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ECO325: Advanced Economic Theory-Macro

Problem Set #3

1. Consider an individual who lives for two periods with preferences given by

$$U(c_1, c_2) = u(c_1) + (1+\rho)^{-1}u(c_2)$$

where $u(c)$ is the felicity (one period utility) function and ρ is the subjective rate of time preference. Each period he inelastically supplies 1 unit of labour to firms and receives w_1 when young (i.e. in period 1) and w_2 when old (i.e., in period 2). Assume that he inherits A units of wealth when he is young and is committed to leaving a bequest of B to his children when he is old.

a. What is the budget constraint in period 1? in period 2? (Let r be the rate of return on savings between period 1 and 2.)

b. What is the lifetime budget constraint of the individual when r is the rate of return on savings between period 1 and 2?

c. Solve for his Euler equation, which gives the trade-off between optimally chosen first and second period consumption, for each of the following felicity (one period utility) functions:

i. $u(c) = \frac{c^{1-\theta}}{1-\theta}$ where $0<\theta$ and $\theta \neq 1$

ii. $u(c) = \ln(c)$

iii. $u(c) = -e^{-\alpha c}$ where $0<\alpha$

d. For each of the felicity functions in part (c) solve explicitly for c_1 and c_2. Show that the optimal levels of consumption in each period depends only upon wealth and the rate of interest.

2. Consider the flow budget constraint defined for all $t \geq 0$ as

$$\dot{K}(t) = A(t)w(t)L(t) + r(t)K(t) - c(t)L(t)$$

Define $R(t) = \int_0^t r(s)\partial s.$ Show that the flow budget constraint can be redefined for all $T \geq 0$

$$e^{-R(T)}K(T) = K(0) + \int_0^T e^{-R(t)}[A(t)w(t)L(t) - c(t)L(t)]\partial t$$

3. Romer 2.1
4. Romer 2.4
5. Romer 2.6 (do only a, b and c)
6. Romer 2.7 (For simplicity, assume that the shift in the production function in part (c) of the question is a parallel shift)