ECO 336Y Handout 7 (10/16/01)

Calculus practice questions (some of which are relevant for econometrics)

A. In each case, what is $\frac{dh}{dx}$? [Just treat the symbols as pure algebra.]

i $h = x^2$

ii $h = x^a$

iii $h = 2x^a$

iv $h = a_0 + a_1 x$

v $h = a_0 + a_1 x + a_2 x^2$

vi $h = a_0 + a_1 x + a_2 (E \times x)$ [that is, E multiplied by x]

B. In econometrics, we usually deal with a sample of observations. Suppose our sample consists of N people. For each person i, we observe his/her

hours of work (h_i)
hourly wage (w_i)
education level (E_i).

So rather than hours of work taking on any value, we just observe the hours worked by the N people in our sample.

Consider the following relations between hours and the determinants of hours, and ε_i is a random error term.

a/ $h_i = \alpha_0 + \alpha_1 w_i + \varepsilon_i$

b/ $h_i = \alpha_0 + \alpha_1 w_i + \alpha_2 E_i + \varepsilon_i$

c/ $h_i = \alpha_0 + \alpha_1 w_i + \alpha_2 E_i + \alpha_3 (E_i \times w_i) + \varepsilon_i$

d/ $h_i = \alpha_0 + \alpha_1 w_i + \alpha_2 E_i + \alpha_3 w_i^2 + \varepsilon_i$

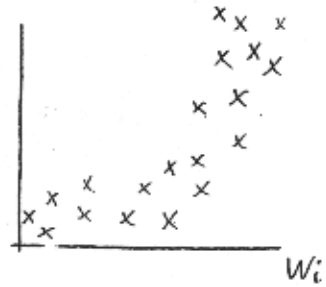
e/ $h_i = \alpha_0 (E_i \times w_i^2) + \varepsilon_i$

In each of cases a/ - e/, work out:

i the intercept of the line relating h_i to the hourly wage w_i. [In other words, think of h_i being on the vertical and w_i being on the horizontal axis.]

ii the slope of the line relating h_i to the wage w_i. [Here, you will have to use a bit of calculus — see Question A.]

C. Suppose we have data on hours of work plotted against the hourly wage, as in the diagram.



i Why might fitting a straight line through these points be less than ideal?

ii Suppose, instead, we fit the following quadratic model

$$h_i = \alpha_0 + \alpha_1 w_i + \alpha_2 w_i^2 + \varepsilon_i$$

where the α's are the 'true' parameters and ε_i is a random error. If our estimates, designated with a "^" symbol, are

$$\hat{\alpha}_0 = 0$$
$$\hat{\alpha}_1 = 2$$
$$\hat{\alpha}_2 = 1$$

then what is the slope of the fitted line in each of the following three cases in (w_i, h_i) space — that is, with w_i on the horizontal and h_i on the vertical axis?

a/ $w_i = 2$
b/ $w_i = 4$
c/ $w_i = 10$

[Hint: work out the general expression for $\frac{dh_i}{dw_i}$, then plug in specific values of w_i.]

iii Based on your answer to part ii, why might estimating a quadratic specification be better than a linear specification, given the nature of the data? [Hint: to answer this well, you should say what happens to the 'response' of hours to wages as the wage goes up.]

D. Suppose our true model is now

$$h_i = \alpha_0 + \alpha_1 w_i + \alpha_2 E_i + \varepsilon_i,$$

where E_i is the person's education and takes on two values:

$$E_i^L = 1$$
$$E_i^H = 5.$$

Suppose also that we estimate $\hat{\alpha}_0 = 1$, $\hat{\alpha}_1 = 3$, $\hat{\alpha}_2 = 1/2$.

i Given these estimates, what is the predicted value for the intercept in (w_i, h_i) space?

ii Use the estimates to predict hours, h_i, for <u>low</u>-education individuals

a/ if $w_i = 4$
b/ if $w_i = 7$.

iii If we didn't have information on education levels, what would be the problem with our predictions of the hours of low education individuals?