1(a)
$$U=\int_{t=0}^{\infty} e^{-\rho t}\left(C(t)^{\alpha}+\Delta\right)\frac{L(t)}{H}\,dt$$

$$=\int_{t=0}^{\infty} e^{-\rho t}\left((A(t)c(t))^{\alpha}+\Delta\right)\frac{L(t)}{H}\,dt$$

$$=\int_{t=0}^{\infty} e^{-\rho t}\left[\left(A(0)e^{gt}\right)^{\alpha}c(t)^{\alpha}+\Delta\right)\frac{L(0)e^{nt}}{H}\,dt.$$

$$=\int_{t=0}^{\infty} e^{-\rho t}\,\frac{A(0)^{\alpha}e^{g\alpha t}L(0)e^{nt}}{H}\,c(t)^{\alpha}\,dt$$

$$+\frac{\Delta L(0)A(0)}{H}\int_{t=0}^{\infty} e^{nt}\cdot e^{-\rho t}\,dt$$

$$=\frac{A(0)^{\alpha}L(0)}{H}\int_{t=0}^{\infty} e^{-t(\rho-\alpha g-n)}c(t)^{\alpha}\,dt$$

$$+\frac{\Delta L(0)A(0)}{H}\int_{t=0}^{\infty} e^{-t(\rho-n)}\,dt=B_0\int_{t=0}^{\infty} e^{\beta t}c(t)\,dt+B_1$$

3 pts

(b) for both parts of U above to be bounded we need

$\beta=\rho-\alpha g-n>0$ & $\rho-n>0$

(c) $$\mathcal{L} = B_0 \int_{t=0}^{\infty} e^{-t\beta} c(t)^{\alpha} dt + B_1$$

$$+ \lambda \left\{ k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t} dt - \int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t} dt \right\}$$

$$\frac{\partial \mathcal{L}}{\partial c(t)} = B_0 e^{-\beta t} \alpha c(t)^{\alpha-1} - \lambda e^{-R(t)} e^{(n+g)t} = 0 \qquad (A)$$

for all t

$$\frac{\partial \mathcal{L}}{\partial \lambda} = k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t} dt - \int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t} dt = 0 \qquad (B)$$

$$(A) \Rightarrow \quad B_0 e^{-\beta t} \alpha c(t)^{\alpha-1} = \lambda e^{-R(t)} e^{(n+g)t}$$

taking ln we get

$$\ln B_0 - \beta t + \ln \alpha + (\alpha-1) \ln c(t) = \ln \lambda - R(t) + (n+g)t$$

since this holds for all t, take the derivative with respect to t to get

$$-\beta + (\alpha-1) \frac{\dot{c}(t)}{c(t)} = -r(t) + (n+g)$$

$$\Rightarrow \quad \frac{r(t) - (\rho - \alpha g - n) - n - g}{1-\alpha} = \frac{\dot{c}(t)}{c(t)}$$

$$\Rightarrow \quad \frac{r(t) - \rho - (1-\alpha) g}{1-\alpha} = \frac{\dot{c}(t)}{c(t)}$$

(d) $\pi(t) = F(K(t), A(t)L(t)) - w(t)A(t)L(t) - r(t)K(t)$

$= A(t)L(t)\, f(k(t)) - w(t)A(t)L(t) - r(t)\,K(t)$

$\Rightarrow \max_{K(t),L(t)} \pi(t)$

F.O.N.C.

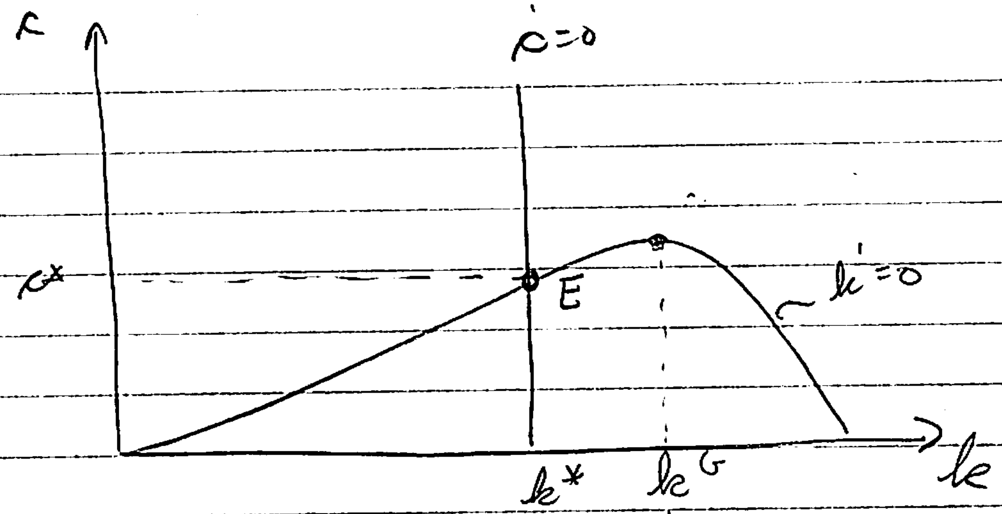
$$\frac{\partial \pi(t)}{\partial K(t)} = A(t)L(t)\, f'(k(t))\, \frac{1}{A(t)L(t)} - r(t) = 0$$

