

# Competition, Incentives, and Public School Productivity\*

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Final version: July 27, 2003

## Abstract

When competition increases, it is often presumed that public schools will be forced to become more efficient. This paper challenges that presumption, showing that in well-defined circumstances, rent-seeking public schools find it optimal to *reduce* productivity when a voucher is introduced. This occurs for incentive reasons alone. More generally, the productivity effects of vouchers are shown to be non-uniform, varying systematically according to household and community characteristics and the form the voucher takes; when the voucher is targeted, perverse productivity outcomes do not arise. The analysis has relevance to the policy issue of voucher design. (*JEL* H41, I28)

*Keywords:* competition, incentives, productivity, voucher, targeting.

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\*This paper is based on Chapter 2 of my Stanford University Ph.D thesis. Thanks to my advisors Tom Nechyba, John Pencavel and Tim Bresnahan and to Pat Bayer, Ettore Damiano, Li Hao, Roger Noll, Kim Rueben, Matt Turner and Pablo Ruiz-Verdu for helpful discussions. Thanks also to seminar participants at the University of Florida for additional suggestions. Steve Coate and two anonymous referees provided extensive comments that improved the paper considerably. All remaining errors are my own. Financial support from the Public Policy Institute of California is gratefully acknowledged. Contact address: Department of Economics, University of Toronto, 150 St. George Street, Toronto, ON M5S 3G7, Canada. Email: `mcmillan@chass.utoronto.ca`.

# 1 Introduction

Concerns over public school quality are voiced with unerring regularity in the media and in popular debate, prompting considerable interest in measures such as vouchers that increase competition. According to the received wisdom, vouchers will force technically inefficient schools to raise productivity and so improve quality. Under this view, public school efficiency will increase unambiguously, and such gains may more than offset any adverse effects on school quality resulting from greater sorting.<sup>1</sup>

The intuition behind the received wisdom is simple: improving rival options raises customers' reservation quality levels, and in order to retain market share in the face of greater competition, previously inefficient suppliers have to increase quality by raising productivity. The soundness of this intuition has not been examined in the prior literature. To make good the deficit, this paper offers the first formal analysis of the incentives faced by public schools to change productivity in response to an increase in competition. It demonstrates that the standard intuition needs to be refined.

The key insight in our analysis is that rent-seeking suppliers may choose to go 'down-market' in response to an increase in competition, *reducing* productivity and service quality. Though they lose market share, suppliers can more than offset the losses by being able to cut costly effort. In the typical incentive environment faced by many public schools, such reductions in productivity are entirely possible.

We demonstrate the insight in a simple model of a local education market. The model features two types of household - low and high income - who choose between public and private schools. Private schools charge tuition and are assumed to behave competitively. Public schools, in contrast, are free. To bring incentive issues to the fore, we assume that they are rent-seeking and that performance contracts are not imposed on them.<sup>2</sup> Public school quality is determined by the public school's effort choice, high effort bringing high enrollment and high revenues but being costly to provide. We suppose a simple form of demand externality operates within public schools. Though high and low income households have different demands for

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<sup>1</sup>Adverse effects of sorting are likely to be magnified if conventional peer effects are important. (See Epple and Romano (1998) for a thorough analysis.)

<sup>2</sup>In practice, the power of unions helps explain why performance contracts are not imposed, allowing public schools to earn positive rents. (See Section 3 for a detailed discussion.)

school quality, students are not completely tracked on the basis of type within a given school.<sup>3</sup> This means that the children of low income households will receive higher quality education if the public school serves both low and high income types.

In this simple framework, it is possible to identify a range of communities in which it pays public schools to win the entire market before a private school voucher has been introduced. Following the voucher's introduction, however, within a subset of these communities, public schools choose to reduce effort (equivalent to a reduction in productivity), let the high income types leave and serve just the low income types.<sup>4</sup> In this case, the losses from smaller market share are more than offset by the saving in effort cost. When the voucher is targeted at poor households, the perverse productivity outcome does not arise: a targeted voucher is productivity-enhancing in all communities. The perverse result also goes away when a uniform voucher is introduced in communities in which the public schools are completely tracked, though in such settings low income households suffer from lower quality education pre-voucher.

The analysis has relevance for policy. It shows that in some communities, greater choice alone will not be a sufficient discipline on rent-seeking public schools. In the case of a uniform voucher, additional regulations are needed to avoid perverse effects - regulations that either rule out reductions in enrollment or reductions in quality directly. To the extent that such regulations are effective in raising quality themselves, so policies to increase competition as a means of raising productivity may be rendered superfluous.

The rest of paper is organized as follows. The next section places the analysis in the context of the prior literature. Section 3 sets out the model; Section 4 discusses possible equilibria that arise, focusing on public school effort-setting before and after the introduction of a voucher. Section 5 considers the productivity implications of a targeted voucher, and Section 6, the impact of a uniform voucher when students are tracked. Section 7 discusses extensions and Section 8 concludes.

## 2 Relation to the Prior Literature

A growing body of theoretical research examines the effects of competition on the level of school quality and the degree of sorting across schools. (See, for instance, interesting recent papers

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<sup>3</sup>This is distinct from conventional peer effects. There are no ability or educability differences in the model.

<sup>4</sup>This arises when the cost of enrolling high types increases relative to the cost of enrolling low types.

by Epple and Romano (1998), Hoyt and Lee (1998), and Nechyba (1996, 1999, and 2000).) In this research, schools have typically been treated as passive technologies for converting inputs into outputs, thus abstracting from potential incentive effects of vouchers that influence public school efficiency. The current analysis models these incentive effects explicitly. Not only do incentives lie at the heart of the arguments surrounding school vouchers; as the analysis shows, they may be perverse.<sup>5</sup>

Other models in the literature have the feature that school quality can fall when competition increases, notably in Epple and Romano (1998) and Nechyba (2000). The underlying mechanisms there are very different: by increasing the degree of sorting, vouchers lead to a reduction in school quality due to a loss of favorable peer effects in public schools. The current paper abstracts from conventional peer effects entirely, making clear that peer effects are not necessary for the ‘falling quality’ result to arise. The incentive mechanism studied in this paper is likely to have effects distinct from those due to conventional peer effects, even if the two are difficult to separate in practice. Adding conventional peer effects to the analysis would provide an additional reason why quality should fall.<sup>6</sup>

The model draws on insights in previous work by industrial organization theorists demonstrating that household heterogeneity can have important consequences for supplier behavior. For instance, Stiglitz and Salop (1977) show that search activity by consumers who are more effective searchers confers an externality on those who do not search in the form of lower prices; having a mixture of types of consumer can thus lead to spillover benefits for the ‘low’ types.<sup>7</sup> These sorts of model are relevant to the supply side of the public sector, where suppliers often enjoy market power and households differ in ways that affect the ability of suppliers to earn rents. The current analysis shows how heterogeneity among households combined with demand spillovers can give rise to perverse incentive effects when competition increases.<sup>8</sup>

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<sup>5</sup>Manski’s (1992) computational study shares this paper’s rent-seeking assumption. In contrast to his findings, the current paper shows that monotone improvements in productivity should not always be expected when competition increases.

