A comment on notation (and investing)

We want to distinguish:

a/ the real return an investor is being offered on an investment at time t, and

b/ the realized (or actual) real return that an investor makes, having invested at time t in a nominal bond.

They need not be the same. So let us denote the former by r_t.

If you invest \$1 at a real interest rate of r_t, then you are guaranteed that real return over the investment horizon. How is that accomplished? The lender is contractually obliged to compensate the investor for any realized inflation over the holding period.

So consider a one-year investment of \$1. At the end of the year, the investor must receive a real return of r percent — that is the deal. Thus, if actual inflation over the holding period is π_{t+1}, then the nominal payment the investor receives is

$$(1+r_t)(1+\pi_{t+1}).$$

Why? Because if we deflate that nominal sum by the actual inflation that occurred over the holding period, we get the real return that the investor earned:

$$\frac{(1+r_t)(1+\pi_{t+1})}{1+\pi_{t+1}} = 1+r_t,$$

which is the precise real amount the investor was promised.

Note well: investing in a financial instrument at time t gives a known real return. But the nominal return over the holding period is not known. In other words, it is uncertain, involving nominal risk.

In contrast, investing in a nominal financial instrument entails no nominal risk, but some real risk, except in the case where inflation is perfectly predictable (in other words, effectively never!)

This brings us to the case considered in class: suppose I invest in a nominal instrument, paying a nominal return of i_t. The financial institution that issued this

is (clearly) obligated to pay me

$$(1 + i_t)$$

on my \$1 investment in one year's time. Now what is my actual real return on that investment? It depends on the actual inflation rate during the year. We have called that π_{t+1}, understood as the retrospective inflation at $t+1$ over the previous year. (Under alternative notation, this might have been written $\pi_t^{\wedge}$, understood as the actual realized inflation rate over the year starting at time t. Such notation would 'fit' better with some other notation I used — namely π_t^E, which referred to the expected inflation rate at time t, looking ahead over the coming year.)

Now, what is the actual (realized) real return on this nominal investment? It is

$$\frac{(1 + i_t)}{(1 + \pi_{t+1})} \equiv \frac{1 + i_t}{(1 + \pi_t^{\wedge})}$$

notation in class — equivalent notation.

What should we call that realized quantity? In class, I wrote

$$\frac{1+i_t}{1+\pi_{t+1}} = 1+r_{t+1}$$

The right hand side is not ideal, as it may be confused with the contractually obligated real rate of return offered, this time at time $t+1$. Instead, we should have written, for clarity, notation

$$\frac{1+i_t}{1+\pi_{t+1}} \equiv \frac{1+i_t}{1+\pi_t^A} = 1+r_t^A,$$

where r_t^A (instead of r_{t+1}) captures the actual real return realized on a nominal investment over period t.

On this basis, i_t, r_t, r_t^A, π_t^A, and π_t^E are all defined at time t, looking ahead over the coming year. The first two are interest rates offered at t, the next two are actualized rates realized over the course of t, and the last is the expected value, looking ahead at t.

So now to investing: whether to invest in a nominal or real financial instrument? It is a good question.

We mentioned above that a nominal instrument gives nominal certainty but real uncertainty in terms of the return.

In contrast, a real instrument (such as indexed debt) gives real certainty but nominal uncertainty.

When investors invest, they are typically trading off risk against returns, as you will know. To simplify, suppose we abstract from risk. This is equivalent to assuming that uncertain future values can be replaced by their expected values.

On this basis — abstracting from risk — agents should invest in the instrument that gives the highest return over the holding period.

To decide between nominal and real investments, we need to put both on the same basis. This can be done either way — nominal or real. Suppose we take nominal units:

Clearly, a nominal investment of \$1 will give $(1+i_t)$ in nominal terms at the end of the year.

What about a real investment of \$1 at a real return of r_t? The nominal return is an <u>expected</u> nominal return that depends on the expected inflation rate over the holding period, which we are denoting π_t^E.

Thus the expected nominal return is

$$1 + i_t^E = (1 + r_t)(1 + \pi_t^E).$$

Abstracting from risk (I'm repeating), the agent would invest in the real instrument if

$$1 + i_t^E > 1 + i_t$$

or if

$$(1 + r_t)(1 + \pi_t^E) > 1 + i_t$$

or if

$$1 + \pi_t^E > \frac{1 + i_t}{1 + r_t}$$

Note that we can define (in a one-period setting) the RHS as $1 + \pi_t^{BEIR}$. Thus the rule would be: invest in an indexed bond if $\pi_t^E > \pi_t^{BEIR}$.