$$\Rightarrow f'(k(t)) = r(t)$$

$$\frac{\partial \pi(t)}{\partial L(t)} = A(t)\, f(k(t)) + A(t)\cancel{L(t)}\, f'(k(t)) \left(\frac{-K(t)}{A(t)L(t)^2} \right) - w(t)A(t) = 0$$

$$\Rightarrow \boxed{w(t) = f(k(t)) - k(t)\, f'(k(t))}$$

2 (a)



point E corresponds to the balanced growth path

k^G is the golden rule level of k

1 pt for well labelled graph

2 points for E
2 pts for k^G
deductions for improper labelling or k^G not greater than k^*

b) i) $\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \delta - \rho - \theta g}{\theta}$

as $\dot{c}=0$ gives: $f'(k^*) = \delta + \rho + \theta g$

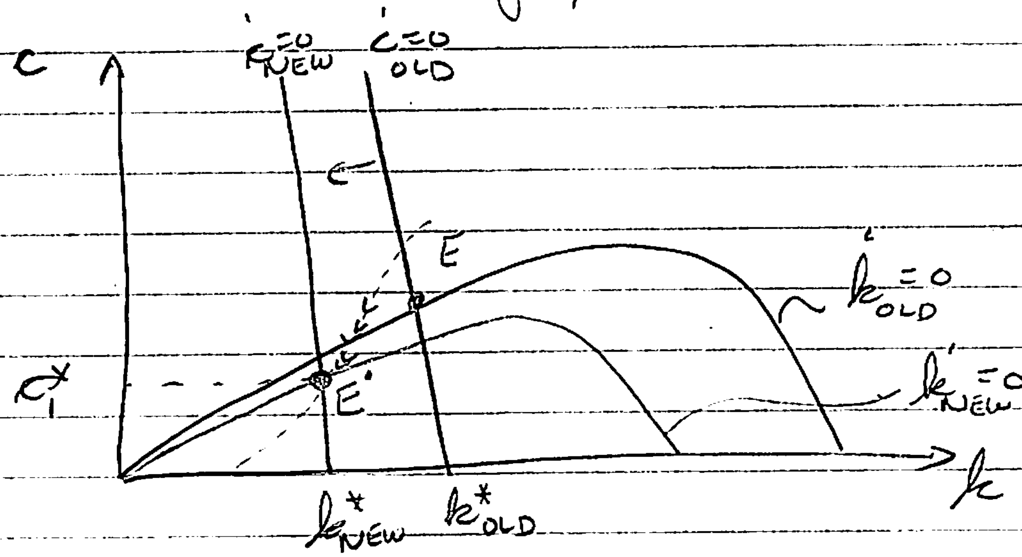
as $f''(k^*)\frac{\partial k^*}{\partial \delta} = 1$ or $\frac{\partial k^*}{\partial \delta} = \frac{1}{f''(k^*)} < 0$

since $f''(k) < 0$ as $\delta \uparrow$ causes $k^* \downarrow$
and $\dot{c}=0$ curve shifts inwards
1 pt for shift 2 pts for derivation.