<sup>6</sup>A more elaborate model in which parents use collective action (‘voice’) could generate a similar result that quality falls in response to an increase in competition, building on Hirschman’s insightful (1970) analysis. Even if parental voice is non-existent, the mechanism at work in the current analysis can operate.

<sup>7</sup>See also Salop (1977) and Tirole’s (1988) discussion of the quality setting choice of a monopolist.

<sup>8</sup>Laffont and Tirole’s 1990 study is similar to the current paper in that it combines both consumer heterogeneity and changes in competition. However, their formulation is quite different, and they focus on the gains to low-demand consumers from greater competition.

The paper has some similarities with interesting recent research on voucher design. Epple and Romano (1998b) consider the effects of targeted vouchers in a setting in which public schools are allowed to be technically inefficient (exogenously), and show how vouchers can increase technical efficiency while avoiding sorting effects. In the current analysis, determinants of public school inefficiency are modeled explicitly. Nechyba (2000) considers the distributional effects of vouchers targeted at low-income households using a general equilibrium framework in which household residential location choice is endogenized. Household location and neighborhood compositions are taken as given in the current model, though as long as the sorting equilibria that emerge have both low and high type households mixing to some degree, then the main result can go through; in practice, mixing both in schools and the residential market is reasonably widespread. Epple, Newlon, and Romano (2002) analyze the consequences of tracking for equilibria in the education market. There, tracking has the plausible effect of allowing public schools to increase enrollment among high ability students. In the current analysis, tracking serves purely as a rent extraction device, as there are no ability differences in the model.

### 3 The Model

This section presents the model. It introduces the players, describes the game they play, and defines an equilibrium in this game, before considering how the game is solved.

#### Households

Consider a community with two types of household: those with high income  $y^H = y$  (henceforth the ‘high types’) and those with low income  $y^L = \mu y$  (the ‘low types’), where  $\mu < 1$ . Households are otherwise identical: they all have school-aged children, all children are of equal ability, and all parents in the community share the same concave utility function  $U(c, Q)$ , defined over consumption  $c$  and the quality of education received by their children  $Q$ .<sup>9</sup> One additional assumption proves convenient:<sup>10</sup>

**Assumption U:**  $U(0, a) < U(\epsilon, a - \epsilon)$  for  $a > 0$  and small  $\epsilon$ .

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<sup>9</sup>The figures shown later in the paper are based on a Cobb-Douglas utility specification  $U = c^\alpha Q^\beta$ . (Details of the fully solved model are available on request.) See also Appendix B.

<sup>10</sup>This is used below in the derivation of optimal private school tuition.

Households choose between public and private schools so as to maximize utility. The high types are found in proportion  $x$  in the community, where  $x$  is an exogenous parameter. For convenience, the size of the market is normalized to 1.

## The Public School

A single public school receives an exogenous amount  $V$  per pupil, as under a state-financed system. To bring the productivity issue to the fore, suppose that the sole determinant of education quality  $Q$  is the effort level  $e$  set by the public school - the effects of student ability, peer effects across students, or parental help in the home (among other things) are thus abstracted from. School quality is determined according to a concave production function  $Q(e)$  that transforms effort into school quality, an increase in effort having the interpretation of an increase in productivity.<sup>11</sup>

Because they are free and entry is restricted, public schools enjoy varying degrees of market power. Whether they will respond to changes in competition that reduce their market power depends critically on their underlying objectives. Here, we adopt the extreme assumption that public schools are rent-seeking. This helps stack the deck *against* finding that incentives have perverse effects, as we are considering the case in which incentives might be expected to be most needed. If they chose instead to maximize quality, then it is unlikely that incentives would have any efficiency effects. We suppose public schools earn rents by cutting costly effort.

It is important to justify why rents can be earned by public schools. In practice, a number of school systems do not reward their schools based on performance, other than via enrollment changes. Flat incentives for public schools and their personnel not only permit rent-seeking in principle; there is also a strong perception that many schools are inefficient in practice. In this analysis, we assume that performance contracts are not imposed either before or after the voucher's introduction.<sup>12</sup> Thus rent-seeking schools still have scope to earn rents after the reform, even though households are made somewhat less captive via the tuition subsidy.

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<sup>11</sup>Though exerting effort is (privately) costly to the school, even after raising effort we suppose that the public school earns higher rents than its next best alternative. Thus higher effort is associated with a more productive use of existing resources.

<sup>12</sup>Given that the government has the option of imposing performance contracts, the most compelling explanation as to why these are not imposed is that unions are powerful, their ability to inflict political damage leading politicians to leave positive rents 'on the table.' To tell this story fully, one would need a political economy model, beyond the scope of the current analysis.

This allows us to focus on the policing capability of competition alone, rather than this in combination with additional regulations.

Under the rent-maximizing objective, the single public school chooses effort so as to maximize its payoff function, written

$$R(e) = (V - C(e)) \times s(e). \tag{1}$$

Accordingly, rents are equal to the product of the mark-up per pupil - given by the difference between  $V$  and the convex disutility of effort per student enrolled  $C(e)$  - and enrollment, given by  $s(\cdot)$ .<sup>13</sup> By reducing effort, the public school can earn a higher mark-up on each pupil enrolled. But lower effort lowers quality, which reduces market share. In the model,  $s$  takes three values (0,  $1 - x$ , and 1) depending on the public school effort choice and the level of private school quality (the precise linkage of  $s$  to  $e$  is developed below). Note that the public school is assumed to have sufficient capacity to enroll everyone in the market. Whether it does so depends on the effort level it chooses. This choice is analyzed in Section 4.

## Private Schools

Competition for student enrollment comes from a competitive private school sector. Private schools are assumed to maximize profits. They charge tuition  $T$  and use the proceeds to provide school quality  $Q_P$ . As with public schools, private school production technology is subject to constant returns to scale. Unlike the public school sector, entry into the private school sector is assumed to be free. Thus private schools provide the maximum possible competition for public schools, making it more likely that vouchers have positive incentive effects.

### 3.1 The Game

The game proceeds as follows: the public school sets quality through its choice of effort, based on its (correct) assessment of the tuition and quality levels private schools will set, and private schools set tuition and determine private school quality. Then households choose among public and private schools based on relative quality and cost.

Equilibrium in this game is characterized by four conditions:

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<sup>13</sup>To simplify the analysis, we assume that there are constant returns to scale in education production. This is reasonable based on evidence in the literature. (See Dewey, Husted and Kenny (2000) for a survey.) We discuss the implications of relaxing this assumption in Section 7.

1. The public school maximizes rents, setting quality through its choice of effort.
2. Private schools set tuition and quality so as to maximize profits, subject to a free entry constraint.
3. Households choose a school so as to maximize utility, given their income.
4. All households' children are enrolled in some school.

In solving the game, we begin by considering private school behavior. Free entry implies that private schools earn zero profits in equilibrium and so have no scope for shirking. Thus we suppress effort as a choice variable from the private school production process. Given that private school quality  $Q_P$  should be increasing in the level of resources devoted to educating students, we assume for simplicity that  $Q_P$  is equal to spending per pupil.<sup>14</sup> Further, private schools break even on every pupil they enroll, given the constant returns to scale assumption and free entry, so  $Q_P = T$ .

