University of Toronto
Economics 336Y – Public Economics

Problem Set #5

1. Ahmed and Bob are the only two residents on a street in a new subdivision, who must decide how much to spend on street lighting, a pure public good. Suppose that they have identical preferences for wattage of street lighting G and composite private consumption x_i given by the utility function

$$u_i(G, x_i) = 50 \log G + x_i \qquad (i = A, B)$$

Street lighting costs 1 unit of private consumption per Watt, and each has the same total income, equal to Y.

(a) Calculate the efficient level of spending of street lighting. Is this level unique, or does it depend on the distribution of utility between Ahmed and Bob? Explain why or why not.
(b) Now suppose that Ahmed and Bob independently decide how much to spend on street lighting, taking the amount supplied by the neighbour as given. Calculate the total amount of lighting that is supplied in equilibrium, and contrast it with your answer to part (a).
(c) Now suppose Bob is richer than Ahmed: $Y_B > Y_A$. How would total spending on street lighting from part (b) change? Would Ahmed contribute a positive amount or not?
(d) Now suppose that there were ten residents of the street, rather than just two. How would the efficient and equilibrium levels of wattage from parts (a) and (b) change? Comment on the difference: do you think the equilibrium prediction of this theory is realistic, and what extensions to the model would change this result?

(a) $\sum MRS_i = 2 \times (50/G^{eff}) = 1$ implies $G^{eff} = 100$.
(b) $MRS_i = 50/G^{eq} = 1$ implies $G^{eq} = 50$.
(c) The previous answer shows G^{eq} is independent of private incomes. In both cases, however, individual contributions are not uniquely determined: Ahmed might contribute or not.
(d) We now have $G^{eff} = 1000$ and $G^{eq} = 50$. The efficient provision level is proportional to number of citizens, while the equilibrium level is independent of the number of citizens. This is a rather extreme version of the free-rider problem. The model would change if: (i) there were a "warm glow effect" of private contributions; and (ii) if we considered different preferences that caused some (e.g. low-income) consumers to reduce their contributions to zero as the number of contributors became large.

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2. The federal government is trying to decide how many hours G of new Canadian programming to produce for broadcast on CBC television next year. Surveys have determined that only three people watch the CBC in total, two with demand curves given by

$$G_i = 15 - p_i \quad (i = 1, 2)$$

where p_i is the price the individual pays for each hour of programming; and the third with demand curve

$$G_3 = 100 - p_3.$$

(a) Calculate the *Pareto efficient* level of new programming G^*, when the marginal cost of an hour of new programming is 100 units of private consumption per hour (that is, the marginal rate of transformation is 100).

(b) Suppose that the government instead holds a referendum among the three CBC viewers to determine how much new programming will be provided. If each voter expects to pay an equal share (one-third) of the total cost through the tax system, what do you expect will be the *majority voting equilibrium* level $\hat{G}$ of new programming? Explain why G^* and $\hat{G}$ differ, and comment on the use of democratic decision-making procedures in such cases.

(a) Summing demand curves vertically gives aggregate marginal willingness to pay (marginal rate of substitution) $W(G) = 130 - 3G$. Thus

$$130 - 3G^* = 100$$

or $G^* = 10$.

(b) The tax price for each voter is 33.33 per hour, which exceeds the marginal willingness to pay for the first two consumers. Hence the majority voting equiibrium level of spending is zero. The problem with voting is that it does not allow voter 3 to express the intensity of his/her preference for the public good.

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3. In the suburb of Harristown, 100 motorists each day drive to their jobs in Toronto. There are two highways connecting Harristown to Toronto, and each motorist independently chooses which highway to drive into the city each day. The highways can be crowded at rush hour, and driving time to the city is longer the more cars there are on the highway. If N_1 motorists take Highway 1, then each driver's travel time is

$$T_1(N_1) = 10 + N_1^{1/2}$$

minutes. Highway 2 is located further from the centre of Harristown, and so travel time in minutes on Highway 2 is

$$T_2(N_2) = 20 + N_2^{1/2}$$

if N_2 motorists take Highway 2.

(a) Suppose that each motorist independently chooses a highway to minimize travel time. What must be true about T_1 and T_2 in equilibrium? How many of the 100 motorists take Highway 1 in equilibrium? (Hint: Instead of solving complicated algebraic expressions, try guessing values for N_1.)

(b) Suppose that government can use command-and-control regulation to force motorists to take one highway or the other, and that government wishes to allocate cars to minimize the aggregate travel time of all drivers. Write down the government's optimization problem in terms of N_1 and N_2. (Be sure to include all constraints!)