2b cont. $\dot{k}(t) = f(k(t)) - c(t) - (n+g+\delta)\,k(t)$

so $\dot{k} = 0 \Rightarrow c(t) = f(k(t)) - (n+g+\delta)\,k(t)$

so $\delta \uparrow$ will cause $\dot{k}=0$ to move downwards as drawn in the graph below



1 pt for correct movement of $\dot{k}=0$
2 pts for justification

ii) At the time of the change in δ, c will fall, rise or stay the same depending on where the new saddle path intersects the old $\dot{c}=0$ line. 1 pt

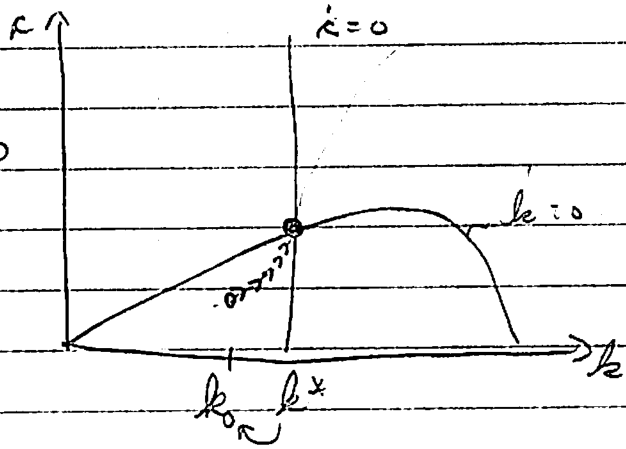
After the initial change, c will travel along the saddle path (falling) towards the new steady state level c_1^*. 1 pt

In the long run, the level of c reaches c_1^* and remains there until another shock occurs.

1 pt

2 c)(ii) Since $K(t) \downarrow$ and $k(t) = \frac{K(t)}{A(t)L(t)}$, $k(t)$ falls below k^* (see graph below) to level k_0. (1pt)

After the initial drop, k increases as the economy moves along the saddle path back to the original steady state value k^*. (1pt)
Once it reaches k^*, it remains there until there is another shock (1pt)



Since $y(t) = f(k(t))$ and $f'(k(t)) > 0$

initially y falls from $y^* = f(k^*)$

to $y_0 = f(k_0)$ (1pt)

After the initial decrease since $\dot{y}(t) = f'(k(t))\,\dot{k}(t)$

y increases as k increases and the economy move along the saddle path. (1pt)

Once the economy reaches the balanced growth path, y is back at its original value y^* (1pt)

c) i) There is no effect on the balanced growth path since the decrease in $K(t)$ does not effect $\delta, n, g, \rho,$ or θ, and hence does not affect $\dot{c}=0$ or $\dot{k}=0$ curves

(3pts)

3 a) $$\int_{T=0}^{\infty} e^{-R(t)} c(t)\, e^{(n+g)t}\, dt \le k(0) + \int_{T=0}^{\infty} e^{-R(t)} w(t)\, e^{(n+g)t}\, dt - \int_{T=0}^{\infty} e^{-R(t)} T(t)\, e^{(n+g)t}\, dt$$

3 pts

b) $$\int_{t=0}^{\infty} e^{-R(t)} c(t)\, e^{(n+g)t}\, dt \le k(0) + b(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t)\, e^{(n+g)t}\, dt - \int_{T=0}^{\infty} e^{-R(t)} T(t)\, e^{(n+g)t}\, dt.$$

3 pts

c) $$\mathcal{L} = B \int_{T=0}^{\infty} e^{-\beta t} \frac{[c(t) + G(t)]^{1-\theta}}{1-\theta}\, dt + \lambda \Big[-\int_{t=0}^{\infty} e^{-R(t)} c(t)\, e^{(n+g)t}\, dt - \int_{t=0}^{\infty} e^{-R(t)} T(t) e^{(n+g)t}\, dt + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t}\, dt + k(0) \Big]$$

2 pts

In the 1st case

and in the second.

$$\mathcal{L} = B \int_{t=0}^{\infty} e^{-\beta t} \frac{[c(t) + G(t)]^{1-\theta}}{1-\theta}\, dt + \lambda \Big[k(0) + b(0) + \int_{T=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t}\, dt - \int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t}\, dt - \int_{t=0}^{\infty} e^{-R(t)} T(t)\, e^{(n+g)t}\, dt. \Big]$$

2 pts

3 c) cont In both cases

$$\frac{\partial \mathcal{L}}{\partial c(t)} = B e^{-\beta t}\left[c(t)+G(t)\right]^{-\theta} - \lambda e^{-R(t)} e^{(n+g)t} = 0$$

so

$$B e^{-\beta t}\left[c(t)+G(t)\right]^{-\theta} = \lambda e^{-R(t)} e^{(n+g)t}$$

$$\Longleftrightarrow \quad \ln B - \beta t - \theta \ln\left[c(t)+G(t)\right] = \ln \lambda - R(t) + (n+g)t$$

so taking derivatives gives us

$$-\beta - \theta \frac{\left[\dot c(t)+\dot G(t)\right]}{\left[c(t)+G(t)\right]} = -r(t) + (n+g)$$