In this competitive environment, private schools will set quality in such a way as to maximize the utility of the households that attend them. They do not condition on the behavior of the public school. If public school quality is less than a critical amount so that households choose to enroll in private school, and if private school quality is a normal good, then we can state

**Lemma 1:** *When private schools emerge, they will be type-specific, serving each type of household separately.*

This follows from the profit maximization objective, the constant returns to scale assumption, and from the assumption that conventional peer effects do not operate. In the analysis below, private school quality *is* a normal good. Thus tuition and school quality will be tailored to each type so as to maximize that type's utility. In view of Lemma 1, denote tuition (and quality) in the high-type private schools as  $T_H$ , and low-type tuition  $T_L$ .

## Utility Payoffs

We now derive the payoffs to households of each type from their different school choices, conditioning on public school quality; public school choice of quality is considered next section.

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<sup>14</sup>It is possible to introduce fixed costs in the private school production process that increase the competitive advantage of public schools. This strengthens the main result in the paper (analysis available on request).



The utility a type- $i$  household receives from attending public school is

$$U^i(e) = U(y^i, Q(e)), \quad (2)$$

where  $i \in \{L, H\}$ . Private schools charge tuition (also equal to quality) conditioning on type, and households receive a tuition subsidy  $\nu$ , possibly equal to zero. The utility the type- $i$  households receive from attending private school in the presence of the voucher<sup>15</sup> is

$$U_P^i = U\left(y^i - (T_i - \nu), T_i\right). \quad (3)$$

where the  $P$ -subscript denotes ‘private.’ Recall from Lemma 1 that separate private schools will emerge to serve high- and low-income students respectively. In high-type private schools, optimal tuition  $T_H^*(y, \nu)$  will be set to maximize high-type utility. Taking the level of the voucher  $\nu$  as given, we assume that this is always greater than the voucher, or

**Assumption V:**  $T_H^* > \nu$ .

For low-type private schools, tuition setting is potentially more involved. Typically, vouchers can be applied only to costs of private school tuition. If this constraint were ignored, then if the low income households were poor enough relative to the high income households, so optimal (utility-maximizing) tuition might be less than the voucher, with some of the voucher being used for consumption. In the unconstrained case, low type tuition is given by

$$T_L^u(\mu y, \nu) \equiv \operatorname{argmax}_{\{T\}} U\left(\mu y - (T - \nu), T\right). \quad (4)$$

The properties of  $T_L^u$  are straightforward to derive.

**Lemma 2:** *Unconstrained low type tuition,  $T_L^u$ , is increasing in income  $y$ .*

Making the assumption that  $-U_{11} + U_{12} > 0$ , this is implied by the concavity of the utility function: private schools will raise tuition (and thus quality) as income rises. By analogous reasoning,  $T_L^u$  is increasing in  $\mu$ , so  $T_H^* > T_L^u$  for  $\mu < 1$ .  $T_L^u$  is also increasing in the voucher  $\nu$ . Thus it will be optimal for private schools to raise tuition once the voucher has been introduced, in order to provide higher quality education. Setting  $\mu = 1$ , these results also apply to high-type tuition,  $T_H^*$  (though  $T_H^*$  is not a function of  $\mu$ ).

We now address the complication presented by the fact that the voucher must be less than or equal to private school tuition, as accords with the types of voucher scheme proposed in

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<sup>15</sup>For now, we do not close the model, ignoring the source of financing for the voucher - endogenizing finance is considered in Section 7. For an elaborate treatment of financing in a general equilibrium setting, see the papers by Nechyba (1996, 1999).

practice. Taking the level of the voucher  $\nu$  as given, we have assumed that optimal private school tuition for the high types is always greater than the voucher (see Assumption V), ensuring that high type tuition is determined by the unconstrained program. For low-type tuition, by Assumption V, we know that  $T_L^u > \nu$  when  $\mu = 1$ , and by Assumption U, we have  $T_L^u < \nu$  when  $\mu = 0$ . As unconstrained tuition is increasing in  $\mu$ , given Lemma 2, we can define a cut-off value as follows:

**Definition M:**  $\underline{\mu} \equiv \mu \in [0, 1)$  satisfying  $T_L^u(\underline{\mu}y, \nu) = \nu$ .

Here,  $\underline{\mu}$  will be a function of the voucher, and for  $\nu = 0$ ,  $\underline{\mu} = 0$ . The optimal tuition function for the low types in the face of the constraint,  $T_L^*$ , can thus be written in two parts, depending on the value of  $\mu$ :

$$T_L^* = \begin{cases} \nu & \text{if } \mu \leq \underline{\mu}(\nu) \\ T_L^u & \text{if } \mu > \underline{\mu}(\nu). \end{cases} \quad (5)$$

Thus for  $\mu \leq \underline{\mu}(\nu)$ , the utility from attending private school is maximized by setting tuition equal to the voucher, giving utility  $U_P^L = U(\mu y, \nu)$ .

With the analysis of optimal tuition in place, the indirect utility of the high types is given by

$$V_P^H(y, \nu) = U\left(y - (T_H^*(y, \nu) - \nu), T_H^*(y, \nu)\right). \quad (6)$$

This is increasing in both income and the voucher, applying the envelope theorem. For the low types, indirect utility depends on  $\mu$  also, so we write

$$V_P^L(\mu y, \nu) = U\left(\mu y - (T_L^*(\mu y, \nu) - \nu), T_L^*(\mu y, \nu)\right). \quad (7)$$

This is increasing in income, the ratio  $\mu$ , and the voucher.

## Market Shares

Given the utility levels for each type from attending public and private school, we can derive the public school market share for different public school effort levels. Assuming switching costs are zero, the public school can at least attract the low types if it supplies effort  $e$  greater than or equal to a minimum level  $l$ , for which the utilities from attending public and private schools are equal. (For convenience, assume that indifferent households choose the public school.) The effort threshold  $l$  required to enroll the low types in public school is defined implicitly by

$$U(\mu y, Q(l)) - U\left(\mu y - (T_L^*(\mu y, \nu) - \nu), T_L^*(\mu y, \nu)\right) \equiv 0. \quad (8)$$

Several results are apparent:

**Lemma 3a:** For  $\mu \leq \underline{\mu}(\nu)$ ,  $l$  is not a function of  $\mu$ . For  $\mu > \underline{\mu}(\nu)$ , the effort threshold  $l$  of the low types is increasing in  $\mu$ .

