University of Toronto
Economics 336Y – Public Economics (Taxation)

Practice exercises #1

1. For the following economies, find an equation for the set of feasible, Pareto efficient allocations and graph it.
   (a) Society has 100 units of a private good available to share between two consumers with utility functions
   $$u_A(x_A) = \sqrt{10x_A}$$
   $$u_B(x_B) = \sqrt{20x_B}$$
   (b) There are two private goods which are in fixed supplies X and Y, and two consumers A and B with utility functions
   $$u_A(x_A, y_A) = x_A$$
   $$u_B(x_B, y_B) = \min\{x_B, y_B\}$$

(a) $x_A + x_B = 100$.

(b) Given utility functions, an allocation is feasible and efficient if and only if $x_A + x_B = X$ and $y_B \geq x_B$ (any bundle for B that is below the 45-degree line in (x, y)-space can be improved by giving more y to B, which B would value and A would not) . Combining these two conditions and $u_A = x_A$, an allocation is Pareto efficient if and only if it satisfies
$$x_B = X - x_A$$
$$Y \geq y_B \geq X - x_A$$

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2. The *utility possibility frontier* is defined as the set of maximum utility levels for all consumers that is feasible in the economy, given resource constraints and preferences. For a two-consumer economy, it is defined formally as the solution to:
   $$u_A^*(\bar{u}_B) = \max u_A(x_A) \text{ subject to } u_B(x_B) \geq \bar{u}_B \text{ and } (x_A, x_B) \text{feasible.}$$
   For each of the economies above, find an equation for the utility possibilities frontier, and graph it.

(a) $2u_A^2 + u_B^2 = 2000$

(b) From the answer to #1, $Y \geq y_B \geq x_B$ at an efficient allocation, so $X - x_A = x_B = u_B \leq Y$. Therefore, the UPF is defined by
$$u_A = X - u_B$$
$$u_B \leq Y$$

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September 22, 2009

3. An economy has two commodities and two consumers with preferences

$$u_A(x_A, y_A) = 2\log x_A + y_A$$
$$u_B(x_B, y_B) = 3\log x_B + y_B$$

Society has an endowment of one unit of each good in total so that $x_A + x_B = y_A + y_B = 1$. Calculate the UPF for this economy and graph it. Why does it have such a special shape, in spite of diminishing marginal utility?

Note the solution must have $x_A = 2/5$ and $x_B = 3/5$ so that redistribution is by reallocating Y only. Formally can express the allocation as a function of $\bar{u}_B$ as in the definition but this is not necessary to determine that the UPF is a straight line with slope -1 and

$$(u_A, u_B) = (2\log(2/5) + 1, 2\log(3/5))$$

is on the UPF. (Note that if we assume $y_i \geq 0$ then the UPF "stops short" of the axes, since $u_A \geq 2\log(2/5)$ and $u_B \geq 3\log(3/5)$.) It is a straight line because of quasi-linear preferences: there is a unique efficient allocation of commodity X, and commodity Y can be allocated between consumers to achieve any profile of utilities along the linear UPF, witout diminishing marginal utility.

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4. A worker's labour supply function is given by

$$H(w) = 100w$$

where H is hours worked per year and W the after-tax hourly wage.

(a) Calculate the worker's Marshallian consumer surplus (or if you prefer "producer surplus") as a function of the wage rate.

(b) Now suppose that the worker can earn a pre-tax wage of \$20 per hour, but must pay a proportional wage income tax of 25 per cent. Calculate the revenue raised by the tax and the Harberger triangle measure of its excess burden. Draw a graph illustrating the results.

(c) Do you think the assumed labour supply function is realistic? Justify your answer.

(a)

$$CS = 50w^2$$

(b)

$$TR = 5 \times 1500 = 7500$$
$$EB = \frac{1}{2} \times 5 \times 500 = 1250$$

(c) Empirical studies normally find that the Marshallian wage elasticity of labour supply at the intensive margin is near zero. The elasticiy here is

$$\epsilon = \frac{w}{H}\frac{\partial H}{\partial w} = \frac{100w}{H} = 1$$

which is very large. But this may better describe the *compensated* elasticity of labour supply – see next Lecture.