so

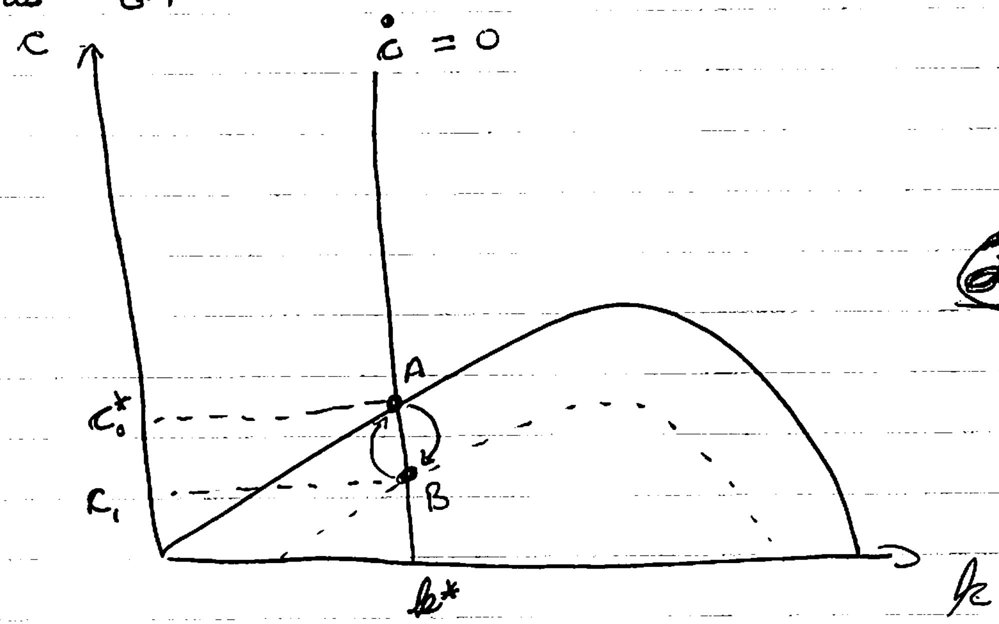
$$\frac{\dot c(t)+\dot G(t)}{c(t)+G(t)} = \frac{-\left(\rho - n - (1-\theta g)\right)] + r(t) - n - g}{\theta}$$

$$= \frac{r(t) - \rho - \theta g}{\theta}$$

5 pts for derivation of $\frac{\dot c(t)+\dot G(t)}{c(t)+G(t)}$

1 point for statement they are the same for both cases.

3d) as G ↑



2 pts

in S.S $\dot{c} = 0$, $\dot{G} = 0$ and $\dot{k} = 0$

change in G lowers $\dot{k} = 0$ curve temporarily since $\dot{k}(t) = f(k(t)) - c(t) - G(t) - (n+g)k(t)$

~~economy moves~~

2

and $G(t)$ increases from G_L to $G_H > G_L$ at t_0

and back to G_L at t_1 — # pts

There is no change in the $\dot{c} = 0$ eq'n. # pts

at time t_0 the economy jumps from A to B
and stays there until time t_1 when it jumps back to A.

2 pt

∴ $k(t)$ does not change in response to the shock.
since $y(t) = f(k(t))$, $y(t)$ remains at its value y^*
and is unaffected by the changes in G. 2 pts

since $r(t) = f'(k(t)) = f'(k^*)$, r is unaffected 2 pts
consumption per unit of effective labour drops from
c_0^* to c_1 at t_0 & remains there until t_1 when
it jumps back to c_0^* 2 pts

3 e) Ricardian Equivalence does hold in this economy since the path the economy takes is affected only by the path of government expenditures — not the path of taxes (since they are lump sum here).

4 pts

You can see this since: (i) (c) shows that the Euler eq'n doesn't depend on taxes,

(ii) in both the case of balanced budget w/ lump-sum taxes & case with bond & lump-sum taxes financing gov't expenditures the household's budget constraint can be written as

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t}\, dt = k(0) + \int_{t=0}^{\infty} e^{-R(t)} w(t) e^{(n+g)t}\, dt - \int_{t=0}^{\infty} e^{-R(t)} G(t) e^{(n+g)t}\, dt$$

and (iii) $\dot{k}(t) = f(k(t)) - c(t) - G(t) - (n+g)\,k(t)$ & independent of the way $G(t)$ is financed.

4 pts for justification.