**Proof:** The first part is apparent given that for  $\mu \leq \underline{\mu}$ ,  $T_L^* = \nu$ . Substituting into (8), we have that  $Q(l) = \nu$ , where  $\nu$  is exogenously given. Thus  $l$  is not a function of  $\mu$  for  $\mu \leq \underline{\mu}(\nu)$ . Turning to the second part, where  $\mu > \underline{\mu}$ , totally differentiating the left-hand side of (8) with respect to both  $l$  and  $\mu$  and rearranging gives

$$\frac{dl}{d\mu} = \frac{-y\left(\frac{\partial U}{\partial c} - \frac{\partial U_P}{\partial c}\right) + \left(-\frac{\partial U_P}{\partial c} + \frac{\partial U_P}{\partial Q}\right)\frac{\partial T_L^*}{\partial \mu}}{\frac{\partial U}{\partial Q} \frac{\partial Q}{\partial l}}. \quad (9)$$

The first term in the numerator, in brackets, is negative. Given  $T_L^* > \nu$  for  $\mu > \underline{\mu}$ , consumption under the private school option is less than consumption under the public school option as  $\nu$  cannot cover all private school tuition. Indifference between the public and private school options implies that private school quality must therefore exceed public school quality  $Q(l)$ . The second term in the numerator is equal to zero, evaluated at the optimum, and the denominator is positive. Thus the whole of the right-hand side is positive. **Q.E.D.**

**Lemma 3b:** The effort threshold  $l$  of the low types is increasing in the voucher.

**Proof:** This can be shown in two parts. For  $\mu \leq \underline{\mu}(\nu)$ ,  $Q(l) = \nu$ , as argued above, and as  $Q(\cdot)$  is an increasing function of  $l$ , we have the result. For  $\mu > \underline{\mu}(\nu)$ , totally differentiating the left-hand side of (8) with respect to both  $l$  and  $\nu$  and rearranging gives

$$\frac{dl}{d\nu} = \frac{\frac{\partial U}{\partial c}}{\frac{\partial U}{\partial Q} \frac{\partial Q}{\partial l}}, \quad (10)$$

which is positive. **Q.E.D.**

**Lemma 3c:** For  $\mu \leq \underline{\mu}(\nu)$ , the effort threshold  $l$  is not a function of  $y$ . For  $\mu > \underline{\mu}(\nu)$ ,  $l$  is increasing in  $y$ .

This follows using similar reasoning to that for Lemma 3a.

We now analyze the properties of the high-type effort threshold. To enroll the high types in addition to the low types, the public school must supply effort  $e \geq h$ , again assuming indifferent households choose public school. This effort threshold is defined implicitly by

$$U(y, Q(h)) - U(y - (T_H^*(y, \nu) - \nu), T_H^*(y, \nu)) \equiv 0. \quad (11)$$

Several results are apparent: First, the high effort threshold exceeds the low effort threshold, or  $h > l$ . This follows from the restriction that  $\mu < 1$  and that  $l$  is increasing in  $\mu$  (from Lemma 3a). Second,  $\frac{\partial h}{\partial \mu} = 0$  and  $\frac{\partial h}{\partial x} = 0$ . This is apparent from (11), as the threshold for the high types  $h$  is not a function of either  $\mu$  or  $x$ . Third, the effort threshold of the high types is increasing in the voucher, which follows given that indirect utility of the high types is increasing in the voucher. Fourth, the effort threshold of the high types is increasing in income  $y$ . This is implied by Lemma 3c, and letting  $\mu = 1$ .

From these conditions on school choice, we can derive the enrollments implied by each effort choice on the part of the public school. Pre- or post-voucher, the public school market shares are

$$s = \begin{cases} 0 & \text{if } e < l(\nu) \\ 1 - x & \text{if } e = l(\nu) \\ 1 & \text{if } e = h(\nu). \end{cases} \quad (12)$$

Thus, if public school effort is low, the public school enrolls only the low types while if effort is high, the school enrolls the whole market. Whether the public school will supply the effort required to enroll the high types or be happy just enrolling the low types depends on the costs of effort relative to the benefits. This decision is addressed in the next section.

## 4 Properties of Equilibrium

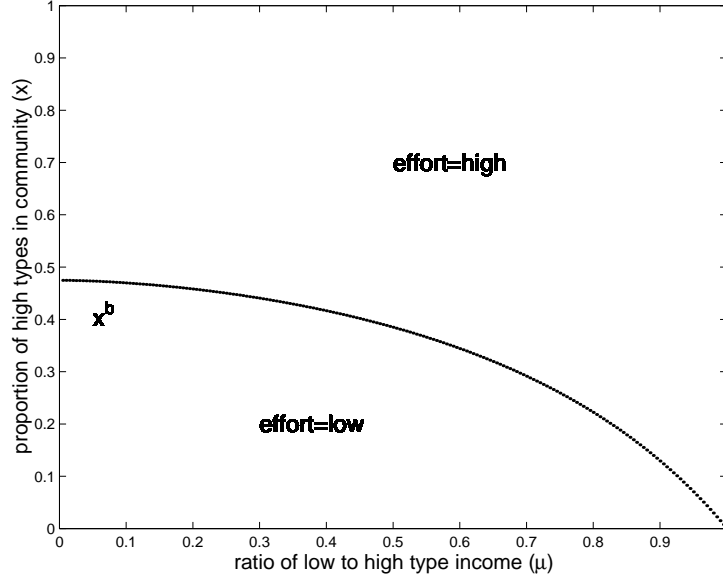
We now focus on the types of equilibria that arise. First, we draw attention to two possible equilibria prior to the introduction of the voucher, summarizing the outcomes in a simple figure in  $[\mu, x]$  space. Then we turn to the changes in school effort following the introduction of a uniform voucher, identifying the cases where public school effort (and thus quality) declines and increases respectively.

### 4.1 Pre-Voucher Equilibria

In the absence of a voucher, high type parents prefer public school over private if the public school exerts high effort, in which case the public school captures the whole market. With high effort, the school payoff is  $R(h) = V - C(h)$ . Similarly, the payoff to the public school if effort is low is  $R(l) = (1 - x)(V - C(l))$ . Throughout, we will make

**Assumption C:**  $V > C(h) > C(l)$ .

Figure 1: Shape of the boundary between the two regions - high and low effort



In the ‘high quality’ equilibrium, everyone enrolls in public school and the public school supplies high quality, which arises if  $R(h) \geq R(l)$  or  $V - C(h) \geq (1 - x)(V - C(l))$ , implying  $x \geq \frac{C(h) - C(l)}{V - C(l)}$ . Through their taste for school quality, the high types generate an externality whereby the low types receive higher quality education.<sup>16</sup> In the ‘low quality’ equilibrium, only the low types enroll in public school, and receive just their threshold quality level, while the high types all enroll in private school. The condition for this equilibrium to hold is simply  $x < \frac{C(h) - C(l)}{V - C(l)}$ .

## 4.2 Illustrating the Pre-Voucher Equilibria

Each possible community can be characterized by the exogenous parameters  $x$  and  $\mu$ ,  $x$  giving the proportion of high types in the community, and  $\mu$  gives the ratio of the income of low to high types. Figure 1 illustrates the two equilibria just described, showing the regions for which it is optimal for the public school to apply high and low effort respectively in the space of communities  $[\mu, x]$ .<sup>17</sup>

To explain the shape, we can solve for the boundary between the high and low effort regions. Assume that the convex cost of effort function is given by  $C(e) = e^\psi$  with  $\psi > 1$ .

<sup>16</sup>In Section 6, we discuss the implications of tracking.

<sup>17</sup>The patterns illustrated in this and the other figures hold more generally, as we show.

