University of Toronto
Economics 336Y – Public Economics

Problem Set #4

1. Adam likes to talk loud in restaurants, but loud talking puts Betty off her food. Specifically, let X_i denote the quantity of restaurant food consumed by consumer $i = A, B$, and let L_A denote the quantity of loud talking consumed by consumer A. B's utility function is

$$U_B(X_B, L_A) = X_B - c(L_A)^2$$

where $c > 1$ is a parameter. A's utility function is

$$U_A(X_A, L_A) = L_A + X_A$$

Assume the amount of restaurant food available for consumption is fixed at R in this exchange economy, and that loud talking has no resource cost. The economy-wide feasibility condition is then

$$X_A + X_B = R$$

Find an algebraic expression for the utility possibilities frontier $U_B^*(\bar{U}_A)$, and graph it. Which levels of L_A are consistent with Pareto efficiency in this case? How does L_A vary with the planner's choice of $\bar{U}_A$?

2. There are two consumers A and B with utility functions over public and private consumption:

$$u_i(x_i, G) = w_i \log G + x_i \quad (i = A, B)$$

where w_i is a parameter measuring willingness to pay for the public good, and $w_A \geq w_B$. The cost of the public good is $C(G) = \gamma G$ and total resources for public and private consumption are given by Y. What is the MRT and the MRS of the two consumers? Calculate the optimal level of public goods provision from the Samuelson condition. Does this level depend on the distribution of income chosen by the planner? Explain why or why not. Finally, draw a graph of demand and supply curves that illustrates the result.

3. Repeat the previous question for the case where the utility functions are

$$u_i(x_i, G) = w_i\sqrt{G} + x_i \quad (i = A, B)$$

and $MRT = 1$.

4. Now suppose that consumers' utility functions are:

$$u_i(x_i, G) = w_i \log G + \log x_i \quad (i = A, B)$$

but everything else is the same as in the preceding question. Show that there is a unique efficient level of G consistent with the Samuelson conditions if $w_A = w_B$. Argue that, if $w_A > w_B$, then a move along the UPF that causes a redistribution from B to A is associated with an increase in the optimal G.

5. Lindahl prices are defined as the share of the cost of the public good paid by each taxpayer that would induce all of them to demand the same (efficient) quantity of the public good. For the utility functions in the previous question, suppose that $w_A = 3$ and $w_B = 4$, and suppose that A's after-tax income is 100 while B's is 50. Calculate the Lindahl prices for taxpyers A and B.

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December 1, 2009