**University of Toronto**
**Economics 336Y – Public Economics**

**Problem Set #3**

1. Equating supply and demand, the equilibrium price is

$$\frac{8}{p^*} = 2(p^* - t)$$

or

$$(p^*)^2 - 2tp^* - 8 = 0$$

which implies that

$$p^* = \frac{t}{2} + \frac{\sqrt{4t^2 + 64}}{4}$$

since only the positive root makes sense for a price.

This means that $p^* = 2$ when $t = 0$ and $p^* = 4$ when $t = 3$. In this sense, two-thirds of the incidence of the tax is borne by consumers.

2. The optimal demand for capital is the one which maximizes the wage (since there are no pure profits under constant returns to scale, and since labour supply is inelastic). Therefore

$$K^*(r) = 10 - r$$

When there is no tax $r = r_w = 4$, $K^* = 6$, and $W(K^*) = 18$. When a tax of $t = 2$ is imposed in a small open economy the pre-tax price of capital rises to $r = 6$: the tax is shifted to domestic workers. Therefore in the new equilibrium $K^* = 4$, $W(K^*) = 8$, and $tK^* = 8$, implying the after-tax wage falls to 16. Since the tax is shifted to domestic workers anyway it cannot create any net gains. In fact it creates losses for domestic workers because of the excess burden of the tax (the Harberger triangle).

3. (a) By direct substitution.
   (b) $C_2 = (1+r)(Y - C_1) = f(C_1)$ implies

$$V(C_1) = U(C_1, f(C_1)) = \log C_1 + \log(1+r) + \log(Y - C_1)$$

   (c) $\max V(C_1)$ implies first-order condition

$$V'(C_1^*) = \frac{1}{C_1^*} - \frac{1}{Y - C_1^*} = 0$$

   or $C_1^*/Y = 1/2$, implying

$$\frac{S}{Y} = \frac{Y - C_1^*}{Y} = \frac{1}{2}$$

4. (a) In this case

$$V(C_1) = 2\sqrt{C_1} + 2\sqrt{1+r}\sqrt{Y - C_1}$$

   so the first-order condition for $C_1^*$ is

$$C_1^{*\,-1/2} = (1+r)^{1/2}(Y - C_1^*)^{-1/2}$$

   or

$$\frac{C_1^*}{Y} = \frac{1}{2+r} \implies \frac{S^*}{Y} = 1 - \frac{C_1^*}{Y} = \frac{1+r}{2+r}$$

   (b) Yes, since $S^*$ is increasing in $Y$.



3-a.tex
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