Then the boundary values are given by

$$x^b = \frac{(h(\nu))^\psi - (l(\nu))^\psi}{V - (l(\nu))^\psi}. \quad (13)$$

It is straightforward to show that the boundary  $x^b$  declines as  $\mu$  increases. Differentiating the right hand side of (13) with respect to  $\mu$  gives

$$\frac{\partial x^b}{\partial \mu} = -\frac{(V - h^\psi)}{(V - l^\psi)^2} \psi l^{\psi-1} \frac{\partial l}{\partial \mu}. \quad (14)$$

In the absence of the voucher, the whole expression is negative, given Assumption C and the result from Lemma 3a that  $\frac{\partial l}{\partial \mu}$  is positive.<sup>18</sup>

With the boundary in place, the region above and to the north-west of the boundary indicates the high effort region. Here, it pays the public school to exert higher effort in order to enroll everyone because there are sufficient high types, given  $\mu$ . In the region below and to the south-west, it pays to set effort low and just enroll the low types.<sup>19</sup>

### 4.3 Introducing a Uniform Voucher

This subsection presents the main results of the paper. First, we analyze the effects of the introduction of a voucher on the borderline  $x^b$ , before turning to changes in effort levels in different communities. Conditions under which the borderline shifts up monotonically are given in the following lemma.

**Lemma 4:** *The borderline  $x^b$  between high and low effort regions shifts up monotonically after the introduction of a uniform voucher if  $\frac{\partial l}{\partial v} < \left(\frac{V-l^\psi}{V-h^\psi}\right) \left(\frac{h}{l}\right)^{\psi-1} \frac{\partial h}{\partial v}$ .*

This is apparent differentiating (13) with respect to  $\nu$ . From Lemma 3b, both  $\frac{\partial l}{\partial v}$  and  $\frac{\partial h}{\partial v}$  are greater than zero. Also, given the assumptions that  $\psi > 1$  and  $V > h^\psi > l^\psi$ , the term  $\left(\frac{V-l^\psi}{V-h^\psi}\right) \left(\frac{h}{l}\right)^{\psi-1}$  is greater than one. Thus the increase in  $l$  is restricted from being much greater than the increase in  $h$  in response to the voucher.

Lemma 4 sets out conditions under which outside options improve for all but the public school chooses to reduce effort in response to the increase in competition. The intuition is as

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<sup>18</sup>Intuitively, as the low types become more like the high types, so the cost of supplying high relative to low types falls, and a lower proportion of high types is needed to make it worthwhile to supply high quality.

<sup>19</sup>We might expect that as the cost of exerting effort rises, so the range of communities in which it pays the public school to exert low rather than high effort grows. A sufficient condition for this is that  $h > 1$ . (Proof available on request.)

follows: As  $h > l$ , and the cost of effort is a convex function, increases in  $h$  in response to the voucher become relatively more costly, thereby raising the cost of supplying the whole market. This convexity allows the increase in  $l$  to exceed the increase in  $h$  and still allow the cut-off value of  $x$  to rise with the voucher. Under the condition in Lemma 4, it now takes a higher proportion of high types in the community to make the school indifferent between supplying high versus low effort, because the cost of supplying the high types relative to the low types has risen.

Placing more structure on the problem, we can examine the determinants of the sizes of the partial derivatives  $\frac{\partial l}{\partial \nu}$  and  $\frac{\partial h}{\partial \nu}$  using a simple Cobb-Douglas example. Considering the initial value of the voucher to be zero, the condition in Lemma 4 is satisfied, as  $\frac{\partial l}{\partial \nu} < \frac{\partial h}{\partial \nu}$ . (See Appendix B.)

The next set of results concern the effects of the voucher's introduction on effort levels (and thus school quality). There are four possible pairs of equilibria, pre- and post-voucher:

1. The public school could provide high effort both before and after.
2. The public school could provide low effort both before and after.
3. The public school could provide high effort before and low effort after.
4. The public school could provide low effort before and high effort after.

Under the condition given in Lemma 4, Figure 2 provides an illustration of these pairs of equilibria in  $[\mu, x]$  space - the corresponding regions are labeled 1, 2, and 3. (Note that if this condition holds, Region 4 does not arise for a uniform voucher.)

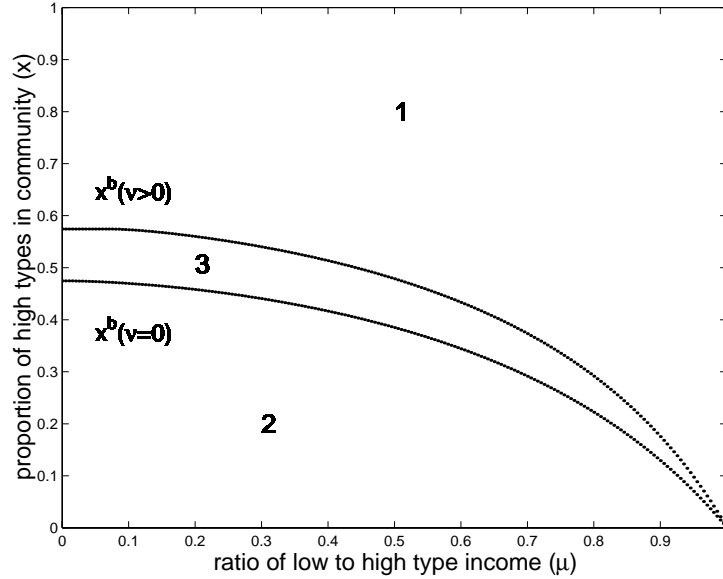
The effects of the introduction of a voucher on public school effort are summarized in the following four propositions.

**Proposition 1:** *If it is optimal for the public school to supply high effort before and after the introduction of the voucher (and thus enroll both low and high types), then the introduction of the voucher must raise school effort.*

Because high type indirect utility from attending private school is increasing in the voucher, so  $h(\nu)$  is increasing in  $\nu$ , and both the low and high types benefit from the increase in school productivity (assuming school quality is non-excludable).

**Proposition 2:** *If it is optimal for the public school to provide low quality both before and after the introduction of the voucher, then the voucher will unambiguously raise school quality.*

Figure 2: The two regions - before and after the introduction of the voucher



This follows from Lemma 3b as  $l(\nu)$  is increasing in  $\nu$ .

In the third region, vouchers lead the public school to switch from providing education to both the high and low types to just the low types. Following the introduction of a uniform voucher, the high types enroll in private school, the optimal response of the public school to the voucher being to allow the high types to leave and focus on serving just the low types. This will usually mean that quality falls, but whether this is the case depends on the size of the voucher and  $\mu$ , the ratio of low- to high-type income. If the voucher is high enough or  $\mu$  is close to one, then even though public school effort ‘falls’ from high to low,  $l$  post-voucher may be greater than  $h$  pre-voucher.

