

## ECO325F: Mid-Term Examination

**Duration: 1.5 hours**

**NO AIDS ALLOWED**

**Note to student:** Enter your name and student number on each examination book that you use. There are three questions on this test for a total of 100 marks. Read each question carefully and note the number of points allocated to each part. Good Luck.

1. (Total 21 points) Assume that households have the following lifetime utility function

$$U = \int_{t=0}^{\infty} e^{-\rho t} (C(t)^\alpha + \Delta) \frac{L(t)}{H} dt$$

where  $\alpha < 1, \Delta > 0, \theta \neq 1, \rho$  is the discount rate,  $C(t)$  is consumption per worker,  $L(t)$  is the number of workers and  $H$  is the number of households.

- a. (3 points) Express the household's lifetime utility in terms of consumption per unit of effective labour,  $c(t)$ , using the fact that  $L(t)=L(0)e^{nt}$ ,  $A(t)=A(0)e^{gt}$
- b. (5 points) Under what conditions is the household's lifetime utility bounded (less than  $\infty$ )?
- c. (6 points) Given the fact that the household has the following lifetime budget constraint

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t} dt \leq k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t} dt$$

Find the household's Euler equation  $\frac{\dot{c}(t)}{c(t)}$ .

- d. (7 points) Let  $w(t)$  be the wage per unit of effective labour,  $r(t)$  be the rate of return on capital at time  $t$ , and  $Y(t) = F(K(t), A(t)L(t))$  Write down the firm's maximization problem for time  $t$ . Prove that when  $Y(t)$  exhibits constant returns to scale the first order necessary conditions from the firm's problem imply that:

$$w(t) = f(k(t)) - k(t)f'(k(t))$$

$$r(t) = f'(k(t))$$

2. (Total 23 points) Let  $c$  and  $k$  be consumption per unit of effective labour and capital per unit of effective labour. Using the models studied in class for the general case where  $f(k(t))$  gives output per unit of effective labour, complete the following parts assuming that  $\delta > 0$ .
- a. (5 points) Draw the Phase diagram for the Ramsey-Cass and Koopmans model and label the point that corresponds to the economy's balanced growth path and the golden rule level of  $k$ .
- b. Assume that the Ramsey-Cass and Koopmans model's economy is initially on its

balance growth path. Suppose that there is a permanent rise in  $\delta$ .

- i. (6 points) How, if at all does this affect the  $\dot{k}=0$  and  $\dot{c}=0$  curves?
  - ii. (3 points) What happens to  $c$  after  $\delta$  changes.
- c. Now assume that there is a one time unexpected decrease in the amount of total capital in the economy.
- i. (3 points) Explain what affect this shock has on the balanced growth path in the Ramsey-Cass-Koopmans model?
  - ii. (6 points) Explain what happens to  $k$  and  $y$  after the shock?
3. (Total 36 points) Assume that households in the Ramsey-Cass-Koopmans model have the following lifetime utility function:

$$U = B \int_{t=0}^{\infty} e^{-\beta t} \frac{[c(t) + G(t)]^{1-\theta}}{1-\theta} dt$$

where  $\theta > 0, \theta \neq 1, B = \frac{A(0)^{1-\theta} L(0)}{H}, \beta = \rho - n - (1-\theta)g > 0, c(t)$  is consumption per unit of effective labour,  $G(t)$  is government expenditures per unit of effective labour,  $k(t)$  is capital per unit of effective labor and  $r(t)$  is the return on capital at time  $t$ .

- a. (3 points) Assume that the government finances  $G(t)$  by levying lump-sum taxes per unit of effective labour,  $Tax(t)$ , on the households and they run a balance budget each period. What is the resulting lifetime budget constraint for the household in terms of the per unit of effective labour variables?
- b. (3 points) Assume instead that the government finances  $G(t)$  by levying lump-sum taxes per unit of effective labour,  $Tax(t)$ , on the households and issuing bonds per unit of effective labour,  $b(t)$ . What is the resulting lifetime budget constraint for the household in terms of the per unit of effective labour variables?
- c. (10 points) Find the household's Euler equation in this model. Does the Euler condition for  $c(t)$  depend on how the government funds the expenditures? Justify your answer.
- d. (12 points) Assume that the economy is initially on its balanced growth path where government expenditures are zero. There is a temporary increase in the amount of government expenditures, such that the amount of government expenditures per unit of effective labour per unit of time is  $G$ , which is a positive constant, and that the date that government expenditures will return to zero is known with certainty. Draw the phase diagram showing the effect of the increase in government purchases on  $k$  and  $c$ , by drawing the path the economy takes as a result of this shock. Discuss what happens to  $k, c, y$  and the real return on capital,  $r$ , both immediately after the shock, and as the economy moves to its balanced growth path.
- e. (8 points) Does Ricardian Equivalence hold in this economy? Justify your answer