do we know that a steady state value will exist?
 - b. (12 points) In this discrete time version of the Solow model, find the steady state values for k, y and c (i.e., k^*, y^* and c^*) as functions of the savings rate, s , the rate of depreciation, δ , Z , g and α .
 - c. (4 points) What is the golden rule level of capital, k^G , as a function of model's parameters?
 - d. Assume that there is a permanent decrease in g .
 - i. (12 points) Show what happens to the steady state levels of k, c , and y , and the level of k^G using a diagram. (Hint: think of the graph in the continuous time version)
 - ii. (12 points) Find and determine the signs of: $\frac{\partial k^*}{\partial g}$, $\frac{\partial c^*}{\partial g}$, $\frac{\partial y^*}{\partial g}$, and $\frac{\partial k^G}{\partial g}$
 - e. Now assume the economy is initially on its balanced growth path and that there is a one time increase in the amount of both capital and labour in the economy in the economy. Answer the following questions.
 - i. (6 points) At the time of the jump, what happens to the level of output per unit of effective labour? Justify your answer.
 - ii. (8 points) After the initial change (if any) in output per unit of effective labour, is there any further change in output per unit of effective labour? If so, does it rise or fall? Why?
 - iii. (6 points) Once the economy has again reached its balanced growth path, is output per unit of effective labour higher, lower or the same as it was before the increases? Justify your answer.

3. (Total 70 points) Assume that there is no growth in the economy, the population and the number of households are normalized to one, and the household faces uncertainty about future income. In this case, the household's expected lifetime utility is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t [(C_t + G_t)^{1/2}]$$

The household also has the following period by period budget constraint:

$$K_{t+1} - (1 - \delta)K_t + C_t \leq (1 - \tau_t)r_tK_t + w_t + T_t$$

where $0 < \gamma < 1$, K_t is the level of capital, C_t the consumption in period t , w_t is the wage rate in period t , r_t is the gross real rate of return on capital, and τ_t is the tax on gross capital income in the economy in period t , and T_t is lump-sum taxes in period t .

- a. (15 points) Set up the Lagrangian for the household's problem. Find the first order necessary conditions for this problem and show that they imply that:

$$\left(\frac{1}{C_t + G_t} \right)^{1/2} = \beta E_t \left(\left(\frac{1}{C_{t+1} + G_{t+1}} \right)^{1/2} (1 - \delta + (1 - \tau_{t+1})r_{t+1}) \right)$$

What is the economic interpretation of this equation?

- b. (17 points) If government purchases are constant and the marginal tax on capital income is zero, what effect does it have on the current level of consumption if the covariance of $(C_{t+1} + G_{t+1})$ and r_{t+1} is positive instead of negative? Justify your answer and provide intuition for your results.
- c. (5 points) From the household's point of view, is their private consumption in time period t and government expenditures in time period t complements, substitutes or neither? Justify your answer.
- d. (5 points) What is the government's period t budget constraint in this model?
- e. (8 points) Would Ricardian Equivalence hold in this economy? Justify your answer.
- f. (10 points) Write down the Social Planners problem for this economy and find the first order necessary conditions for the Social Planner's problem.
- g. (10 points) Assume that the taxes described above are used to finance government expenditures and that the government runs a balanced budget each period. Assume that output is produced according to the function $Y_t = Z(K_t^\alpha)(A_t L_t)^{1-\alpha}$. Show that in a competitive economy, if each household satisfies their period by period budget constraint, the economy's resource constraint is satisfied in equilibrium