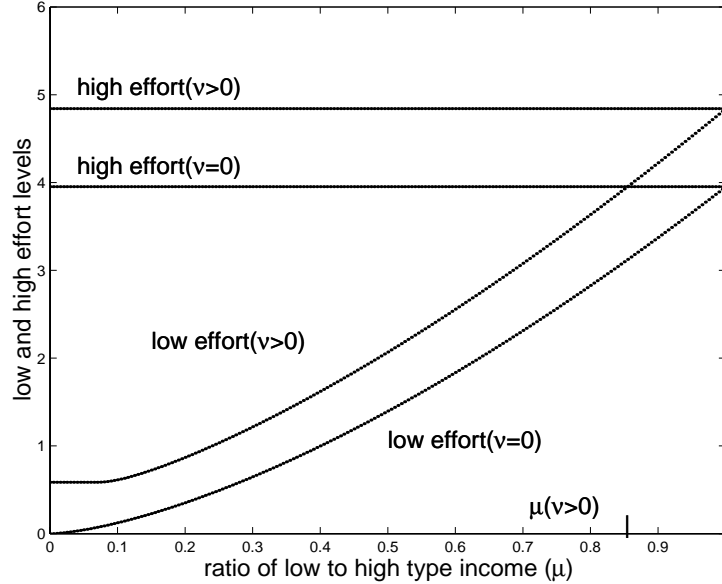
Proposition 3 summarizes the changes in quality in Region 3. Let  $\bar{\mu}(\nu > 0)$  be that  $\mu$  value for which  $h(\nu = 0) = l(\mu, \nu > 0)$ .

**Proposition 3:** *Public school quality falls following the introduction of a voucher if  $x$  lies on or is just greater than the pre-voucher borderline value  $x^b(\nu = 0)$  and  $\mu < \bar{\mu}(\nu > 0)$ .*

**Proof:** Consider a point in  $[\mu, x]$  space lying in Region 3. For a given increase in the voucher (indexed by  $\nu > 0$ ), and holding  $\mu$  fixed, there exist some  $x \in [x^b(\nu = 0|\mu), x^b(\nu > 0|\mu)]$ , noting that the  $x^b$  boundary shifts out under the conditions assumed here. Given such an  $x$ , pre-voucher effort will be high, at level  $h(\nu = 0)$ , as we are on or above the borderline  $x^b(\nu = 0)$



Figure 3: Effort levels against  $\mu$  - increasing the (uniform) voucher



and post-voucher effort will be low. When a voucher is introduced, this shifts the borderline between low and high effort up so that the public school switches to providing low effort. Low effort  $l$  is monotonically increasing in  $\mu$  (from Lemma 3a), and rises to  $h$  when  $\mu = 1$ . It is also monotonically increasing in the voucher (from Lemma 3b). But as long as  $\mu < \bar{\mu}(\nu > 0)$ , low effort post-voucher must be less than high effort pre-voucher. **Q.E.D.**

The proof of Proposition 3 is immediate from Figure 3, which provides an illustration of low and high effort levels pre- and post-voucher against  $\mu$  on the horizontal axis. Once the voucher is introduced, the intersection of the pre-voucher horizontal line for  $h(\nu = 0)$  and the new  $l(\nu)$  curve must be to the left of  $\mu = 1$ . That intersection occurs at the value of  $\bar{\mu}(\nu > 0)$  referred to in the proposition, and as long as  $\mu$  is less than this, so  $l(\nu > 0) < h(\nu = 0)$ .

For a discrete increase in the voucher, it is apparent that the ‘falling quality’ result requires that  $\mu$  does not get too close to 1. In other words, it requires sufficient dissimilarity between high and low types, which we can summarize in the following

**Corollary:** *For school quality to fall for the low types, there needs to be a large enough difference between high and low types in terms of income. In particular, the ratio of low to high income must be strictly less than  $\bar{\mu}(\nu > 0)$  in Region 3.*

Low types suffer when high types leave because they no longer enjoy the higher quality pre-

viously supplied to the high types. But as  $\mu$  rises, so the low types become less and less distinguishable from the high types and the reduction in quality gets smaller:  $(h - l(\mu))$  declines as  $\mu$  rises. At the same time, an increase in the voucher raises the threshold  $l$  of the low types, by Lemma 3b. When  $\mu$  rises above  $\bar{\mu}(\nu > 0)$ , the reduction in quality due to the exit of the high types is more than offset by the gain in quality due to the introduction of the voucher. Thus some difference between the types is required for the ‘falling quality’ result.

For incremental changes in competition, moving from two types to a continuum of types preserves the result that school productivity can fall once competition increases.<sup>20</sup> In the continuous case, the impact of competition on school effort hinges on the sign of a cross-partial derivative:  $\frac{\partial^2 s(e, \nu)}{\partial e \partial \nu}$ . If this cross-partial is sufficiently negative, a non-pathological case, then an increase in the voucher can lead to a reduction in effort. The model provides intuition as to why this cross-partial should matter. A voucher creates incentives to reduce effort when the reservation quality levels of the higher types rise more rapidly than those of the lower types, because the relative costs of enrolling higher types increase. Differential changes in reservation qualities of this sort are what would give rise to the negative cross-partial.

In the fourth case - the one most favorable to vouchers - effort and school quality rise unambiguously, and by possibly substantial amounts. Here, the public school begins by supplying low quality and enrolling just the low types. Following the voucher’s introduction, it pays the public school to raise effort, not only to retain the low types but also to enroll the high types as well.

**Proposition 4:** *The case in which the public school provides low effort before and high effort after the introduction of a uniform voucher (Region 4) only arises if  $\frac{\partial l}{\partial \nu}$  is sufficiently large. Specifically, we need  $\frac{\partial l}{\partial \nu} > \left(\frac{V-l\psi}{V-h\psi}\right) \left(\frac{h}{l}\right)^{\psi-1} \frac{\partial h}{\partial \nu}$ .*

This is the converse of Lemma 4. In the next section, we show how targeting of the voucher gives rise to the condition in Proposition 4.

## 4.4 Welfare

The simple structure of the model makes welfare analysis regarding the introduction of a uniform voucher straightforward. Public school rents must fall unambiguously following the voucher’s introduction. This is obvious in the pairs of equilibria where the public school supplies

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<sup>20</sup>Appendix A sets out the reasoning in more detail.

either high effort or low effort both before and after the voucher’s introduction: in each case, effort must rise (referring to Propositions 1 and 2) but enrollment remains unchanged. In the pair of equilibria where the public school provides high effort before and low effort after the voucher is introduced, rents must also be lower, by a revealed preference argument: pre-voucher, the public school rejected supplying just the low types as this yielded lower rents than supplying the whole market. But post-voucher, the rents from supplying just the low types must be lower than they were pre-voucher, as  $l$  is increasing in the voucher, by Lemma 3b.

The welfare of households of each type before and after the voucher’s introduction is also clear. High income households always gain from the voucher, regardless of whether they remain in public school, switch to private school once the voucher is introduced, or always choose private school.<sup>21</sup> For low income households, they always remain in public school, and given that public schools are free, their utility depends solely on the public school’s effort choice. In the case where public school effort falls, so low type welfare declines.

## 5 Targeting

The analysis so far has assumed that vouchers are available to all households, as with Proposition 174, the 1993 voucher initiative in California. Alternative proposals involve vouchers targeted at low income families.

