**University of Toronto**
**Economics 336 – Public Economics**

**Problem Set #2**

1. A steel firm produces smoke as a byproduct of production, and smokes production costs of a neighbouring laundry. The cost functions of the two firms are

$$C_s(y_s) = \frac{1}{2}y_s^2$$
$$C_l(y_l, y_s) = \frac{1}{2}y_l^2 + \frac{1}{3}y_s y_l$$

   where $y_s$ is steel output and $y_l$ is laundry output.

   Assume that both firms sell their output in a competitive market for a price of one. Calculate the level of steel output $\hat{y}_s$ that maximizes the profit of the steel firm; the level of steel output $y_s^*$ that maximizes the joint profits of steel and laundry firms; and the Pigouvian tax on steel output $t^*$ that would cause a private steel firm to choose the efficient output $y_s^*$ as its profit maximizing output.

2. Acid rain pollution is produced from two sources, a power plant and a smelter. Studies show that the marginal cost of reducing acid pollution at the power plant is given by the function

$$MC_1(A_1) = 10 + A_1$$

   where $A_1$ is the units of acid rain reduction (or "abatement") at the power plant; and the marginal cost at the smelter is

$$MC_2(A_2) = 20 + A_2.$$

   Suppose that the initial level of pollution was 120 units, but the government has limited pollution to 70 units by selling pollution permits to the two polluters – so that the total amount of abatement is 50 units.

   Calculate the total demand for permits to pollute at any permit price $p$ per unit of acid rain. If the permit market is competitive, what is the equilibrium price of a permit? How many permits are bought by the power plant, and how many by the smelter? Draw a graph illustrating your answer.

3. There are two consumers $A$ and $B$ with utility functions over public and private consumption:

$$u_i(x_i, G) = w_i \log G + x_i \quad (i = A, B)$$

   where $w_i$ is a parameter measuring willingness to pay for the public good, and $w_A \geq w_B$. The cost of the public good is $C(G) = \gamma G$ and total resources for public and private consumption are given by $Y$. What is the MRT and the MRS of the two consumers? Calculate the optimal level of public goods provision from the Samuelson condition. Does this level depend on the distribution of income chosen by the planner? Explain why or why not. Finally, draw a graph of demand and supply curves that illustrates the result.

4. Repeat the previous question for the case where the utility functions are

$$u_i(x_i, G) = w_i\sqrt{G} + x_i \quad (i = A, B)$$

   and $MRT = 1$.

5. Now suppose that consumers' utility functions are:

$$u_i(x_i, G) = w_i \log G + \log x_i \quad (i = A, B)$$

but everything else is the same as in the preceding question. Show that there is a unique efficient level of $G$ consistent with the Samuelson conditions if $w_A = w_B$. Argue that, if $w_A > w_B$, then a move along the UPF that causes a redistribution from $B$ to $A$ is associated with an increase in the optimal $G$.

6. Lindahl prices are defined as the share of the cost of the public good paid by each taxpayer that would induce all of them to demand the same (efficient) quantity of the public good. For the utility functions in the previous question, suppose that $w_A = 3$ and $w_B = 4$, and suppose that $A$'s after-tax income is 100 while $B$'s is 50. Calculate the Lindahl prices for taxpayers $A$ and $B$.