University of Toronto
Economics 336Y – Public Economics

Problem Set #5

1. Ahmed and Bob are the only two residents on a street in a new subdivision, who must decide how much to spend on street lighting, a pure public good. Suppose that they have identical preferences for wattage of street lighting G and composite private consumption x_i given by the utility function

$$u_i(G, x_i) = 50 \log G + x_i \qquad (i = A, B)$$

Street lighting costs 1 unit of private consumption per Watt, and each has the same total income, equal to Y.

(a) Calculate the efficient level of spending of street lighting. Is this level unique, or does it depend on the distribution of utility between Ahmed and Bob? Explain why or why not.
(b) Now suppose that Ahmed and Bob independently decide how much to spend on street lighting, taking the amount supplied by the neighbour as given. Calculate the total amount of lighting that is supplied in equilibrium, and contrast it with your answer to part (a).
(c) Now suppose Bob is richer than Ahmed: $Y_B > Y_A$. How would total spending on street lighting from part (b) change? Would Ahmed contribute a positive amount or not?
(d) Now suppose that there were ten residents of the street, rather than just two. How would the efficient and equilibrium levels of wattage from parts (a) and (b) change? Comment on the difference: do you think the equilibrium prediction of this theory is realistic, and what extensions to the model would change this result?

(a) $\sum MRS_i = 2 \times (50/G^{eff}) = 1$ implies $G^{eff} = 100$.
(b) $MRS_i = 50/G^{eq} = 1$ implies $G^{eq} = 50$.
(c) The previous answer shows G^{eq} is independent of private incomes. In both cases, however, individual contributions are not uniquely determined: Ahmed might contribute or not.
(d) We now have $G^{eff} = 500$ and $G^{eq} = 50$. The efficient provision level is proportional to number of citizens, while the equilibrium level is independent of the number of citizens. This is a rather extreme version of the free-rider problem. The model would change if: (i) there were a "warm glow effect" of private contributions; and (ii) if we considered different preferences that caused some (e.g. low-income) consumers to reduce their contributions to zero as the number of contributors became large.

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2. Adnan and Biff have preferences for a public good G and a private good $x_i, i = A, B$ given by

$$U_A(G, x_A) = 20\sqrt{G} + x_A$$
$$U_B(G, x_B) = 10\sqrt{G} + x_B$$

If Adnan and Biff voluntarily and simultaneously choose a private contribution from their incomes to finance the public good, and the cost of each unit of the public good is one unit of the private good, then what is the Nash equilibrium level of public goods provision?

B contributes 0 and A chooses G such that

$$\frac{1}{2} 20 G^{-1/2} = 1$$

So $G = 100$.

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3. Alice and Bo get utility from private goods consumption $x_i, i = A, B$ and from music concerts C. Both have identical utility functions

$$U_i(x_i, C) = 3 \log x_i + \log C$$

Both have income $y_i = 70$, and holding a concert costs one unit of income. Alice and Bo independently choose how many concerts to finance from their own private incomes.

(a) Calculate the Nash equilibrium number of concerts $\hat{C}$, and compare it to the level C^* implied by the Samuelson conditions.

(b) Suppose that the government chooses to finance 15 concerts in addition to those voluntarily provided, and it taxes Alice and Bo equally to pay for them. Calculate the total level of C provided in Nash equilibrium. Is it the efficient level? Explain.

(c) Suppose instead that the 15 concerts are provided by an anonymous benefactor (not Alice or Bo). Calculate the new level of equilibrium provision. Is it the same as in part (b)? Explain.

(a)

$$MRS_i = \frac{x_i}{3C}$$

Since A and B have identical preferences and the MRS is increasing in private consumption, we know that the unique Nash equilibrium involves both contributing the same amount and setting $MRS_i = 1$. Therefore

$$\frac{70 - \hat{C}/2}{3\hat{C}} = 1$$

or $\hat{C} = 20$.

We can check this argument is correct by solving for the best response functions $c_A^*(c_B)$ and $c_B^*(c_A)$: They solve

$$\frac{70 - c_A^*}{3(c_A^* + c_B)} = \frac{70 - c_B^*}{3(c_A + c_B^*)} = 1$$

so

$$c_A^* = \frac{70}{4} - \frac{3}{4}c_B = \frac{1}{4}\frac{70}{4} + \frac{9}{16}c_A^* = 10$$

and similarly $c_B^* = 10$ so $\hat{C} = 20$.

The efficient level solves

$$\frac{x_A}{3C^*} + \frac{x_B}{3C^*} = 1 \implies 3C^* = x_A + x_B = 140 - C^* \implies C^* = 35$$

(b) Define $\tilde{C} = \hat{C} + 15$ as total provision. Since each pays a tax of 7.5, the Nash equilibrium solves

$$\frac{70 - \hat{C}/2 - 7.5}{3(\hat{C} + 15)} = \frac{70 - \tilde{C}/2}{3\tilde{C}} = 1$$

Therefore $\tilde{C} = 20$: there is no change in total provision.

(c) Since there is no tax for the extra concerts, now the equilibrium solves

$$\frac{70 - \tilde{C}/2 + 7.5}{3\tilde{C}} = 1 \implies \tilde{C} = 22\frac{1}{7}$$

The anonymous benefactor finances some of the concerts which is equivalent to an increase in community income. So concerts rise somewhat, as concerts are a normal good.