Extending the model presented in Section 3 to the case where the voucher is targeted solely at the low income types is straightforward. As only the low types receive the voucher,  $h$  is no longer a function of  $\nu$ . Hence the boundary function becomes

$$x^b(\nu) = \frac{h^\psi - (l(\nu))^\psi}{V - (l(\nu))^\psi}. \tag{15}$$

It is straightforward to demonstrate

**Proposition 5:** *The boundary  $x^b$  shifts down once the voucher is introduced.*

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<sup>21</sup>In the former case, high type utility must increase as public school quality rises (as in Proposition 1). In the case of switching, the private school option post-voucher yields higher utility than the public school option pre-voucher; otherwise, the public school would be able to preempt this. For households always choosing private school, high type utility is increasing in the voucher.

**Proof:** Differentiate  $x^b$  given by (15) with respect to  $\nu$  yielding

$$\frac{\partial x^b}{\partial \nu} = -\frac{(V - h^\psi)}{\left(V - (l(\nu))^\psi\right)^2} \psi l^{\psi-1} \frac{\partial l}{\partial \nu}. \quad (16)$$

This expression is negative, given Lemma 3b and Assumption C. **Q.E.D.**

This proposition eliminates the possibility that public schools start by supplying high and then switch to providing low effort. The rationale is clear: supplying low effort becomes less attractive as the voucher rises because the voucher raises required effort for enrolling low types. Thus fewer high types are needed to make supplying high effort optimal.

The significance of this proposition is that targeting overturns the result that the voucher can reduce school productivity in certain communities. In addition, there is a range of communities for which it is now optimal for the public school to switch from supplying just low effort to high effort following the voucher's introduction.<sup>22</sup>

## 6 Tracking

In this model, there are no ability differences. But tracking based on household type could emerge in public schools, serving as a way of making school quality excludable and thus as a rent-extraction device.

To explore the productivity implications of tracking, we introduce a tracking parameter  $t \in [\underline{t}, 1]$ , which is defined as the proportion of the high effort level enjoyed by the low types. (Here,  $\underline{t}$  denotes that  $t$  value for which  $\underline{t} \times h = l$ .) When  $t = 1$ , low and high types in the same school are supplied the same (high) effort level, consistent with there being perfect mixing; when  $t = \underline{t}$ , the low types are supplied effort level  $l$  at the same time as the high types receive  $h$ . In terms of the previous framework, for a given  $t$  we can write the rents earned by the public school when it tracks as:

$$R(h; t) = V - xh^\psi - (1 - x)(t \times h)^\psi = V - \left(x + (1 - x)t^\psi\right)h^\psi. \quad (17)$$

Suppose the public school faces the choice between, on the one hand, perfect tracking and supplying everyone or versus supplying just the low types on the other. Tracking will be

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<sup>22</sup>An anonymous referee noted the following interesting possibility: under targeting, where the low types are similar to the high types ( $\mu$  is close to 1), so the effort required to supply these low types post-voucher can exceed  $h$ . In this case, after the voucher's introduction, the public school may choose to supply just the high types, the low types exiting to the private sector.

chosen if  $V - (x + (1 - x)t^\psi)h^\psi > (1 - x)(V - l^\psi)$ , implying that

$$x > \frac{(t \times h)^\psi - l^\psi}{V - l^\psi - (1 - t^\psi)h^\psi}. \quad (18)$$

It is clear that perfect tracking always dominates supplying just the low types. Using the fact that  $\underline{t} \times h = l$ , the numerator in (18) equals zero. As long as  $x > 0$  and  $V > h^\psi$ , the public school makes additional rents on each high type student enrolled, so perfect tracking must dominate.

Now consider the introduction of a voucher. With perfect tracking,  $x^b = 0$  for all  $\mu$ , before or after the voucher's introduction. Thus regardless of whether a voucher has been introduced, perfect tracking will dominate supplying just the low types. In the light of this, we can state

**Proposition 6:** *With perfect tracking, the introduction of a voucher always causes productivity to rise for both low and high types.*

**Proof:** The low types are now always supplied with effort level  $l$  pre- or post-voucher, and from Lemma 3b, this is strictly increasing in the voucher. Now, there are no spillovers from high to low types. In addition, it is never optimal to switch from supplying both low and high to supplying just low types, the mechanism by which quality fell in the earlier analysis.

**Q.E.D.**

In short, if low and high types are fully segregated within-school, the low types cannot benefit from the high quality demanded by the high types prior to the introduction of the voucher. This prevents the low types from suffering a quality reduction once the voucher is introduced. However, while perfect tracking avoids the perverse result, the low types *are* given a poorer quality education prior to the introduction of the voucher than they would receive without tracking, at least in the case where both types are served in the same school.

## 7 Extensions

This section considers several extensions.

The main analysis has abstracted from financing issues, and it is worth considering how an explicit treatment of these would affect the main results. Suppose that each public school is viewed as operating in its own district, and the local jurisdiction faces a balanced budget constraint: all spending on the public school and any voucher must be financed by a tax on income. For convenience, suppose the tax rate adjusts to bring about budget balance.

With reference to the regions in Figure 2, it is clear that there will be no effect in Region 1. Here, both high and low types enroll in public school before and after the introduction of the voucher, so there is no change in government expenditure. In Region 2, public school quality will fall. This is because the high types choosing private school are now subsidized, requiring an increase in the tax rate. Reservation quality for the low types will now be lower, allowing the public school to cut effort. However, Region 3 is likely to shrink. As the high types switch to private school, so expenditures on public school and the voucher will fall, allowing a reduction in the tax rate. This will raise the reservation quality of the low types via their budget constraint, meaning that the saving in effort cost will be smaller than previously.

This latter effect will tend to weaken the ‘falling quality’ result. Just how strong this offsetting effect will be depends on how large a reduction in the tax rate results from the introduction in the voucher. We can envisage circumstances in which the change in the tax rate will be negligible, preserving the main perverse effect - for instance, when only one public school occupies Region 3 but the tax base includes many other communities.

The basic model adopts a constant returns to scale cost function, both because this is supported by empirical evidence and simplifies the analysis. If marginal costs decline with increases in enrollment, this provides a new incentive not to cut enrollment by allowing high types to leave when the voucher is introduced. If reductions in enrollment cause costs to rise substantially, then the perverse result could again be overturned.

While identifying the productivity effects of increased competition is difficult in practice, the model has empirical relevance to that policy-relevant task. It provides testable implications as to the circumstances under which vouchers and increases in competition more generally will be productivity-enhancing. If each public school is very homogeneous in terms of the quality thresholds of households attending it, then perverse effects on productivity are unlikely. Where public schools serve more heterogeneous populations, then it becomes possible that public schools opt to move downmarket when competition increases. This possibility is worth investigating empirically. It is interesting to note that a recent study of the effects of Chile’s voucher program (see Hsieh and Urquiola (2002)) finds that the voucher’s introduction did lead households from higher socioeconomic groups to exit public schools, while lower income households remained behind. Measured public school performance also declined in areas where the exodus was highest. This pattern of sorting is consistent with the type of effort setting mechanism analyzed in the current paper.

## 8 Conclusion

This paper has provided the first formal analysis of the effects of an increase in competition on public school incentives to raise productivity. It shows that, even when a voucher improves outside options for all, it can still become optimal for public schools to reduce effort (equivalent to a reduction in productivity) and lower school quality in response to the increase in competition. More generally, the productivity effects of a voucher are likely to be non-uniform, varying systematically according to household and community characteristics, features of the production technology, and the form of the voucher itself.

