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## ECO325F: Advanced Economic Theory-Macro

### Problem Set #1

1. Which of these gross production functions generally display constant returns to scale in labour and capital (i.e. if  $K$  and  $L$  double, does  $Y$  double)? For those that do not display constant returns to scale for all parameter values, find the parameter values that make them satisfy constant returns to scale.
  - a.  $\ln Y = A + \alpha \ln K + \beta \ln L$  (Cobb-Douglas)
  - b.  $Y = A \min(K, L)$  (Leontieff)
  - c.  $\ln Y = A + B \ln(\alpha_0 K^\rho + \alpha_1 L^\rho)$  (CES)
  - d.  $\ln Y = \left\{ \begin{array}{l} A + \alpha_0 \ln K + \alpha_1 \ln L \\ + \frac{1}{2}(\alpha_3 (\ln K)^2 + 2\alpha_4 \ln K \ln L + \alpha_5 (\ln L)^2) \end{array} \right\}$  (Translog)
  - e.  $Y = A + \alpha_0 K + \alpha_1 L$  (Linear)
2. Let  $F(K, L) = G(K, L) + H(K, L)$ .
  - a. Prove that  $F$  is homogeneous of degree one if both  $G$  and  $H$  are.
  - b. Give a counter-example to show that the fact  $F$  is homogeneous of degree one does not always imply that  $G$  and  $H$  are homogeneous of degree one.
  - c. Let  $H(K, L) = \alpha_0 L$ . Prove that  $F$  is homogeneous of degree one *iff*  $G$  is also homogeneous of degree one.

3. Romer 1.1
4. Romer 1.2 Assume in this question that during the time  $t_1$  to  $t_2$  that the growth rate increases linearly from 0 to  $a$ .
5. Mathmatically describe the function  $X(t)$  in Romer 1.2. Hint: What is the growth rate of the function  $Y(t) = e^{\frac{b}{2}t^2}$ ?