

University of Toronto
Economics 336Y – Public Economics

Practice Problems #2

1. Let compensated demand for x be a general linear function of price p :

$$x = a - bp$$

where a, b are fixed, and let the marginal cost be fixed at c . Calculate an approximate formula the excess burden of a specific tax t on x . (Hint: Draw the graph.) How does the excess burden change with a and b ? How does a change in b affect the elasticity of demand for x ?

2. A taxpayer has utility function $U(x, h) = \sqrt{x} - h$ where h is hours of labour supply and x is consumption. The taxpayer earns a wage of \$4 per hour worked.

- (a) Suppose that the government imposes a proportional tax at rate t on labour income, so that the taxpayer's budget constraint is

$$x = (1 - t)4h$$

Solve for the optimal labour supply and consumption (h^*, x^*) as a function of t . What is the revenue raised by the tax? What is the taxpayer's level of utility, as a function of t ?

- (b) Suppose that the government imposes a lump-sum tax T so that the taxpayer's budget constraint is

$$x = 4h - T$$

Solve for the optimal labour supply and consumption (\hat{h}, \hat{x}) as a function of R in this case. Calculate the value of T (as a function of t) that equivalent for taxpayer utility to the proportional tax of the last part.

- (c) Based on your answers, compute the excess burden of any proportional tax rate t . Could you have obtained this answer by integrating above the labour supply curve? Explain why or why not.
3. A person spends E years in school and then $R - E$ years working before retirement age. If the person choose E years of education, she receives pre-tax labour income $W(E)$ for each year she is working thereafter, where $W(E)$ is an increasing, concave function. The government levies a proportional income tax at rate τ on income, and pays a benefit of B to everyone, regardless of whether they are working or not. After-tax income in year t is therefore

$$Y(E, t, B, \tau) = \begin{cases} B & \text{if } t \leq E \\ B + (1 - \tau)W(E) & \text{if } E < t \leq R \end{cases}$$

- (a) Suppose that people chose E to maximize lifetime total income

$$U(E) = \sum_{t=1}^R Y(E, t, B, \tau)$$

Write down the first-order condition for the optimal E^* and explain what each part means. How does it depend on the tax system? Provide an intuition for your answer.

- (b) Now suppose people incur an additional "psychic" cost K for each year they are in school, relative to years they are working, and they choose E to maximize lifetime income minus psychic costs. How does this change your answers to part (a)?
- (c) Politicians (usually conservative ones) sometimes argue that income taxes discourage people from pursuing education and entering high-wage occupations. Do you agree or disagree?

4. A consumer supplies labour and demands goods X and Y . Let the price of labour be fixed at one. Let the compensated demand functions for goods X and Y be

$$X = \frac{1}{p_X^2(1/p_X + 2/p_Y + 1)^2}$$

$$Y = \frac{4}{p_Y^2(1/p_X + 2/p_Y + 1)^2}$$

where (p_X, p_Y) are the after-tax prices. If the optimal tax rate on good Y is 25 per cent of the pre-tax price, then what is (approximately) the optimal tax rate on good X ?