The results draw attention to a new mechanism whereby household heterogeneity in the presence of demand spillovers can lead to perverse incentive effects when the degree of competition changes. The perverse productivity effects are likely to be robust: the underlying mechanism requires only some heterogeneity across households within a given community and less than complete tracking on the basis of type, conditions that are likely to hold in a variety of communities and their public schools. This mechanism is likely to apply to public sector supply more generally.

From a policy perspective, the analysis shows that it is by no means automatic that stronger market forces alone must improve incentives. This is at odds with the prescription that greater parental choice will solve the public school performance problem.<sup>23</sup> Instead, in some settings, additional regulations may be needed. The paper's findings also have relevance to the issue of voucher design. The introduction of a uniform voucher creates incentives in some communities for public schools to reduce quality, especially if public schools face steeply increased costs in order to retain market share. A voucher targeted at low-income households avoids the perverse outcome.

The incentive mechanism studied in this paper is likely to have effects distinct from those due to conventional peer effects, even if the two are difficult to separate in practice. By abstracting from peer effects, the analysis makes it easier to generate the result that public school quality rises following the introduction of a voucher. But school quality is not just a function of school effort, and vouchers are likely to induce greater sorting and lead to a loss of favorable peer effects in public schools. Thus even in communities where productivity increases, any productivity gain may be offset by quality reductions due to greater sorting.<sup>24</sup>

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<sup>23</sup>In the words of Chubb and Moe (1990), for instance, "Choice is a panacea."

<sup>24</sup>Our analysis also demonstrates that the standard dichotomy between efficiency and sorting effects is hard to

## Appendix A

This appendix demonstrates that the ‘falling quality’ result still arises when moving to a model with continuous types, as opposed to two types. Starting from the rent function considered in the text,

$$R(e, \nu) = (V - C(e))s(e, \nu), \quad (19)$$

now suppose that the market share function is continuous in both  $e$  and  $\nu$ . Optimal effort  $e^*(\nu)$  satisfies the condition that marginal costs and benefits should be equalized, or

$$(V - C(e)) \frac{\partial s(e, \nu)}{\partial e} - \frac{\partial C(e)}{\partial e} s(e, \nu) = 0. \quad (20)$$

The second-order condition holds assuming the strict concavity of  $s$  and the strict convexity of  $C$  in effort. Totally differentiating (20) with respect to both  $e$  and  $\nu$  and rearranging, we have

$$\frac{de^*}{d\nu} = \frac{-\left(V - C(e)\right) \frac{\partial^2 s(e, \nu)}{\partial e \partial \nu} + \frac{\partial C(e)}{\partial e} \frac{\partial s(e, \nu)}{\partial \nu}}{\left(V - C(e)\right) \frac{\partial^2 s(e, \nu)}{\partial e^2} - \frac{\partial^2 C(e)}{\partial e^2} s(e, \nu) - \frac{\partial C(e)}{\partial e} \frac{\partial s(e, \nu)}{\partial e}}. \quad (21)$$

The sign of the total derivative depends on the sign of  $\frac{\partial^2 s(e, \nu)}{\partial e \partial \nu}$ : if this is negative enough, then the numerator will be positive, making the whole expression negative. In this case, the slope of the enrollment function (which is increasing in effort) becomes flatter as the voucher is increased. As more effort is costly, the net benefits of higher enrollment relative to lower enrollment fall, creating incentives to reduce effort (and enrollment).

## Appendix B

This appendix sets out an example based on the Cobb-Douglas utility function  $U = c^\alpha Q^{1-\alpha}$ .<sup>25</sup> Assuming  $Q(e) = e^\omega$ ,  $\omega < 1$ , we can solve explicitly for the threshold effort levels  $l$  and  $h$ , and in turn the boundary between the high and low effort regions. Noting that  $l$  is in two portions, the general expression for the boundary is:

$$x^b(\nu) = \begin{cases} \frac{\left(\alpha^{\frac{1-\alpha}{1-\alpha}} (1-\alpha) y^{-\frac{1-\alpha}{1-\alpha}} (y+\nu)^{\frac{1}{1-\alpha}}\right)^{\frac{\psi}{\omega}} - \nu^{\frac{\psi}{\omega}}}{V - \nu^{\frac{\psi}{\omega}}} & \text{if } \mu \leq \underline{\mu}(\nu) \\ \frac{\left(\alpha^{\frac{1-\alpha}{1-\alpha}} (1-\alpha) y^{-\frac{1-\alpha}{1-\alpha}} (y+\nu)^{\frac{1}{1-\alpha}}\right)^{\frac{\psi}{\omega}} - \left(\alpha^{\frac{1-\alpha}{1-\alpha}} (1-\alpha) (\mu y)^{-\frac{1-\alpha}{1-\alpha}} (\mu y + \nu)^{\frac{1}{1-\alpha}}\right)^{\frac{\psi}{\omega}}}{V - \left(\alpha^{\frac{1-\alpha}{1-\alpha}} (1-\alpha) (\mu y)^{-\frac{1-\alpha}{1-\alpha}} (\mu y + \nu)^{\frac{1}{1-\alpha}}\right)^{\frac{\psi}{\omega}}} & \text{if } \mu > \underline{\mu}(\nu), \end{cases} \quad (22)$$

where  $\underline{\mu}(\nu) = \frac{\alpha}{1-\alpha} \frac{\nu}{y}$ . This expression is used to plot the figures appearing in the main text for different values of the voucher.

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sustain conceptually, given that sorting is likely to have efficiency consequences, partly for the reasons captured in our model.

<sup>25</sup> Full workings are available on request.



The determinants of the sizes of the partial derivatives  $\frac{\partial l}{\partial \nu}$  and  $\frac{\partial h}{\partial \nu}$  can be found by differentiating the expressions for  $l$  and  $h$  with respect to  $\nu$ , evaluating these at a certain initial value of the voucher  $\nu_o$ . Given

$$h(\nu_o) = \left( \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) y^{-\frac{\alpha}{1-\alpha}} (y + \nu_o)^{\frac{1}{1-\alpha}} \right)^{\frac{1}{\omega}}, \quad (23)$$

so

$$\frac{\partial h}{\partial \nu} = \frac{1}{\omega(1-\alpha)(y + \nu_o)} h(\nu_o). \quad (24)$$

Similarly,

$$l(\nu_o) = \left( \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) (\mu y)^{-\frac{\alpha}{1-\alpha}} (\mu y + \nu_o)^{\frac{1}{1-\alpha}} \right)^{\frac{1}{\omega}}, \quad (25)$$

and

$$\frac{\partial l}{\partial \nu} = \frac{1}{\omega(1-\alpha)(\mu y + \nu_o)} l(\nu_o). \quad (26)$$

Supposing the initial value of the voucher is zero, or  $\nu_o = 0$ , the condition for the boundary  $x^b$  to shift up in Lemma 4 requires that

$$\mu^{\frac{\psi}{\omega}-1} < \frac{V - l^\psi}{V - h^\psi}. \quad (27)$$

This holds, as the left-hand side of the inequality is strictly less than one, and the right-hand side is strictly greater than one.

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