ECO 336Y

Solutions for Third Set of Sample Questions

Questions A. and B. below are designed to give you some calculus practice. Questions C. and D. will give you an idea if the sorts of questions that might appear on a term test or final exam.

A.

i $\frac{dh}{dx} = 2x$
ii $\frac{dh}{dx} = ax^{a-1}$
iii $\frac{dh}{dx} = 2ax^{a-1}$
iv $\frac{dh}{dx} = a_1$
v $\frac{dh}{dx} = a_1 + 2a_2x$
vi $\frac{dh}{dx} = a_1 + a_2E$

B. In this question, we provide the intercepts and slopes of the corresponding lines in (w_i, h_i) space.

a/ $h_i = \alpha_0 + \alpha_1 w_i + \epsilon_i$
i The intercept is given by α_0.
ii The slope of the line is given by α_1.
b/ $h_i = \alpha_0 + \alpha_1 w_i + \alpha_2 E_i + \epsilon_i$
i The intercept is given by $\alpha_0 + \alpha_2 E_i$.
ii The slope of the line is given by α_1.
c/ $h_i = \alpha_0 + \alpha_1 w_i + \alpha_2 E_i + \alpha_3 E_i w_i + \epsilon_i$
i The intercept is given by $\alpha_0 + \alpha_2 E_i$.
ii The slope of the line is given by $\alpha_1 + \alpha_3 E_i$.
d/ $h_i = \alpha_0 + \alpha_1 w_i + \alpha_2 E_i + \alpha_3 w_i^2 + \epsilon_i$
i The intercept is given by $\alpha_0 + \alpha_2 E_i$.
ii The slope of the line is given by $\alpha_1 + 2\alpha_3 w_i$.
e/ $h_i = \alpha_0 E_i w_i^2 + \epsilon_i$
i The intercept is given by 0.
ii The slope of the line is given by $2\alpha_0 E_i w_i$.

C.

i Fitting a straight line would not work well in this situation because the underlying relationship between wages and hours given in the diagram is non-linear. Specifically, the line that minimizes the sum of squared residuals (the 'least squares' line) would tend to underpredict hours at low values of the wage, overpredict at intermediate values of the wage, and underpredict again at high values of the wage. We would do better trying to fit a quadratic curve, as this would allow the slope to vary with the wage rate, getting steeper as the wage rate rose.

ii Substituting the estimates into the quadratic model, we have
$h_i = 2w_i + w_i^2 + \epsilon_i$.
The general expression for the slope in this case is
$\frac{dh_i}{dw_i} = 2 + 2w_i$.
Substituting the different values for w_i, we have
a/ $\frac{dh_i}{dw_i} = 6$
b/ $\frac{dh_i}{dw_i} = 10$
c/ $\frac{dh_i}{dw_i} = 22$
in each of the three cases.
iii The quadratic model allows the slope of the line in (w_i, h_i) space to vary for different wage levels. Given the estimates obtained in part i above, it appears that the slope *does* vary, as the estimated coefficient on α_2 is non-zero. (NB note that it would be constant if α_2 were zero. Thus a quadratic specification will allow us to do a better job of fitting the data than a linear specification. As shown in part ii, the slope of the line increases as the wage rate rises, which is what we would like, given the nature of the data.

D.
i First, substitute in the estimates, to give
$h_i = 1 + 0.5E_i + 3w_i + \epsilon_i$.
The predicted value of the intercept is then 1.5 for low-education and 3.5 for high-education individuals.
ii For low-education individuals, $E_i = 1$. Subsituting this into the expression in part i, we have
$h_i = 1 + 0.5 + 3w_i + \epsilon_i$.
This expression can now be used to predict hours worked, given wages. We assume (as is common) that the error term is random noise, with a mean of zero. Thus it can be ignored in making predictions - our best guess is that it is zero.
a/ If $w_i = 4$, then predicted $h_i = 13.5$.
b/ If $w_i = 7$, then predicted $h_i = 22.5$.
iii Note that the intercept in (w_i, h_i) space is given by
$\alpha_0 + \alpha_2 E_i$,
or with the estimates given in the question, the intercept is
$1 + 0.5E_i$.
This implies that the intercept varies with education, being lower for low-education individuals. (Aside: the interpretation of this is that low-education individuals work less than high-education individuals, whatever the level of the wage.) If we did not have information on an individual's education levels, and if both low- and high-education individuals were observed in our data (though not, by assumption, their education levels), then we would tend to *over*predict the hours worked by low-education individuals.