(c) Show that, at the efficient allocation (N_1^*, N_2^*) of part (b), the following condition is satisfied:

$$T_1(N_1^*) + N_1^* \frac{\partial T_1}{\partial N_1} = T_2(N_2^*) + N_2^* \frac{\partial T_2}{\partial N_2}.$$

The solution to the above expression is approximately $N_1^* = 91$ and $N_2^* = 9$. Why is this outcome different from the equilibrium of part (a)?

(d) Suppose that each motorist valued his or her time at \$1 per minute (\$60 per hour: this is a rich suburb!). Calculate the toll rates for the two highways that would cause motorists to choose the efficient allocation of part (d) themselves, without the use of command-and-control-regulation.

(a) Equilibrium occurs at the point (N_1, N_2) where (average) time on each road is equalized; otherwise, a driver could gain by changing his or her route. Note $T_1(N_1) = T_2(100 - N_1)$ implies $N_1 = 100$.

(b) Government solves

$$\min N_1 T_1(N_1) + (100 - N_1) T_2(100 - N_1)$$

where I have substituted the constraint $N_2 = 100 - N_1$.

(c) Immediate from differentiation of the above objective function. The second term on each side of the condition defining the efficient allocation is the marginal external cost associated with a driver choosing the corresponding highway. (Each driver's travel time rises by $\partial T_i / \partial N_i$ and there are N_i drivers on each road.) Drivers ignore this cost in the unregulated equilibrium. As a result there are too many drivers on the road for which the *fixed cost* (the time taken when there are no other drivers) of travelling is lower.

(d) There are many toll rates (t_1, t_2) which lead to efficiency. In each of them

$$t_1 - t_2 = N_1^* \frac{\partial T_1}{\partial N_1} - N_2^* \frac{\partial T_2}{\partial N_2} \approx 3.27.$$

(For example, government could levy no toll on Highway 2 and a toll of \$3.27 on Highway 1.)

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4. Consider two people, Pollux and Victor. Pollux chooses the level of a damaging activity P, and Victor chooses a level of investment I. The greater the investment I, the greater the damage caused by P. Victor's profits $\pi_v(P, I) = I(2 - I - P)$. Pollux's profits: $\pi_p(P) = P(A - P/2)$ where $2 \geq A \geq 1$.

   (a) What is efficient investment and pollution (I^*, P^*)?

   (b) Suppose that Victor has property rights, and Pollux makes an offer of compensation for pollution which Victor wither accepts or rejects. If the offer is rejected, then no pollution is allowed. How much compensation $C(I, P)$ must Pollux offer Victor to accept a level of pollution P if the investment is at level I?

   (c) Suppose I is chosen before compensation is offered: What is equilibrium investment and pollution $(\hat{I}, \hat{P})$?

Answers:

(a) $I^* = 2 - A, P^* = 2(A - 1)$.

(b) $C = IP$.

(c) If $C = IP$ then Victor solves $\max_I I(2 - I)$ implying $\hat{I} = 1$. Pollux then solves

$$\max_P AP - \frac{1}{2}P^2 - \hat{I}P$$

so $\hat{P} = A - 1$. Notice $\hat{P} < P^*$ and $\hat{I} > I^*$.

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5. The capacity of a subway system is measured by the maximum number of passengers per direction per hour (ppdph) it can carry. Suppose that the capacity of the TTC subway system is fixed at K ppdph. The capital cost of the subway is cK, and operating costs are zero. The TTC has estimated that the inverse demand curve for total trips T on the subway at rush hour is given by the inverse demand function

$$w_r(T) = A_r - T$$

whereas at non-rush hour times it is given by

$$w_n(T) = A_n - T$$

If $A_r > K > A_n$, what price should be charged for trips on the TTC at rush hour? at non-rush hour times? Now suppose that the TTC has chosen capacity optimally. Will revenues be sufficient to cover the TTC's capital costs?

Since marginal cost is zero and there is excess capacity outside rush hour, it is optimal to set the fare then to $f_n = 0$. At rush hour, a fare of zero would result in excess demand, so fares should be used to ration trips among consumers (why use fares instead of queues, by the way?) If a "peak-load fare" of $f_r = w_r(K) = A - K$ is chosen, then supply equals demand at rush hour. For the last part of ther question, observe that the net benefit to society of subway capacity K is

$$B(K) = \frac{1}{2}K^2 + (A_r - K - c)K + \frac{1}{2}A_n^2$$

(Draw the graph and use the Harberger triangle formula to verify this). Therefore the optimal capacity sets $B'(K^*) = 0$ or $K^* = A_r - c$, and the optimal fare at rushhour is $f_r = c$. Revenues are $R = cK^*$ which exactly cover the capital costs of the system.